Astronomy for Colleges.

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A TEXT BOOK ON ASTRONOMY

(General & Mathematical)

BY

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FOREWORD

Dr. H. Subramani Aiyar has asked me to write a foreword to his book on mathematical astronomy. It gives me much pleasure to do so as I have become well acquainted with the author and his work during the time I have been in Trivandrum.

His book should I think find a very useful place between say, Godfray's "Astronomy" and Ball's "Spherical Astronomy", furnishing as it does the right material for collateral reading. The bookwork gives the main practical principles in a straight-forward and clear manner; and, as an introduction to the subject, it will, I think, fully meet the requirements of the student. The chapter on eclipses seems to me to be particularly welldone.

I trust that the work will meet a demand for a really helpful text on the usual range of matter which the student is expected to know, and that the result will encourage Dr. H. Subramani Aiyar to make any additions or amendments which may be found needed, should, as I hope will be the case, a second edition be called for.

W. A. GARSTIN,
Lieut. Colonel.
Agent to the Governor General,
Madras States, Trivandrum.

Trivandrum,
23rd April 1936.

INTRODUCTION.

I write these words by way of introducing to the student and teachers of Astronomy in the Indian Universities the book on Theoretical Astronomy by my friend, former pupil and present colleague Dr. H. Subramani Aiyar. Seventeen years' experience in teaching the subject has left its imprint on the book. It is essentially a text book, written for the students and teachers, in the lecture room style, and planned with such care and thought as long acquaintance with teaching the subject can alone give. Though written primarily for the B. A. (Hons.) standard, it will be equally useful to students of the Pass Degree, who can advantageously omit the articles referred to by the author in his Preface. While the book is very handy, no essential topic has been omitted; while apparently mathematical, there are chapters giving all that is necessary to an average student about the general aspects of the subject.

There are excellent treatises on the subject in the western countries, but an Indian publication of this kind has been a long-felt want. I congratulate the author on the successful manner in which he has tried to fill in this gap.

I trust the book will have the wide circulation and demand which it richly deserves.

H. H. The Maharaja's College of Science, Trivandrum, 20th June 1936.

R. SRINIVASAN,
Professor of Mathematics.

PREFACE.

The present volume of Astronomy is more or less intended to be a text book, and I have endeavoured to cover in it all the portions of the field of astronomy required for the B. A., B. Sc., B. A. (Hons.), and M. A. Degree courses of the Indian Universities. Students appearing for the B. A. and B. Sc. Examinations may omit some of the more advanced articles of the different chapters indicated in the contents and take them up for Post Graduate study later on. I hope an acquaintance with the chapters dealt with in this volume, will enable the reader to understand easily the more advanced portions of both practical and theoretical astronomy treated in other standard and original publications on the subject.

It is indeed difficult to be original in writing a text book on a subject like astronomy on which great men have bestowed their attention from very early times, and my attempt is only to present facts in the most simple and easily understandable form. The treatment of the subject has necessarily to be mathematical and as far as possible, the mathematical portions are made clear and self-explanatory. A knowledge of elementary plane trigonometry and calculus is assumed, and the first chapter on 'The sphere' is specially intended to be an introductory chapter deriving the fundamental formulae of spherical trigonometry, which are made use of in the later chapters of the book.

The chapters on the 'Sun' 'Stellar Universe' and 'Thirty constellations' have been specially included, though they do not properly belong to the subject of mathematical astronomy. Without a knowledge of the matter contained in these chapters, the study of the subject is bound to

be incomplete. As an exhaustive treatment of the above portions is quite out of place in a mathematical book like this, I have only tried to put in a nutshell the most important and up-to-date facts regarding the sun and the stellar universe.

I cannot sufficiently emphasise the indebtedness I owe to such valuable publications as Ball's 'Spherical Astronomy', Godfray's 'Treatise on Astronomy', Spencer Jones' 'General Astronomy,' and Smart's 'Spherical Astronomy' all of which I have been using for the past so many years in teaching astronomy in the college classes.

I must record my grateful thanks to Lt. Col. Garstin, Agent to the Governor-General, Madras States, for the deep interest he took in the publication of this book and for the foreword which he so readily and kindly wrote. I am also thankful to Professor R. Srinivasan, M. A., for the encouraging words he has written in the Introduction.

I am thankful to my assistants in the Observatory, Messrs. A. Krishnan Nair, B. A., and V. Krishna Kurukkal, who were helping me throughout, in preparing the diagrams and in reading the proofs, and also to other friends for their wise and useful suggestions regarding the get-up of the book.

Lastly, it is a pleasure to express my heartfelt thanks to the Manager and staff of the A. R. V. Press, Trivandrum, for the excessive pains and care taken by them in getting such a book printed so neatly in their press.

In spite of the shortcomings and slight errors that may have crept in, I hope that the readers will appreciate this work, and from them I heartily invite their valuable suggestions, which may be made use of in a second edition.

TRIVANDRUM,
20th June 1936.

H. Subramania Aiyar.

CONTENTS

* Denotes the sections which the pass course students could afford to omit.

LIST OF PLATES.

CHAPTER I.

THE SPHERE.

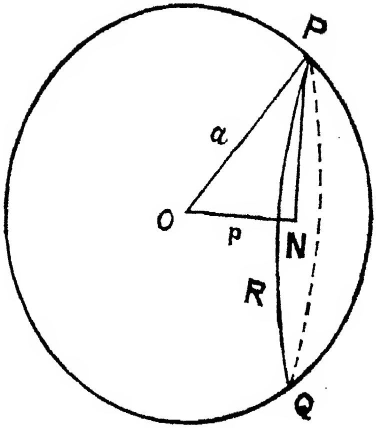


Fig. 1.

1. Plane Sections of a sphere.

A sphere is a solid bounded by a surface, all points of which are equidistant from a fixed point, called its centre.

Every plane section of a sphere is a circle.

Let a plane cut a sphere, centre O, along the curve P Q R. Draw O N perpendicular to the plane P Q R and join any point P on the curve to N and O.

Since O N P is a right-angle,

$$NP^2 = OP^2 - ON^2.$$

$= a^2 - p^2$ where a is the radius of the sphere and p is the distance of the plane from the centre.

Hence the locus of P is a circle whose centre is N and radius is $\sqrt{a^2 - p^2}$.

A plane section of a sphere passing through the centre is called *a great circle.* All other sections are *small circles.* The diameter of the sphere perpendicular to a plane section is called *the axis of the section.* The extremities of the axis are called the *poles.* A great circle passing through the poles of a circle is called *a secondary to the circle.*

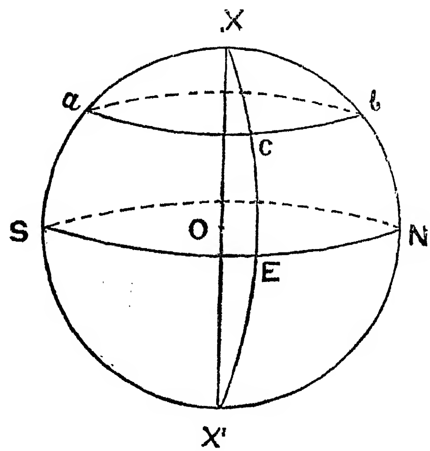


Fig. 2.

In Fig. 2.	S E N	is a great circle.
	a c b	is a small circle.
	X O X′	is the axis of S E N or *a c b.*
	X and X′	are the poles of S E N or *a c b.*
	X E X′	is a secondary circle of S E N.

It will be noted that the circle S E N can have any number of secondary circles and that their planes are perpendicular to the plane S E N.

2. Angular distance.

The distance between any two points on a sphere is measured by the smaller arc of the great circle joining them. This arc is measured by the angle it subtends at the centre of the sphere and not by the length unit.

Hence the distance between two points is called the angular distance between them.

(i) All points of a small circle are at a constant angular distance from either pole. The distance from the nearer pole is called the spherical radius of the circle.

(ii) All points on a great circle are distant 90^c from either of its poles.

The angle between two great circles is the angle between their tangents at either point of intersection. It is also equal to the arc intercepted by them on a great circle to which they are both secondaries or to the angle between their poles.

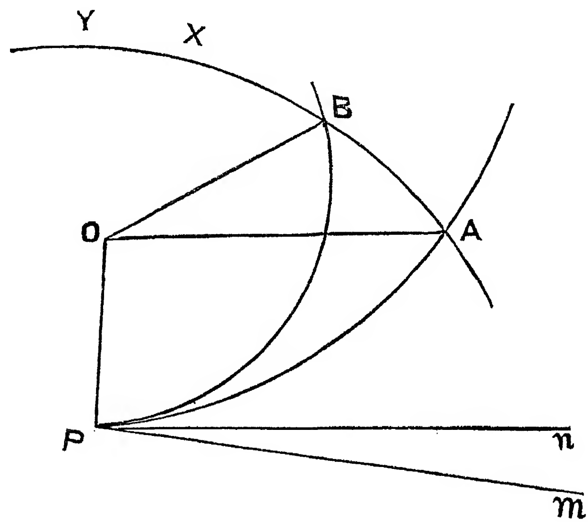


Fig. 3.

Let P A and P B be two great circles and X, Y their poles. Draw the great circle through X Y to cut P A and P B in A and B. Join A, B, and P to O the centre of the sphere. Let Pm and Pn be the tangents to the circles PA and P B respectively.

Since X and Y are the poles of P A, and P B.

$PX = PY = 90°$: $\therefore$ P is the pole of XY.

i.e. $\angle POA = 90°$.

$\therefore$ Pm and OA are parallel.

Similarly Pn and OB are parallel.

$\therefore \angle AOB = \angle mPn.$

= angle between the planes of PA and PB.

Again since $AX = BY = 90°$,

$AB = XY$.

$\therefore$ XY is also equal to the angle between the great circles PA and PB.

The length of the arc of a small circle subtending a given angle at its pole bears a constant ratio to the sine of its angular radius.

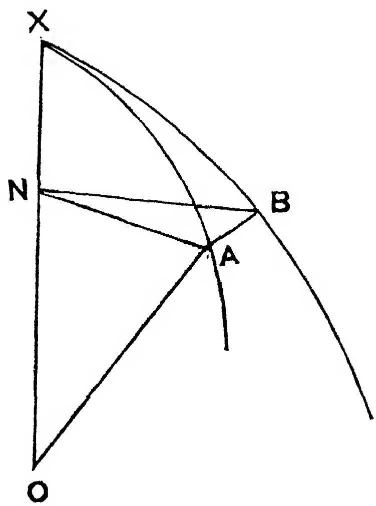


Fig. 4.

Let AB be an arc of a small circle whose pole is X. Let AB subtend at X an angle equal to θ, *i.e.* θ is the angle between the great circles XA and XB. Let R be the angular radius XA or XB and a the radius of the

sphere. Join X and A to O the centre of the sphere and let X O cut the plane of A B at N. Join N A and N B.

Since X O is perpendicular to the plane A B N,

$\angle ONA = \angle ONB = 90°$.

$\therefore NA = a \operatorname{Sin} R$.

Also N A and N B are parallel to the tangents at X to the circles X A and X B respectively.

$$\therefore \angle ANB = \theta.$$

$$\text{arc } AB = NA.\ \theta.$$

$$= a\,\theta \operatorname{Sin} R.$$

Since $a\,\theta$ is the length of an arc of a great circle subtending at its pole an angle equal to θ, it is constant.

3. Spherical triangle.

A spherical figure is a portion of the surface of a sphere enclosed by arcs of great circles, unless otherwise stated. Thus a *lune* is bounded by the arcs of two great circles, *a spherical triangle* by the arcs of three great circles, and *a spherical quadrilateral* by the arcs of four great circles. The arcs are the sides and their angles of inclination are the angles of the figure.

A general treatment of spherical figures is beyond the scope of the book.

A spherical triangle has six elements, the three sides and the three angles. Generally, if three of these be known, the remaining elements may be calculated, *i. e.* a spherical triangle is known when three of its elements are given. The following theorems relating to a spherical triangle are in close analogy with those of a plane triangle.

(*a*) Two sides of a spherical triangle are together greater than the third side.

If A B C be a spherical triangle and O the centre of the sphere,

$\angle BOC + \angle COA > \angle AOB$ ($\because$ when a solid angle is formed by three angles, the sum of any two is greater than the third.)

Hence $BC + CA > AB$.

(*b*) The angles at the base of an isosceles spherical triangle are equal.

(*c*) If two angles of a spherical triangle are equal, the opposite sides are equal.

(*d*) If one angle of a spherical triangle is greater than another, the side opposite the greater angle is greater than the side opposite the smaller angle.

(*e*) If one side of a spherical triangle is greater than another the angle opposite the greater side is greater than the angle opposite the smaller side.

(*f*) Two spherical triangles on the same sphere are equal in all respects,

1. When two sides and the included angle of the one are respectively equal to two sides and the included angle of the other.

2. When three sides of the one are respectively equal to three sides of the other.

3. When two angles and the adjacent side of the one are respectively equal to two angles and the adjacent side of the other.

4. When the three angles of one are respectively equal to the three angles of the other.

[The first three may be easily proved by the method of superposition.]

4. Polar triangle.

Certain relations between the elements of a spherical triangle may be deduced from known relations by means of the polar triangle.

Let A B C be any spherical triangle and X,Y,Z the poles of B C, C A, A B respectively such that A and X lie on the same side of B C, and B and Y on the same side of C A. X Y Z is called the *polar triangle* of A B C, and with respect to X Y Z, A B C is called the *primitive triangle*. (The reader may show that A B C is the polar triangle of X Y Z). If the sides B C, C A and A B and angles B A C, C B A, A C B of the triangle A B C be denoted by a, b and c and A, B and C respectively, the sides and angles of the polar triangle are $180°-A$, $180°-B$, $180°-C$ and $\pi-a$, $\pi-b$, and $\pi-c$.

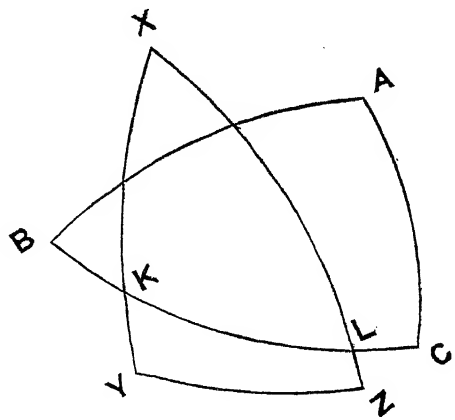


Fig. 5.

Let B C meet X Y in K and XZ in L.

Since X is the pole of B C, angle Y X Z = K L.

Also B L = 90° = K C :

$\therefore$ B L + K C = 180°

But B L + K C = B L + K L + L C

= B C + K L.

$$i.\,e.\ 180^\circ = a + X.$$
$$\therefore\quad X = 180^\circ - a.$$

Similarly $Y = \pi - b$ and $Z = \pi - c$.

Again consider A B C as the polar triangle of X Y Z.

Then $A = \pi - x$, $B = \pi - y$ and $C = \pi - z$.

$\therefore\ x = \pi - A$, $y = \pi - B$, and $z = \pi - C$.

The relations connecting the six elements of the polar triangle with those of the primitive triangle show that from a result involving sides and angles of a triangle, it is possible to infer those of another, by changing the angles into supplements of the corresponding sides and the sides into supplements of the corresponding angles.

Thus in (*f*), (3) and (4) may be inferred from (1) and (2).

5. Trigonometrical formulae relating to spherical triangles.

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$$

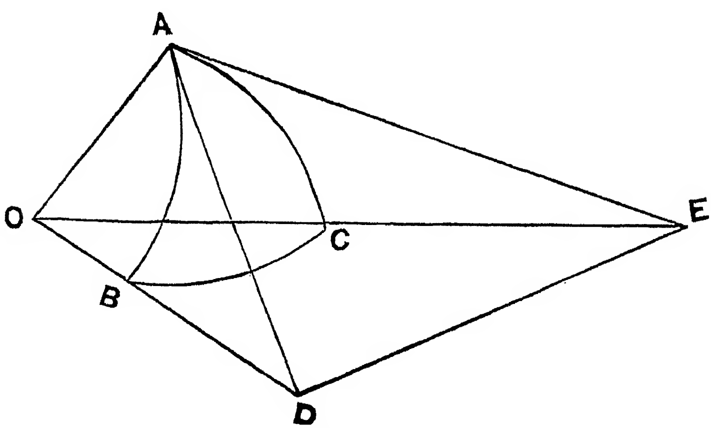


Fig. 6.

Let A B C be a spherical triangle on a sphere whose centre is O and radius is r. Join O A, O B and O C. Let the tangents at A to A B and A C cut O B and O C respectively at D and E. Join D E.

$DE^2 = OD^2 + OE^2 - 2\,OD.\,OE.\cos DOE$ (from triangle DOE)

$= AD^2 + AE^2 - 2\,AD.\,AE\cos DAE$ (from triangle DAE).

Now $\angle DOE = a$ and $\angle DAE = A$.

Also $OD = r\sec c$, $OE = r\sec b$.

$AD = r\tan c$, $AE = r\tan b$.

Substituting these in the above relation and cancelling r^2,

$$\sec^2 c + \sec^2 b - 2\sec c\sec b\cos a$$
$$= \tan^2 c + \tan^2 b - 2\tan b\tan c\cos A.$$
$$\therefore (\sec^2 c - \tan^2 c) + (\sec^2 b - \tan^2 b)$$
$$= 2[\sec c\sec b\cos a - \tan b\tan c\cos A].$$

ie, $1 = \sec c\sec b\cos a - \tan b\tan c\cos A$.

Multiplying by $\cos b\cos c$,

$$\cos b\cos c = \cos a - \sin b\sin c\cos A.$$

$$\cos A = \frac{\cos a - \cos b\cos c}{\sin b\sin c} \quad\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Similarly, $\cos B = \dfrac{\cos b - \cos c\cos a}{\sin c\sin a}$ and

$$\cos C = \frac{\cos c - \cos a\cos b}{\sin a\sin b}$$

By changing A to $\pi - a$, $a = \pi - A$ etc,

$$\cos a = \frac{\cos A + \cos B\cos C}{\sin B\sin C}$$

$$\cos b = \frac{\cos B + \cos C\cos A}{\sin C\sin A} \quad\text{and}$$

$$\cos c = \frac{\cos C + \cos A\cos B}{\sin A\sin B}$$

If $2S = a + b + c$,

(a) $\sin\frac{A}{2} = \sqrt{\left\{\dfrac{\sin(s-b)\sin(s-c)}{\sin b\sin c}\right\}}$

(b) $\cos \frac{A}{2} = \sqrt{\dfrac{\sin s.\ \sin (s-a)}{\sin b\ \sin c}}$ etc.

(c) $\tan \frac{A}{2} = \sqrt{\left\{\dfrac{\sin (s-b)\ \sin (s-c)}{\sin s.\ \sin (s-a)}\right\}}$

(d) $\sin A = \dfrac{}{\sin b\ \sin c} \sqrt{\left\{ \sin s.\ \Pi \sin (s-a) \right\}}$

(e) $\dfrac{\sin A}{\sin a} = \dfrac{\sin B}{\sin b} = \dfrac{\sin C}{\sin c} : \begin{vmatrix} 1 & \cos b & \cos c \\ \cos a & 1 & \cos c \\ \cos a & \cos b & 1 \end{vmatrix} \div \Pi \sin a.$

As the proofs of these formulae are exactly similar to those for a plane triangle, it is hoped that the reader will work these out. Also corresponding to the formulae (a), (b), (c) and (d), the reader may apply the principle of duality and obtain the formulae for $\sin a/2$, $\cos a/2$, $\tan a/2$ and $\sin a$ in terms of the angles only. They are :—

(f) $\sin a/2 = \sqrt{\left\{ -\dfrac{\cos S\ \cos\ (S-A)}{\sin B\ \sin C} \right\}}$

(g) $\cos a/2 = \sqrt{\left\{ \dfrac{\cos (S-B)\ \cos\ (S-C)}{\sin B\ \sin C} \right\}}$

(h) $\tan a/2 = \sqrt{\left\{ -\dfrac{\cos S\ \cos\ (S-A)}{\cos (S-B)\ \cos (S-C)} \right\}}$

(i) $\sin a = \dfrac{2}{\sin B\ \sin C} \sqrt{\left\{ -\cos S.\ \Pi \cos (S-A) \right\}}$,

where $A+B+C=2\,S$.

Another formula involving two sides and two angles is, $\cos b \cos C = \sin b \cot a - \sin C \cot A$.

Proof :— $\cos a = \cos b \cos c + \sin b \sin c \cos A$.

Also $\cos c = \cos a \cos b + \sin a \sin b \cos C$

and $\sin c = \dfrac{\sin C}{\sin A} \sin a$.

Substituting these values for $\cos c$ and $\sin c$,

$$\cos a = \cos b\,(\cos a \cos b + \sin a \sin b \cos C) + \sin b\ \cos A.\ \frac{\sin C}{\sin A} \sin a.$$

$$\therefore \cos a\ (1-\cos^2 b) = \cos b \sin a \sin b \cos C) \\ + \sin a \sin b \sin C \cot A.$$

Dividing by $\sin a \sin b$,

$$\cot a \sin b = \cos b \cos C + \sin C \cot A.$$

$$\therefore \cos b \cos C = \sin b \cot a - \sin C \cot A. \quad \text{.........(2)}$$

Note. This formula involves two sides, the included angle and another angle. For convenience, b may be called the included side and a the other side. Similarly C is the included angle and A the other angle. The formula then reads :—

cos (included side) cos (included angle)

= sin (included side) cot (other side)

− sin (included angle) cot (other angle).

6. Right angled triangles.

When one of the angles of a spherical triangle is a right angle, the triangle is called a right angled spherical triangle. In this case the general formulae for a spherical triangle take simpler forms.

If angle C be right angled, from result (1) obtained earlier $\cos c = \cos a \cos b$. (i)

Again from (e) $\sin B = \dfrac{\sin b}{\sin c}$ (ii)

and from (2), $\cos B. \cos a = \sin a \cot c$

$$\therefore \quad \cos B = \frac{\tan a}{\tan c} \quad \text{(iii)}$$

From ii & iii, $\tan B = \dfrac{\sin b}{\sin c} \cdot \dfrac{\tan c}{\tan a} = \dfrac{\sin b}{\sin c} \dfrac{\sin c \cos a}{\cos c \sin a}$

$$= \frac{\sin b}{\sin c} \cdot \frac{\sin c}{\cos a, \cos b} \frac{\cos a}{\sin a}$$

$$= \frac{\tan b}{\sin a} \quad \text{(iv)}$$

Similarly $\tan A = \dfrac{\tan a}{\sin b}$.

$$\therefore \tan A \tan B = \frac{1}{\cos a \,.\, \cos b} = \frac{1}{\cos c}.$$

$$\text{or } \cos c = \cot A.\ \cot B \qquad \text{(v)}$$

$$\text{Also } \cos a = \cos b.\ \cos c + \sin b.\ \sin c \cos A$$

$$= \cos b.\ \cos a \cos b + \sin b \sin c \cos A$$

$$\therefore \cos a\,(1 - \cos^2 b) = \sin c \ \sin b.\ \cos A$$

$$\text{or } \cos a = \frac{\sin c}{\sin b} \cdot \cos A = \frac{\sin C}{\sin B} \cos A$$

$$= \frac{\cos A}{\sin B}\ (\because C = 90^\circ) \qquad \text{(vi)}$$

Napier's rule.

These six formulae are included in two rules called Napier's rule of circular parts. Leaving out the right angle, the sides containing the right angle, the complement of the hypotenuse and the complements of the other angles are called the circular parts of the triangle. These five parts are cyclically arranged in the order in which they occur in the triangle. If C is the right angle fig 8 gives the order in which the circular parts are placed round a circle. Any one of the five parts is selected and called the middle part; the two parts on either side of the middle part are then called adjacent parts and the remaining two, opposite parts. The rule is :—

sin (middle part) = product of the tangents of adjacent parts.

= product of cosines of opposite par ts.

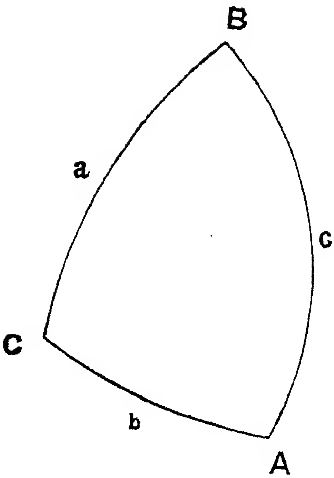


Fig. 7.

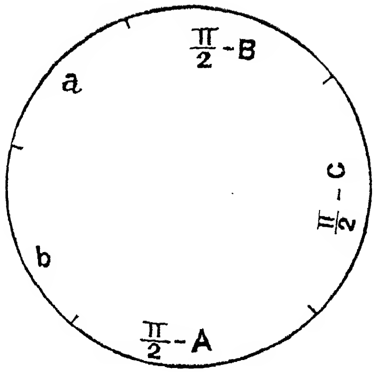


Fig. 8.

Note. To get the formulae connecting any three quantities, (say b, B and a) a is to be middle-part:

$$\sin a = \tan b \, \tan(90° - B)$$

$$= \tan b . \cot B.$$

7. Solution of spherical triangles.

Given any three elements of a spherical triangle, the remaining elements can be calculated with the help of the formulae of section **5**. The different problems that arise in the solution of triangles are:—

Given (1) the three sides, a, b, c;

(2) the three angles, A, B, C;

(3) Two sides and the included angle, a, C, b;

(4) Two angles and the adjacent side, B, c, A;

(5) Two sides and an angle opposite to either, a, b, A;

(6) Two angles and a side opposite to either, A, B, a;

For (1) and (2) the appropriate formulae are

$$\tan \tfrac{A}{2} = \sqrt{\left\{ \frac{\sin (s-b) \sin (s-c)}{\sin s \sin (s-a)} \right\}} \text{ and}$$

$$\tan \tfrac{a}{2} = \sqrt{\left\{ - \frac{\cos S \cos (S-A)}{\cos (S-B) \cos (S-C)} \right\}}$$

For (3) the formula $\cos c = \cos a \cos b + \sin a \sin b . \cos C$, gives two values for c, one of which is to be taken. For logarithmic computation, defining the angle l by the relation

$$\tan l = \tan b \cos C,$$

$$\begin{aligned} \cos c &= \cos b \, [\cos a + \sin a \tan b \cos C] \\ &= \cos b \, [\cos a + \sin a \tan l] \\ &= \cos b \cos (a-l) / \cos l. \end{aligned}$$

Again $\cos a \cos C = \sin a \cot b - \sin C \cot B$ gives :—

$$\begin{aligned} \cot B &= \frac{1}{\sin C} \left\{ \sin a \cot b - \cos a \cos C \right\} \\ &= \frac{1}{\sin C} \left\{ \sin a \frac{\cos C}{\tan l} - \cos a \cos C \right\} \\ &= \frac{\cot C}{\sin l} \left\{ \sin (a-l) \right\} \end{aligned}$$

Similarly $\cot A = \dfrac{\cot C}{\sin l} \sin (b-l)$

Thus A and B may be calculated.

To get a definite solution for the above, three equations are required, though the quantities to be found are only two. They are:—

$$\cos c = \cos a \cos b + \sin a \sin b . \cos C \qquad (1)$$
$$\sin c \sin A = \sin C \sin a \qquad (2)$$
$$\sin c \cos A = \cos a \sin b - \sin a \cos b \cos C \qquad (3)$$

The third formula is obtained by producing the side AC to C′ so that AC′ = 90° and then taking the cosine of BC′ from both the triangles ABC′ and BCC′.

From (1) $\cos c$ is determined uniquely, but not c.

From (2) & (3), sin c can be known, and hence c can be determined uniquely. So also A is known definitely since sin A and cos A are known when c is known.

4. Here A and B and the side c are given. To determine C and a we have the following set of equations using the property of polar triangles.

$$\cos C = -\cos A \cos B + \sin A \sin B. \cos c$$

$$\sin C \sin a = \sin A \sin c$$

$$\sin C. \cos a = \cos A \sin B + \sin A \cos B \cos c.$$

5. The formulae for this case are $\sin B = \dfrac{\sin b}{\sin a} \sin A.$ and $\cos C = -\cos A \cos B + \sin A \sin B \cos a$, giving the value of B (with an ambiguity) and that of C. Then c is readily calculated. A detailed investigation of the ambiguous case is beyond the scope of the book.

6. The last one is reduced to (5) by means of the polar triangle.

In the case of right angled spherical triangles, numerical work involved is considerably less as is shown by the following table:

	Given :—	Formula :
(1)	a and b :	$\cot A = \cot a \sin b$, $\cot B = \cot b \sin a$ $\cos c = \cos a \cos b$.
(2)	A and B :	$\cos A = \cos a \sin B$, $\cos B = \cos b \sin A$. $\cos c = \cot A \cot B$.
(3)	c and a :	$\sin A = \sin a / \sin c$; $\cos B = \tan a \cot c$ $\cos b = \cos c / \cos a$.
(4)	c and A :	$\sin a = \sin A \sin c$, $\tan b = \cos A \tan c$ $\cot B = \tan A \cos c$.
(5)	a and B :	$\cos A = \cos a \sin B$; $\tan b = \tan B \sin a$; $\cot c = \cot a \cos B$.
(6)	a and A :	$\sin b = \tan a \cot A$, $\sin c = \sin a / \sin A$ $\sin B = \cos A / \cos a$.

8. Spherical co-ordinates.

The circumference of a great circle is divided into 360 equal parts, each called a degree. Each degree is divided into smaller sub-divisions of minutes and seconds so that $60'' = 1'$ & $60' = 1^\circ$. The pole of the great circle lying to the left side of the direction of increasing degrees along the arc is called its *nole* and the pole on the right side is called the *antinole.* If we consider longitudes measured east of Greenwich as positive, then the north pole is the nole of the equator, and the south pole, the antinole.

Co-ordinates of a point on a sphere.

Any point on a sphere is specified by two co-ordinates α and δ, these co-ordinates being measured, along a great circle from a point O on it and along a direction perpendicular to it.

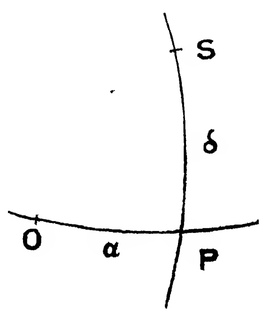


Fig. 9.

Let OP be the great circle and S any point; draw the great circle passing through S and the nole of OP cutting the great circle at P. Then if O be the origin $OP = \alpha$ & $SP = \delta$.

δ is $-$ve for points along SP produced.

Cosine of the arc between two points.

Let the co-ordinates of D and D′ be α, δ, α', δ', with reference to the great circle AB and P the nole. Let DD′ be θ.

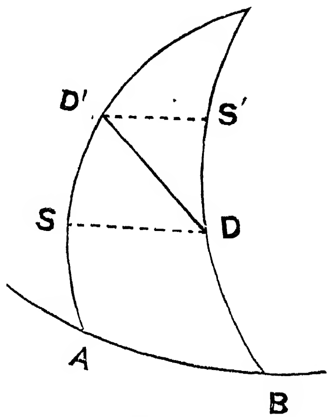


Fig. 10.

$$\begin{aligned}\cos\theta &= \sin\delta\sin\delta' + \cos\delta\cos\delta'\cos(\alpha-\alpha')\\ &= \sin\delta\sin\delta'\left[\cos^2\frac{\alpha-\alpha'}{2}+\sin^2\frac{\alpha-\alpha'}{2}\right]+\\ &\qquad \cos\delta\cos\delta'\left[\cos^2\frac{\alpha-\alpha'}{2}-\sin^2\frac{\alpha-\alpha'}{2}\right]\\ &= \cos^2\frac{\alpha-\alpha'}{2}\cos(\delta'-\delta)-\sin^2\frac{\alpha-\alpha'}{2}\cos(\delta+\delta')\\ &= \cos^2\frac{\alpha-\alpha'}{2}\cos(\delta-\delta')-\sin^2\frac{\alpha-\alpha'}{2}\cos(\delta+\delta') \quad (1)\end{aligned}$$

Subtracting (1) from

$$1=\cos^2\frac{\alpha-\alpha'}{2}+\sin^2\frac{\alpha-\alpha'}{2},\text{ we get}$$

$$2\sin^2\frac{\theta}{2} = 2\cos^2\frac{\alpha-\alpha'}{2}\sin^2\frac{\delta-\delta'}{2} + 2\sin^2\frac{\alpha-\alpha'}{2}\cos^2\frac{\delta+\delta'}{2} \quad (2)$$

When θ is very small, we have

$$\theta^2 = (\delta-\delta')^2 + (\alpha-\alpha')^2\cos^2\frac{\delta+\delta'}{2} \quad (3)$$

It is easy to see that (3) follows from a geometrical consideration of $\triangle$s D D' S and D D' S' which can be considered plane right angled triangles.

$$(\delta-\delta')^2+(\alpha-\alpha')^2\cos^2\delta'=DD'^2=(\alpha-\alpha')^2\cos^2\delta+(\delta-\delta')^2$$

For an approximate solution, we can write for $\cos^2\delta$ or $\cos^2\delta'$ the value $\cos^2\frac{\delta+\delta'}{2}$, and get the result (3),

9. Rectangular co-ordinates.

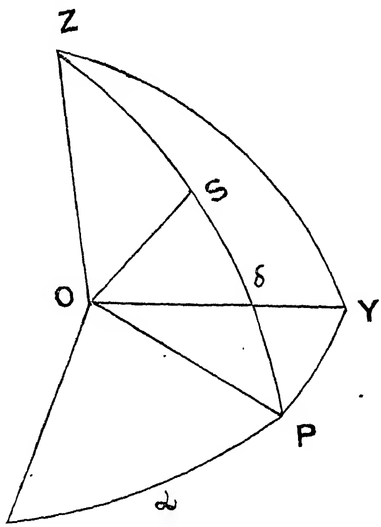


Fig. 11.

Choosing as rectangular axes of x, y, and z the lines from the centre of the sphere to the points (0, 0) (90, 0) and (0, 90), the co-ordinates of a point S (α, δ) on a sphere of radius r are found to be

$$x=r\cos\alpha\cos\delta$$

$$y=r\sin\alpha\cos\delta$$

$$z=r\sin\delta.$$

The cosines of the angles which OS makes with the three rectangular axes are $\cos\alpha\cos\delta$, $\sin\alpha\cos\delta$ and $\sin\delta$ respectively.

10. Transformation of co-ordinates.

Given the co-ordinates of a point with respect to one fundamental great circle, it is useful to determine the co-ordinates of the same point with regard to a different great circle.

Let OA be the original great circle, graduated in the direction OV and having its nole at N. Let O′A′ be the other graduated great circle with nole at N′ and origin O′, to which the co-ordinates are to be transferred.

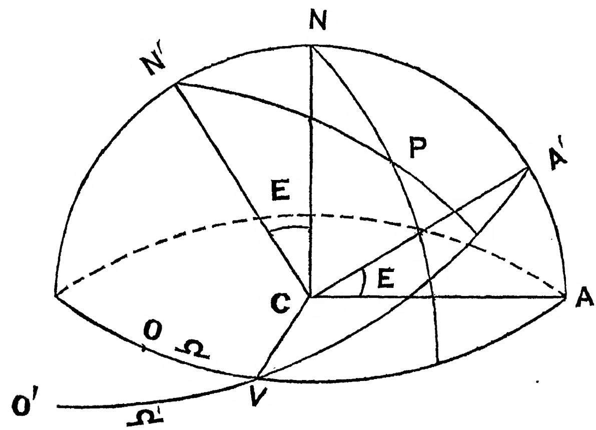


Fig. 12.

Let Ω, Ω' be the distances from O and O′ respectively of V, the ascending node of the second great circle on the first, and let E be the inclination of the two great circles. The co-ordinates of V, A, and N are known in the two systems. Let the spherical co-ordinates of P be α, δ and α', δ' in the two systems. Then with regard to the axes CV, CA and CN the rectangular co-ordinates of P are:—

$$\cos\delta\cos(\alpha-\Omega),\ \cos\delta\sin(\alpha-\Omega) \text{ and } \sin\delta.$$

Transforming these to new set of axes CV, CA′ and CN′, we get by projection, that

$$\cos\delta\cos(\alpha-\Omega)=\cos\delta'\cos(\alpha'-\Omega') \quad \ldots \quad \ldots \quad (1)$$

$\cos\delta'\sin(\alpha'-\Omega')$ along CA′ is equivalent to

$\cos\delta'\sin(\alpha'-\Omega')\cos E$ along CA and

$\cos\delta'\sin(\alpha'-\Omega')\sin E$ along CN.

Also $\sin\delta'$ along CN′ is equivalent to

$\sin\delta'\cos E$ along CN and—$\sin\delta'\sin E$ along CA.

$\therefore$ $\cos\delta\sin(\alpha-\Omega)$, the co-ordinate of P

$$\text{along CA}=\left.\begin{array}{l}\cos\delta'\sin(\alpha'-\Omega')\cos E\\ \quad -\sin\delta'\sin E\end{array}\right\} \quad \ldots \quad (2)$$

and $\sin\delta$, the co-ordinate of P along CN

$$=\sin\delta'\cos E+\cos\delta'\sin(\alpha'-\Omega')\sin E \ldots \quad (3)$$

Similarly projecting α, δ along CV, CA′ and CN′ we get

$$\cos\delta'\cos(\alpha'-\Omega')=\cos\delta\cos(\alpha-\Omega) \ldots \quad \ldots \quad \ldots \quad (i)$$

$$\cos\delta'\sin(\alpha'-\Omega')=\cos\delta\sin(\alpha-\Omega)\cos E+\sin\delta\sin E \quad (ii)$$

$$\sin\delta'=\sin\delta\cos E-\cos\delta\sin(\alpha-\Omega)\sin E \quad \ldots \quad \ldots \quad (iii)$$

For logarithmic computation formulae (ii) and (iii) can be put in another form. Let m be a positive quantity and M an angle between 0° and 360° such that

$$\sin\delta=m\cos M \text{ and}$$

$$\cos\delta\sin(\alpha-\Omega)=m\sin M.$$

$$\text{Then, } \tan M=\sin(\alpha-\Omega)\cot\delta.$$

Formulae (ii) and (iii), now become

$$\cos\delta'\sin(\alpha'-\Omega')=m\sin(M+E) \quad \ldots \quad \ldots \quad (iv)$$

$$\text{and } \sin\delta'=m\cos(M+E) \quad \ldots \quad \ldots \quad (v)$$

CHAPTER II.

THE CELESTIAL SPHERE.

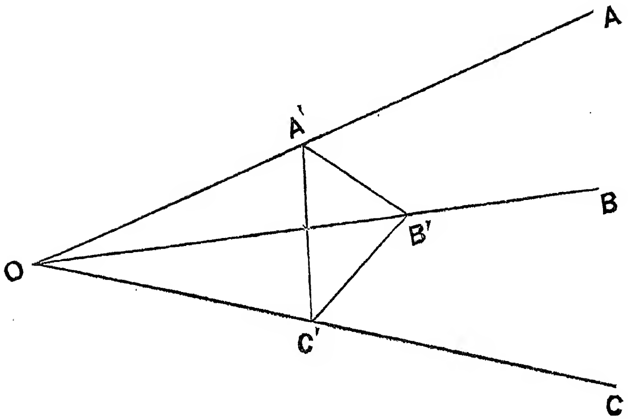


Fig 13.

11. The celestial sphere.

Let A,B,C be the positions in space of three heavenly objects and O, the observer's position. Describe a sphere of any radius OA′ cutting these directions OA, OB, OC at A′, B′ and C′ respectively. The angles AOB, BOC and AOC are measured by the sides of the spherical triangle A′B′C′. Thus A′B′ is the apparent distance between A and B. Similarly B′C′ is the apparent distance between B and C. These quantities, of course, don't give any idea of the real distances between the bodies A and B or B and C. To determine the relative positions of heavenly bodies it is enough if we know the corresponding positions of the objects on a sphere with centre O and any radius. Such a sphere is called the celestial sphere and for convenience we may suppose the stars and other heavenly bodies to lie on such a sphere.

The centre of a celestial sphere is the station of the observer and for different stations there will be different celestial spheres. But so far as the fixed stars are concerned, the celestial spheres on which they are represented, for different positions on the earth are practically the same. This is because the direction of a star would be nearly the same, whatever be the position of the observer on the earth. This is a consequence of the enormous distances of stars from the earth. The diameter of the earth subtends at the nearest star an angle less than $\frac{1}{10000}$ of a second of arc. But the apparent places on the celestial sphere of the sun and the planets to a certain extent and that of the moon to a large extent are affected by the observer's position on the earth, because these bodies are not so remote as the stars.

The celestial horizon.

It is the great circle of the celestial sphere cut by the tangent plane to the earth at the observer's position. This may also be defined as the great circle where the plane through the observer parallel to the surface of a liquid at rest cuts the celestial sphere.

Zenith and Nadir.

The points on the celestial sphere to which the plumbline points, when continued in both directions are the zenith and the nadir points, the zenith being the point overhead and the nadir, the point down below.

Celestial poles.

The earth is rotating round an axis from west to east as can be proved by physical experiments. The period of this rotation of the earth is called a sidereal day (23 hrs. 56 min. 4 sec.) This gives an apparent rotation to the celestial sphere in the opposite direction round an axis parallel to the axis of the earth. *The north and south poles* of the celestial sphere are the points where this axis cuts the celestial sphere. These are fixed points and do not participate in

the diurnal motion of the rest of the points on the celestial sphere. Of these, one is called the north pole, since it is visible to dwellers on the northern hemisphere and its position is conveniently indicated to us by its proximity to the bright star Polaris. The south pole is not capable of being so easily located.

Celestial equator.

The great circle having for its poles the celestial poles is called the celestial equator. It is easily seen that the plane of the earth's equator when extended in all directions meets the celestial sphere in the celestial equator.

The meridian and the cardinal points.

The meridian of a place is the great circle passing through the zenith, the nadir and the celestial poles. The intersections of the meridian with the horizon are the north and south points. The intersections of the equator with the horizon are the east and west points.

Verticals are the secondaries to the horizon and the vertical through the east and west points is called the *Prime Vertical.*

12. Sidereal Day.

The celestial horizon divides the celestial sphere into the visible and invisible hemispheres and a star is said to rise when it comes from the invisible to the visible hemisphere and it is said to set when it disappears from the visible to the invisible hemisphere. If we look at the night sky, at different instants, we shall find that the stars are all apparently rotating round the celestial axis from east to west. This apparent daily motion of the stars is due to the earth's rotation from west to east in a period of what is called a sidereal day. Each sidereal day is equally divided into 24 sidereal hours, each sidereal hour into 60 sidereal minutes and each sidereal minute

into 60 sidereal seconds. Sidereal time is kept by what is called a sidereal clock in an Observatory.

13. Foucault's pendulum experiment to prove the rotation of the Earth round its axis.

In the year 1851, M. Foucault invented a method by which the rotation of the earth about its axis could be verified. A large metal ball was suspended by a fine wire from a fixed point of the roof of a high building. The weight was drawn to one side and suddenly released. Foucault found that the plane of oscillation of this pendulum rotated with reference to the objects around. The direction of its apparent motion and measurements of its magnitude gave him direct evidence of the earth's rotation.

Principle of the above experiment :—

Let the latitude of the place of observation P be l and let v be the angular velocity of rotation of the earth round its axis ON.

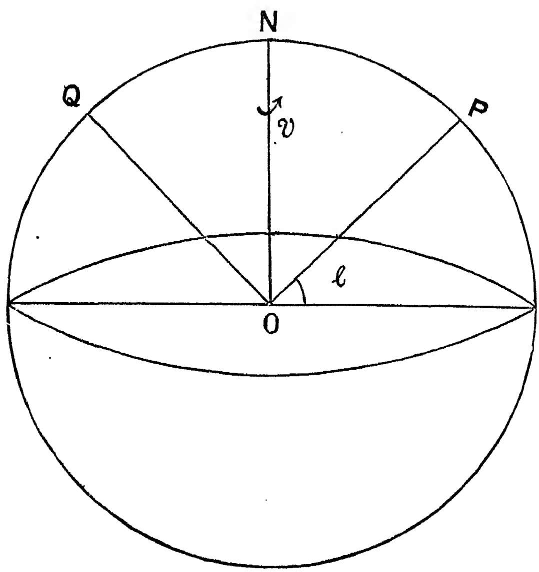


Fig. 14.

The rotation of the earth may be resolved into $v \sin l$ about OP and $v \cos l$ about OQ, perpendicular to OP. Now rotation about OQ has only translatory effects on points near P, all to the same extent. Therefore this could not be easily noticed. The other component $v \sin l$ about OP produces a rotation of the plane of the horizon about OP to the same extent. Now all the fixed objects in the neighbourhood of P possess this rotation except Foucault's pendulum which is simply a mass suspended from a point. The plane of oscillation of such a pendulum should keep the same direction so long as no external force other than gravity acts upon it. Therefore the apparent rotation of the plane of oscillation of the pendulum is due to the rotation of the ground underneath in the opposite direction. From the magnitude of this rotation and the latitude of the place the angular velocity of rotation of the earth could also be obtained.

14. The Ecliptic and the Obliquity.

We have already seen that the positions of the stars are represented by definite points on the celestial sphere which change very little from time to time. This is not the case with the sun, the moon and the various planets that go round the sun. Continued observations of the stars that rise in the eastern horizon a little before sunrise lead us to infer that in different months the sun is found in the direction of different groups of stars. The same inference could also be drawn by observing the stars that set immediately after the sun. Thus the sun seems to move amidst the stars from west to east and takes a (sidereal) year to complete the circuit. This annual motion of the sun is found to be on a great circle of the celestial sphere inclined to the equator at an angle of $23°\text{-}26'\text{-}51{\cdot}86''$. This great circle is called the ecliptic and its inclination to the equator is called the obliquity. The exact position of the ecliptic on the celestial sphere can be

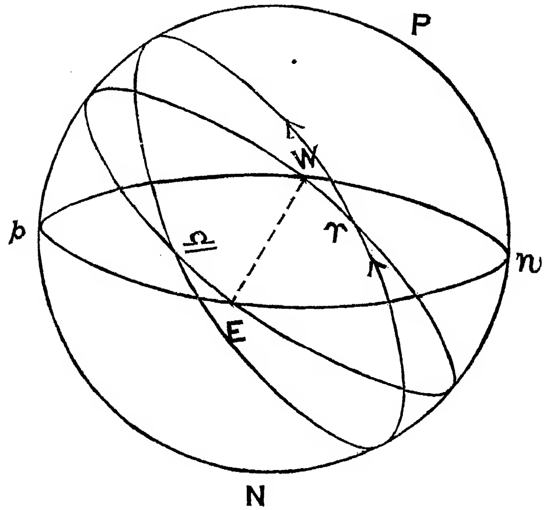


Fig. 15.

determined by accurately determining the position of the sun on the celestial sphere every day, by methods to be described later and joining all these points obtained for the whole year. By the same method, the paths of the moon and any of the planets can be plotted on the celestial sphere.

Equinoctial points.

The points of intersection of the ecliptic and equator are called the equinoctial points. That point through which the sun passes while crossing the equator from south to north is called the first point of Aries and is denoted by the symbol ♈ and the other point of intersection is called the first point of Libra and is denoted by the symbol ♎

15. The celestial co-ordinates.

Just as the position of a point in space is defined by its co-ordinates measuring its distances from three planes

mutually intersecting at right angles, a point lying on a sphere can be fixed by means of two co-ordinates depending upon the reference circle chosen.

(1) *Altitude and azimuth.* The altitude of a star S is its angular distance from the horizon measured along a vertical. The complement of this is the zenith distance of the star.

The azimuth is the angle made by the vertical of the star with the meridian measured from the north point.

Either, the altitude and the azimuth, or the zenith distance and the azimuth would fix the position of a star on the celestial sphere. These co-ordinates are used when the position of a celestial object has to be defined relative to fixed objects on the earth. These co-ordinates are changing due to the rotation of the Earth.

(2) *Right Ascension and Declination.* Another mode of specifying the position of a celestial body is by its Right Ascension and Declination. The Right Ascension (RA) of the star S is the arc of the equator measured eastwards from the first point of Aries to the foot of the secondary to the equator, through the star. The Declination of the star S is its angular distance measured from the equator along a secondary to the equator. The Declination is said to be north or south according as the star is north or south of the equator. The great circle, passing through the star and the celestial poles is called the declination circle. These co-ordinates of a star undergo very little change and are therefore the most suitable for fixing the relative positions of stars on the celestial sphere.

(3) *Hour angle and polar distance.* A third mode of specifying the position of a star is by its hour angle and declination. The hour angle of a star S is the angle made by the star's declination circle with the meridian measured westwards from the meridian. Sometimes it is usual to

give for the second co-ordinate of the star, its distance from the nearest pole and call it the north or south polar distance as the case may be. This is only the complement of the star's declination.

In this system of co-ordinates the hour angle of a star changes uniformly at the rate of 15° per sidereal hour owing to the rotation of the earth but the north polar distance remains constant. These co-ordinates also serve to denote the position of a celestial object relative to observer's meridian.

16. Celestial latitude and longitude.

When the ecliptic is chosen as one of the planes of reference we have still another system of co-ordinates. According to this system one of the co-ordinates of the star is the celestial latitude which is defined as its angular distance measured along a secondary to the ecliptic. The second co-ordinate of this system is the celestial longitude which is the arc of the ecliptic intercepted between this secondary and the first point of Aries. This arc is measured eastwards from ♈. These co-ordinates of a celestial body are not affected by the rotation of the earth. They are particularly useful in locating on the celestial sphere such bodies as the sun, moon, planets etc., which move always very near the ecliptic and have little or small latitudes.

Terrestrial latitude and longitude.

To fix the position of an observer on the surface of the earth two co-ordinates are required. One of the co-ordinates usually taken is the latitude of the observer's position which denotes the angular distance of his zenith from the point where the meridian is cut by the equator. It may be also defined as the altitude of the celestial pole. The second co-ordinate is the longitude which is the angle between the meridian through that place and any

fixed meridian on the earth, called the prime meridian. The meridian through Greenwich Observatory is usually taken to be the prime meridian. The longitudes are measured ordinarily from $0°$ to $180°$ only, both eastwards and westwards from the prime meridian. If it is east of the prime meridian, it is called east longitude and if west, west longitude.

17. The astronomical clock and the sidereal time.

We have already seen that the celestial sphere with all the stars on it is apparently rotating with reference to the observer's meridian and horizon in a sidereal day. This day is less (by about 4 min.) than a day of 24 hours which we use for ordinary business purposes. But this period of time is indicated by the revolutions of the hour hand of an astronomical clock through twenty-four hours. The hands of the clock which indicates sidereal time should point out 0 h. 0 m. 0 sec., when the first point of Aries passes the meridian of a place and for that place the clock time at any instant is the hour angle of the first point of Aries, so that when the sidereal day has ended, the clock should read 24 h. 0 m. 0 sec., and the first point of Aries should have again come to the meridian.

All the stars are apparently revolving at a uniform rate about the pole in a sidereal day and hence, the hour angle of a star is proportional to the time that has elapsed since its transit or since the time when it crossed the meridian. In 24 hours of sidereal time, each star describes $360°$ about the pole and therefore, the hour angle of a star, increases at the rate of $15°$ per sidereal hour or $15'$ per sidereal minute or $15''$ per sidereal second of time. Very often, the hour angle of a star is given in terms of the sidereal time that has elapsed since the star had crossed the meridian and then it is said to be expressed in time.

Example. (1) Express in angular measure the hour angle of ♈ when the sidereal time is 7 h. 22 min. 43 sec.

$$\begin{array}{rrr} 7 & 22 & 43 \\ & & 15 \\ \hline 110^\circ & 40' & 45'' \end{array}$$

The required result is obtained by multiplying the hours, minutes and seconds given, by 15.

(2) Find the sidereal time of transit of a star of R. A. 102°-12′-15″.

When the star is on the meridian its R. A. expresses the hour angle of the first point of Aries or the sidereal time of its transit when this is converted into time, by dividing it by 15. Therefore the sidereal time of transit is obtained as follows.

15 102°. 12.′ 15′

6 hrs. 48 min. 49 sec.

In the Nautical Almanac and other catalogues of stars, the R. A. of a star is usually expressed in time, as it indicates at once the sidereal time of its transit.

Since the hour angle of a star is the distance measured westwards along the equator from the meridian to the foot of the secondary through the star, and since the R. A. of the star, is equal to the distance along the equator measured from there to the first point of Aries, we have, when both these are expressed in time, the following well-known relation

Star's hour angle + its R. A = Sidereal time.

18. The time of transit of a celestial body.

The time of transit of a celestial body undergoing no change in declination is the arithmetic mean between the times when it had equal altitudes on either side of the meridian and the meridian bisects the two verticals through the body in these two positions.

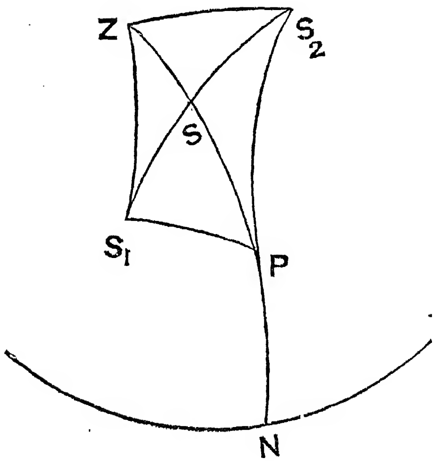


Fig. 16.

Let S_1 and S_2 be the two positions of the body when it has the same altitude.

If Z, P, N are the zenith, pole and the north point respectively, we can see that the two triangles PZS_1 and PZS_2 are congruent, since the three sides of the one are each equal to the corresponding sides of the other.

$$S_1 \hat{P} Z = S_2 \hat{P} Z \qquad (1)$$

$$S_1 \hat{Z} P = S_2 \hat{Z} P \qquad (2)$$

From (1) it is seen that if t_1, t and t_2 be the times when the body was at S_1, S and S_2 respectively

$$t - t_1 = t_2 - t$$

$$\therefore \ t = \tfrac{1}{2}(t_1 + t_2)$$

From (2) we see that the azimuths of the body at the times when it has equal altitudes are the same when measured in opposite sides from the north point. i.e. the meridian plane bisects the angle between the two verticals through S_1 and S_2. If the body observed be the sun, it

has to be noticed that PS_1 is not equal to PS_2 unless the interval between S_1 and S_2 is short. For the sun's R. A. and declination are changing during any interval of time.

19. The Gnomon and its use.

In determining the meridian of a place, the sun's R. A. the latitude of a place and the obliquity of the ecliptic, a simple device known as a gnomon is employed. The gnomon is simply a rod erected vertically on a horizontal floor. A circle of any radius is described with the foot of the rod as centre. By means of such a rod the altitude of the sun at any time can be known, by measuring the length of the shadow cast by the rod. If z be the zenith distance, $\tan z = \dfrac{\text{Length of shadow}}{\text{Length of rod}}$

Mark on this circle the two points where the extremity of the shadow falls both before and after noon. The sun's altitudes on these two occasions being the same, the meridian is the direction midway between the directions joining these two points to the foot of the rod in the centre of the circle.

If the two corresponding instants of time by a sidereal clock be t_1 and t_2 then $\dfrac{t_1 + t_2}{2}$ is the sidereal time of the sun's meridian passage or apparent noon. Now the sidereal time gives the hour angle of ♈ and hence this corresponds to the sun's R. A. at the time of apparent noon. If the observer is equipped with an instrument such as an altazimuth or a theodolite (see chapter on astronomical instruments) he could get the meridian determined very accurately, by knowing the azimuth readings of a star when it has the same altitude on either side of the meridian and also the star's R. A. if the corresponding sidereal times are also known; for, the arithmetic mean of these two times gives the sidereal time of transit of the star which is also its R. A.

Obliquity of the ecliptic and latitude of observer.

A simple way of obtaining the obliquity of the ecliptic is to get the maximum and minimum lengths of the shadow cast by the gnomon at the time of apparent noon. The lengths divided by the height of the gnomon gives the tangents of the zenith distances of the sun on these occasions. Half the sum of the zenith distance gives the latitude of the place and half the difference gives the obliquity.

20. Transit Instrument and its use in fixing stellar positions.

When the meridian of a place has been determined it is very useful to fix a telescope there, so that the telescope axis may describe the meridian plane. Such a telescope is what is called the Transit Instrument. Attached with this and turning with it, is a graduated circle which

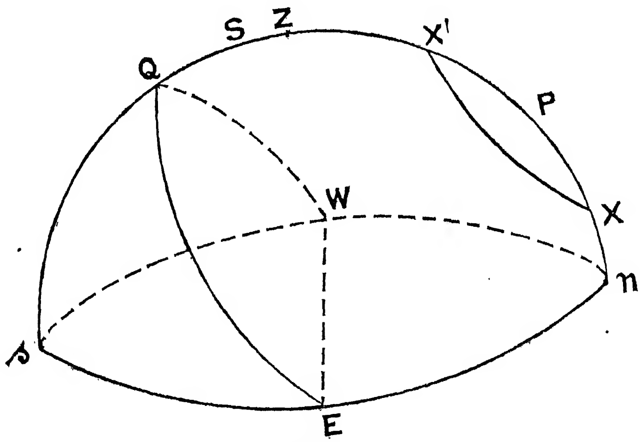


Fig. 17.

would give the direction towards which, the telescope is pointed when a star is observed to go across the centre of its field. This direction gives the zenith distance of the star, since the reading of the circle when the telescope points to the zenith (or zenith reading) can be found, The sidereal clock can also give the exact instant when the star crosses the centre of the field of the telescope.

i. e. when the star crosses the meridian. This would give the R. A. of the star. The declination of the star is obtained from the observed zenith distance, if the latitude of the place is known. Let the star S transit south of the zenith.

$$l = QZ = QS + SZ$$

$$= \text{N. decl} + \text{zenith distance} = d + z$$

If the star had south declination, then $l = z - d$

If the transit was north of the zenith then $l = d - z$

In this manner the R. A. and declination of all the stars that could be seen through the Transit Instrument could be found, provided the sidereal clock gives the correct time, and the latitude of the place also is correctly known. These stars are then designated as known stars.

21. Circumpolar stars and the determination of the Latitude of a place by observations with a Transit Instrument.

At any place north or south of the equator there would be generally some stars which could be seen always. These never go below the horizon as a result of the apparent rotation of the celestial sphere. Such stars have their polar distances less than the latitude of the place and they are called circumpolar stars. Polaris, the so-called Pole Star is itself a circumpolar star for all places in north latitude above (1°.30′.) which is approximately its distance from the celestial pole and it describes a small circle of angular radius 1°. 30′ round the pole in a sidereal day.

These circumpolar stars cross the meridian twice in a sidereal day and both these transits could be observed by the transit circle. These two transits are called the upper and lower culminations of the star. By the culmination of a body is meant the instant at which the body attains to the maximum or minimum altitude. In the case of

bodies whose polar distances are constant, culmination takes place at the time of transit only.

Let X and X′ denote the lower and upper transit of a circumpolar star (Fig. 17.)

If P is the pole, $PX' = PX$

$$\therefore \quad \frac{ZX' + ZX}{2} = ZP = 90° - \text{latitude of the place.}$$

The readings of the transit circle would give ZX′ and ZX and therefore the latitude of the place is known. Observations of as many circumpolar stars as are available would enable one to arrive at the best determination of the latitude of a place.

The same observations would also give the polar distance and therefore the declination of the star, for $\frac{ZX - ZX'}{2} = PX' = PX =$ star's polar distance.

Stars invisible at a place:—In any latitude except on the equator, there will be a few stars about the pole which are always visible, as that portion of the celestial sphere always remains above the horizon. There is a corresponding portion of the celestial sphere about the opposite pole which remains invisible. Any star here would have its distance from the opposite pole less than the latitude of the place.

22. The annual motion of the sun in the ecliptic.

If the R. A. and declination of the sun be determined at apparent noon every day it will be found to be changing. But the changes are not uniform. If the different points representing the various positions of the sun every day be all joined, the curve would give the apparent path of the sun among the stars. This path called the ecliptic, is inclined to the equator at an angle of $23°.27\frac{1}{2}'$ nearly. The points of intersection of this with the equator are called the *equinoctial points.*

On the 21st March the sun's declination is found to be zero and its R. A. is also zero and therefore it is at ♈. This is called the *vernal equinox*. From this date the sun's R. A. and north declination are daily increasing until on the 21st June, when the R. A. of the sun is 6 hrs and the north declination is maximum, being equal to the obliquity of the ecliptic ($23°.27\frac{1}{2}'$). This position of the sun is called the *summer solstice*. The sun's longitude is then 90°. Afterwards the sun's declination is decreasing but the R. A. is still increasing daily till the 23rd September, when the R. A. and declination of the sun become 12 hrs and 0° respectively, the sun being again on the equator on that day. This is called the *autumnal equinox*, or the first point of Libra. On this day the longitude of the sun is 180° and the sun crosses the equator from the north to the south. The sun's south declination and R. A. are daily increasing after the 23rd September till the 22nd December, when the south declination of the sun reaches a maximum ($23°\text{-}27\frac{1}{2}'$) and its R. A. is 18 hours. This position is called the *winter solstice*. The sun's longitude at this time is 270°. From this date the sun's south declination is decreasing and the R. A. increasing until the sun crosses the first point of Aries again on the 21st March next year.

The declination circle passing through the equinoctial points is called the *equinoctial colure* and that passing through the solstitial points of the ecliptic is called the *solstitial colure*.

Longitude of the sun and obliquity of the ecliptic.

The longitude of the sun on any day can be calculated from the observed R. A. and declination for the day. Let λ be the longitude of the sun and α and δ the R. A. and declination of the sun respectively. Let ω be the obliquity of the ecliptic.

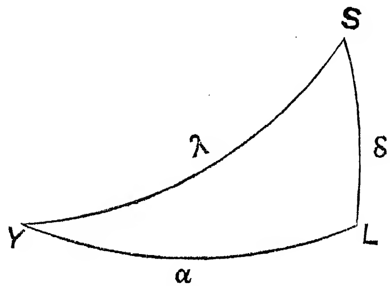


Fig. 18.

Then, for any position of the sun on the ecliptic between ♈ and the summer solstice, we have

(1) $\cos\lambda = \cos\alpha . \cos\delta$

or (2) $\cos\omega = \dfrac{\tan\alpha}{\tan\lambda}$ giving the longitude of sun.

and $\tan\omega = \dfrac{\tan\delta}{\sin\alpha}$ giving ω.

23. Earth's motion round the sun.

The sun's apparent motion on the ecliptic is the result of the actual motion of the earth round the sun and it will be seen later (chapter VI) that the earth moves round the sun in an ellipse and so its motion is not uniform. Therefore the sun's rate of change of longitude cannot also be uniform. The rate of change of R. A. of the sun is also not uniform. The daily motion of the sun in R. A. is the difference in sidereal time between a sidereal day and the interval between two consecutive transits of the sun. This is found to be less than 4 minutes. For rough purposes, we may take the sun's R. A. and longitude to be changing at the rate of 1° per day, provided we don't calculate the change in R. A. or longitude for many days and provided it is reckoned from the nearest solstice or equinox. If however the sun's R. A. and declination are required accurately at any time they have to be deduced

from the Nautical Almanac, where the sun's R. A. and declination are tabulated for a year; and from the hourly variation of the sun's R. A. or declination given there, these quantities can be calculated for any other instant of time.

24. Solar time.

The interval between two consecutive transits of the sun across the meridian of a place is called a *solar day.* We have seen that it exceeds the sidereal day by the amount represented by the sun's daily motion in R. A. Taking the commencement of the solar day, at apparent noon, ie. at the time of the sun's transit we get the solar time at any instant equal to the hour angle of the sun expressed in time. Also the sun's R. A. at any time is equal to the difference between sidereal time and solar time.

25. Morning and Evening stars and Length of sidereal year.

Mention has already been made that the annual motion of the sun among the stars could easily be verified by observation of the so-called morning and evening stars. A star whose R. A. is slightly less than that of the sun but whose declination is the same, could be seen to rise earlier than the sun and in the same part of the horizon. This is called a morning star. Similarly a star whose R. A. is slightly greater than that of the sun, but whose declination is the same would set later than the sun and in the same part of the horizon where the sunset takes place. This is called an evening star. Now the various stars round about the ecliptic could be seen as morning and evening stars according as their R. A. are less or greater than that of the sun and these phenomena are found to recur after a period of one sidereal year which is the period taken by the sun to go through 360° with respect to the stars. Therefore if we note the different morning and evening stars that could be seen at different times of the year, we have a method of determining not only the

annual path of the sun among the stars but also the length of the sidereal year; for, after a year the same morning and evening stars would be observed.

26. The constellations of the zodiac.

It was by observations of the morning and evening stars that the ancient astronomers determined the course of the sun amidst the stars. Now this annual path of the sun known as the ecliptic runs through twelve groups of stars, known as the zodiacal constellations. These stars occupy a region of about 8° on either side of the ecliptic and each of the zodiacal constellations extends to about 30° of the ecliptic. The sun takes a month to pass through the space covered by a single constellation. According to Hindu Astronomy, the ecliptic is divided into twelve equal parts called Rasis and the time taken by the sun to pass through a Rasi is called a solar month and there are twelve solar months in a year. The following is a list of the Zodiacal Constellations and the corresponding portion of the ecliptic occupied by them in terms of Hindu signs or Rasis.

Zodiacal constellations		Hindu names of the signs	Representative figures
(1)	Aries	Mesha	*Ram*
(2)	Taurus	Vrishabha	*Bull*
(3)	Gemini	Mithuna	*Twins*
(4)	Cancer	Karkataka	*Crab*
(5)	Leo	Simha	*Lion*
(6)	Virgo	Kanya	*Virgin*
(7)	Libra	Thula	*Balance*
(8)	Scorpio	Vrischigom	*Scorpion*
(9)	Sagittarius	Dhanus	*Archer*
(10)	Capricornus	Makara	*Goat*
(11)	Aquarius	Kumbha	*Water bearer*
(12)	Pisces	Meena	*Fishes*

27. Diagrammatic representations of celestial bodies.

Before specifying the positions of celestial bodies of known R. A. and declination in an astronomical diagram for a given place at any time of the year, one should draw the celestial sphere and mark on it the position of the visible pole, the cardinal points, the equator, the horizon and other circles that remain fixed for the place. Before marking the position of a star of known R. A. and decl. one has to determine the position of the first point of Aries. This can be done from any one of the following data (1) sidereal time at the instant (2) The hour angle of a known star (3) The apparent solar time and the date of observation. When once ♈ is fixed, the ecliptic can be drawn and a known star, the sun or the moon can be marked on it.

CHAPTER III.

THE SHAPE OF THE EARTH AND CERTAIN PHENOMENA DEPENDING UPON IT.

28. The shape of the earth.

Even from the time of the early astronomers, the globular form of the earth was an ascertained fact. The appearance presented by an approaching ship to an observer on a sea coast, the variation of the stellar configuration depending upon the position of the observer on the surface of the earth, and the form of the earth's shadow during a lunar eclipse gave conclusive evidence that the earth should be spherical in shape. It is the rotation of the earth round an axis pointing to the neighbourhood of the star polaris that brings about the apparent rotation of the heavens once in a sidereal day. Therefore the observer's zenith, horizon, meridian etc. are all rotating about this axis from west to east. This has been directly demonstrated by the pendulum experiment of Foucault.

The points of the earth where the axis of rotation meets its surface are called the *terrestrial poles* and the great circle of the earth to which these points are poles is called the *terrestrial equator.* All sections of the earth passing through the poles are called *terrestrial meridians.* It is found by observation, that the altitude of the pole undergoes a change proportional to the distance travelled along the terrestrial meridian through that place. Now if the earth is assumed spherical, this observation shows that the latitude of a place is proportional to its distance from the equator, for the latitude of a place on the surface of the earth is the angle subtended at the earth's centre by the arc of the meridian drawn to the equator from that place.

29. Methods of determining the radius of the earth.

(i) By measuring the linear distance between two places on the same meridian and finding the difference in the altitudes or the zenith distances of the celestial pole, or any star on the meridian, we have a method of arriving at the radius of the earth, supposing it to be spherical in shape.

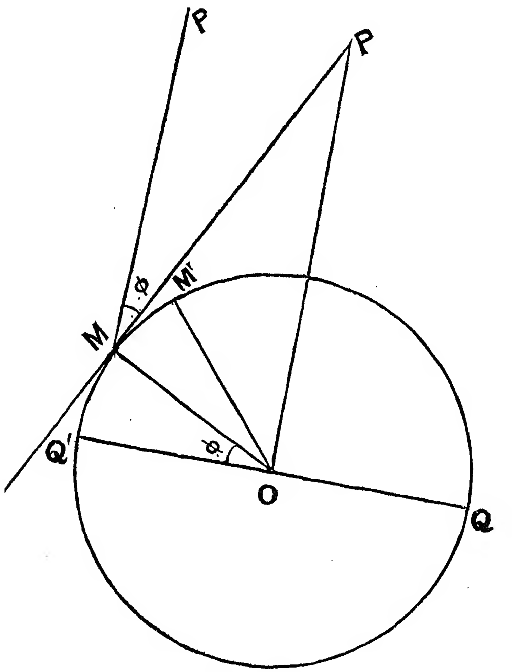


Fig. 19.

If the change in the altitude of P as seen from two places M and M′ be $(\varphi'-\varphi)^\circ$ we have the change of latitude from M to M′ given by $(\varphi'-\varphi)^\circ$. If the line or distance between M and M′ be x and the radius of the earth be R

$$\frac{R\,(\varphi'-\varphi)\,\pi}{180} = x.$$

$$\text{or } R = \frac{x\,.\,180}{\pi\,(\varphi'-\varphi)^\circ} \quad \frac{x\,.\,180}{\pi\,(a'-a)} = \frac{x\,.\,180}{\pi\,(z-z')}$$

n the result derived on the previous page, a and a' are the altitudes and z and z' are the zenith distances.

The zenith sector.

A very convenient form of telescope used in this connection is the zenith sector which is used in observing the meridian zenith distance of a star.

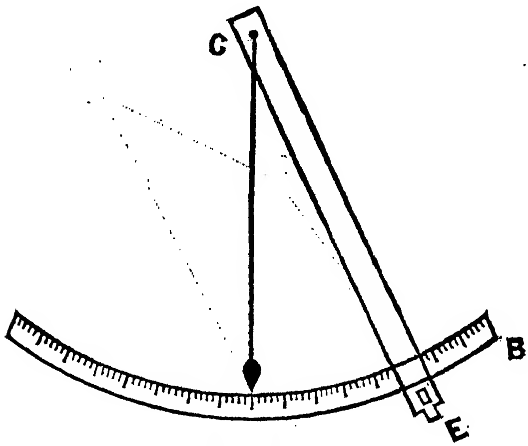


Fig. 20.

This instrument is simply a telescope DE mounted so as to turn about a horizontal axis C pointing east and west, near its object glass. Attached to its eye end there is a graduated arc A B of a circle having C for its centre. Now a plumbline attached to the axis C, would read from the scale AB the zenith distance of a star to which the telescope is directed at any time, provided the telescope is so adjusted as to have the zero of the scale against the plumbline, when it points to the zenith.

(ii) *The Method of Triangulation.*

When the radius of the earth has to be known with great accuracy the distance between the two stations M and M′ on the same meridian should be a large quantity and as this is not generally measurable directly it has to be derived by the method of triangulation. In this

method, at first, the distance between two points A and B lying on a level tract of country is measured with great accuracy and using this known length as the base line, the distance between the two points M and M′ is calculated from the observed angles given by a theodolite. This is done as follows :—

Let M be a point whose bearings from A and B are known from theodolite observations. Then both A M and

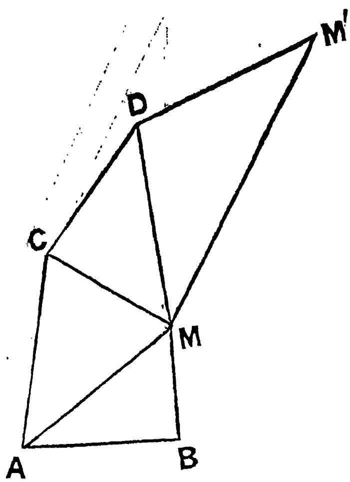


Fig. 21.

M B are known. Again from A M as base line A C and M C are similarly derived from theodolite observations on C from A and M. Similarly from the triangle C D M, C D and M D are known. Finally if M′ is observed from M and D we get M M′ and D M′. If M and M′ happen to lie on the same meridian we have thus secured the distance between them with an accuracy depending upon the reliability of the correctness of the angles observed and the measured base line AB. A direct measurement of the distance M M′ may be difficult not only owing to its much greater distance than AB but owing also to the inaccessibility of the regions between them.

(iii) *Another method of determining the earth's radius.*

Among other methods used in finding the radius of the earth, the method suggested by A. R. Wallace is simple.

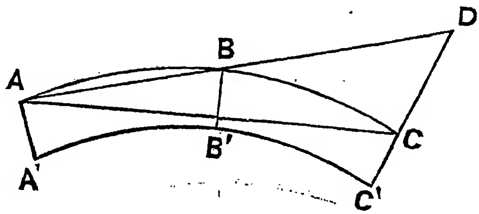


Fig. 22.

A, B, C are the ends of three posts set up in a line and at equal distances in a straight canal and they project to equal heights above the water level. If A′ B′ C′ represent the water surface of the canal, it should be an arc of a circle, owing to the globular form of the earth and so also should A B C. An observer will have to look from D higher than C in order to see A and B in the same direction. This height D C is measured and the distances of A B and B D are also measured. From these, we see that if a be the radius of the earth,

$$\begin{aligned} DB.\ DA &= DC\,(2a + 2\,CC' + DC) \\ &= DC.\ 2a. \text{ nearly} \\ a &= \frac{DB.\ DA}{2\,DC} \end{aligned}$$

As it is difficult to measure correctly the small value DC and as the observation is affected by refraction, this method does not give a very accurate value for the radius.

30. The actual shape of the earth.

The shape of the earth thus determined, is not found to be exactly globular. Its form approximates to the spheroid of revolution obtained by the rotation of an

ellipse about its minor axis. The lengths of the semi-axes of this ellipse are approximately 3963·3 miles $= a$, and 3949·8 miles $= b$. The quantity $\frac{a-b}{a}$ is called the ellipticity and $\frac{\sqrt{a^2-b^2}}{a}$ is called the eccentricity

31. The geocentric and geographic latitudes of a place on the earth.

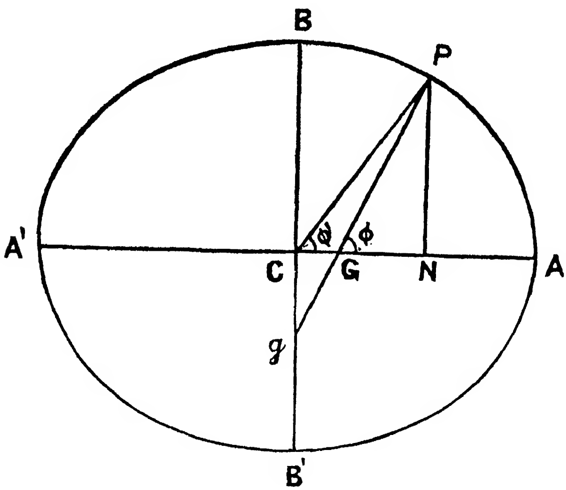


Fig. 23.

Let BPA represent a section of the earth through the poles B and B′, and P any point on it. Let PG be the normal to the elliptic section at P. PC is joined, C being the centre of the earth. Then PGA (φ) is called the geographical latitude of P and PCA (φ') the geocentric latitude of P.

Relation between the geocentric and geographic latitudes.

$$\frac{GN}{CN} = \frac{PG}{Pg} = \frac{b^2}{a^2}$$

$$\tan \varphi' = \frac{PN}{CN} = \frac{PN}{GN} \cdot \frac{GN}{CN} = \tan \varphi . \frac{b^2}{a^2} \text{ —(1)}$$

$$\therefore \tan(\varphi-\varphi') = \frac{\tan\varphi - \tan\varphi'}{1+\tan\varphi\tan\varphi'} = \frac{\tan\varphi\left(1-\frac{b^2}{a^2}\right)}{1+\tan^2\varphi.\frac{b^2}{a^2}}$$

$$= \frac{a^2 e^2 \tan\varphi}{a^2 + b^2\tan^2\varphi} = \frac{a^2 e^2 \sin\varphi.\cos\varphi}{a^2 - a^2 e^2 \sin^2\varphi}$$

$$= \frac{e^2 \sin\varphi\cos\varphi}{1-e^2\sin^2\varphi}$$

$$= \frac{e^2}{2}\sin 2\varphi \text{ neglecting } e^4 \text{ and higher powers of } e$$

The geocentric distance of a place in terms of the equatorial radius and latitude of the place.

If λ is the eccentric angle of P, we have

$$\tan\varphi = \frac{a}{b}\tan\lambda \left(\because \text{ the normal at P is } \frac{ax}{\cos\lambda} - \frac{by}{\sin\lambda} = a^2-b^2\right)$$

$$CP^2 = a^2\cos^2\lambda + b^2\sin^2\lambda$$

$$= \frac{a^2}{\sec^2\lambda}\left(1+\frac{b^2}{a^2}\tan^2\lambda\right)$$

$$= a^2 \frac{\left(1+\frac{b^2}{a^2}\cdot\frac{b^2}{a^2}\tan^2\varphi\right)}{1+\frac{b^2}{a^2}\tan^2\varphi}$$

$$= \frac{a^4\cos^2\varphi + b^4\sin^2\varphi}{a^2\cos^2\varphi + b^2\sin^2\varphi}$$

$$= a^2\frac{\cos^2\varphi + (1-e^2)^2\sin^2\varphi}{1-e^2\sin^2\varphi}$$

$$= a^2(1-2e^2\sin^2\varphi)(1+e^2\sin^2\varphi)$$

neglecting higher powers of e than e^2

$$= a^2(1-e^2\sin^2\varphi).$$

The radius of the earth as determined by the method described earlier, is the radius of a circle that best fits into the shape of the earth at the place of observation. This may be called the radius of curvature of the earth at that place. The centre of this circle is called the centre of curvature. The centre of curvature of any curve is also defined as the point of intersection of two consecutive normals to it. In the case of the earth, any meridional section gives an ellipse as the curve.

32. Radius of curvature along the meridian.

The equation to the normal at P whose eccentric angle is λ is, $ax \sin \lambda - by \cos \lambda = (a^2 - b^2) \sin \lambda \cos \lambda$ (1)

$$\text{where } \tan \varphi = \frac{a}{b} \tan \lambda$$

The equation to the consecutive normal is obtained by differentiating (1) with regard to λ,

$ax \cos \lambda + \text{by} \sin \lambda = (a^2 - b^2) \cos 2 \lambda$. ....(2)

Solving (1) and (2), we get for the co-ordinates of the centre of curvature,

$$x = \frac{(a^2 - b^2)}{a} \cdot \cos^3 \lambda \text{ and } y = \frac{(b^2 - a^2)}{b} \sin^3 \lambda.$$

Also, the radius of curvature at P is given by

$$\rho^2 = (x - a \cos \lambda)^2 + (y - b \sin \lambda)^2.$$

$$= \frac{(a^2 \sin^2 \lambda + b^2 \cos^2 \lambda)^3}{a^2 b^2} \text{ or}$$

in terms of the latitude φ

$$\rho^2 = a^4 b^4 (b^2 \sin^2 \varphi + a^2 \cos^2 \varphi)^{-3}$$

If s be the distance between two points on the same meridian, whose geographical latitudes expressed in radians are φ_1 and φ_2, we have

$$s = \int_{\varphi_1}^{\varphi_2} \rho \, d\varphi = \int_{\varphi_1}^{\varphi_2} \frac{a^2 b^2}{(b^2 \sin^2 \varphi + a^2 \cos_2 \varphi)^{3/2}} \cdot d\varphi$$

33. Length of an arc on the earth parallel to the equator.

Any small arc of the earth parallel to the equator is called a *parallel of latitude.* The length of such an arc can be known, provided the longitudes of its extremities are given. Let A′ and B′ differ by L° in longitude and let the latitude of either of them be $l°$. Then if a is the radius

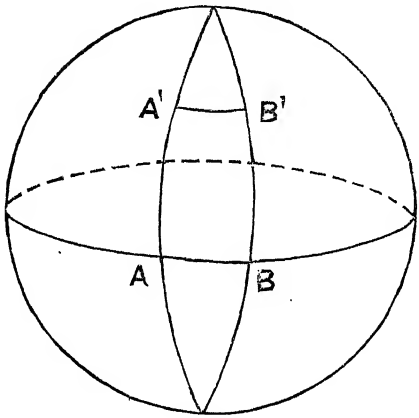


Fig. 24.

of the earth we have arc A′ B′ = cos l. arc AB

$$= \cos l . \frac{2\pi . a . L}{360}$$

$$= \frac{\cos l . \pi . a . L}{180}$$

34. The Geographical and Nautical Miles.

The length of a minute of arc of the earth's equator is defined to be a *geographical mile.* Therefore, the corresponding parallel arc in latitude l measures cos l of a geographical mile.

A *nautical mile* is the length of a minute of arc of the meridian. If the earth were spherical, the nautical and geographical miles would have been equal.

35. Phenomena depending upon the longitude of a place.

Since the earth is rotating from west to east at the rate of 15° per sidereal hour, two observers in places of the

same latitude, but differing in longitude by L°, would see the same aspect of the sky at times differing by $\frac{1}{15}$ L hours. Their local times would differ by $\frac{L}{15}$ hours, the station of more eastern longitude, having its time, faster. It therefore follows, that the longitude of a place west of Greenwich=15 (Greenwich time − Local time). If the two places be also different in latitude, though the same aspect of the sky is never brought about by the lapse of time, their local times still differ by $\frac{L}{15}$ hrs. If the two places be on the same meridian, though differing in latitude, their local times will be the same. Stars in the same declination circle, transit simultaneously for both the observers.

36. Dip of the horizon.

Assuming that the earth is spherical in shape, it is easily seen that an observer stationed at a height h from the surface of the earth would be able to command a view of all the regions of the earth enclosed by a spherical cap, whose size would depend upon the height of the place of observation. The boundary of the visible portion of the earth is called the *visible horizon* or *the offing.* The offing will be circular in shape, unless the earth is taken to be nonspherical in shape.

Owing to the height of the observer's position, the horizontal plane through the observer's eye and the tangent from his eye to the earth are inclined at a certain angle D (say). This angle HPT (Fig. 25) is called the dip of the visible horizon. From the height of the observer's position and the known radius of the earth, it is possible to determine the distance of the offing and the dip of the visible horizon.

Let the radius of the earth be a, and the height of the observer's position be h. Let d be the distance PT of the offing and let D be the dip of the visible horizon.

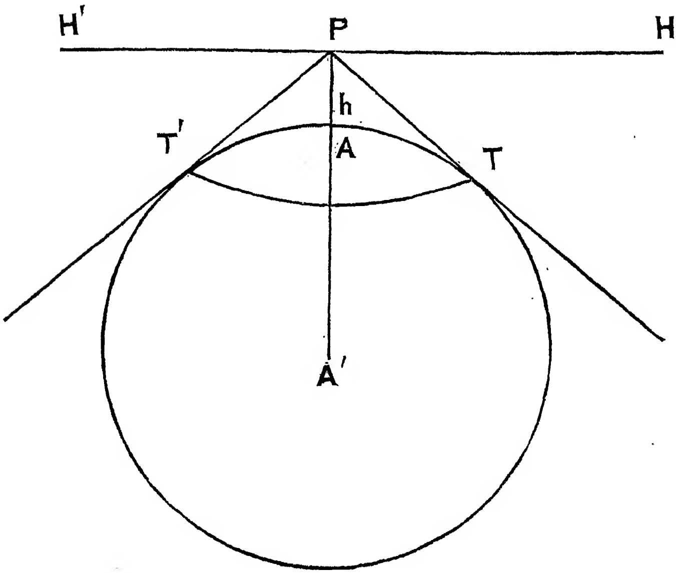


Fig. 25.

Now $PT^2 = h(2a+h)$

or $d^2 = 2ah$. approximately

$d = \sqrt{2ah}.$

Also D $P\hat{A'}T$ and $\tan D = \frac{PT}{A'T} = \frac{d}{a}$

For a correct determination of dip, we can use the formula :—

$$D = \frac{\sqrt{2ah + h^2}}{a}$$

The value of the dip for any place is usually a small quantity. If it is D″, then

tan D=D″ expressed in circular measure

$$= \frac{D.\ \pi}{180 \times 60 \times 60}$$

∴ D (the value of the dip in seconds for any place) $= \sqrt{2h/a}\,\frac{180 \times 60 \times 60}{\pi}.$

The distance between two points, that are just visible from one another at a great distance, can now be known, provided their heights h and h' above the sea-level be known. The distance is approximately given by $\sqrt{2a}\,(\sqrt{h}+\sqrt{h'})$ where a is the radius of the earth.

37. Acceleration of rising and retardation of setting of a heavenly body due to dip.

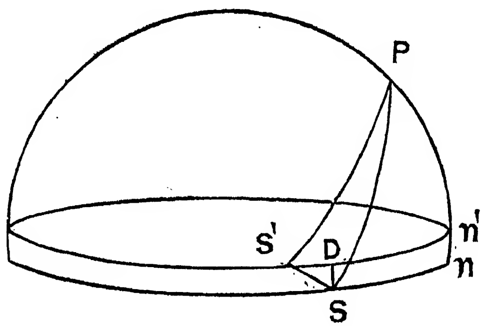


Fig. 26.

At a place, where the offing is well defined, when the observer is stationed at a higher level, he sees a rising body a little before it reaches the true horizon, since the visible horizon is D$''$ below the true horizon, if D$''$ is the dip for the place.

Let φ be the latitude of the place and let n' S$'$ and n S represent the two horizons.

S and S$'$ are the positions of a star of decl. δ when the star is at the two horizons respectively. If t seconds of time lapse between the two positions of the star, then, $15\,t =$ S P S$'$

$$\text{Also } D'' = SD = SS' \cos n'S'P$$
$$= 15\,t \cos\delta . \cos n'S'P$$
$$\therefore t = \tfrac{1}{15} D'' \sec\delta \sec n'S'P$$

$$\cos n' S' P = \sqrt{1 - \sin^2 n' S' P} = \sqrt{1 - \sin^2 \varphi / \cos^2 \delta} = \frac{\sqrt{\cos^2 \delta - \sin^2 \varphi}}{\cos \delta}.$$

$\therefore t = \frac{1}{15} \frac{D'}{\sqrt{\cos^2 \delta - \sin^2 \varphi}}$, which is the acceleration in the time of rising or retardation in the time of setting of a heavenly body, due to dip.

CHAPTER IV

PHENOMENA DEPENDING ON THE SUN'S APPARENT MOTION ON THE ECLIPTIC AND PROBLEMS ON SUN AND MOON.

38. The seasons and the phenomena connected with the diurnal motion.

Reference has already been made to the sun's apparent annual motion among the stars. This motion, combined with the apparent diurnal rotation of the celestial sphere brings about variations in the lengths of day and night at a place in the course of the year, and the division of the year into four seasons is based upon the position of the sun on the ecliptic.

The four seasons into which the year is divided are the *Spring*, commencing from the time when the sun is on the first point of Aries or at the Vernal Equinox (21st March), the *Summer* commencing from the time when the sun is at the Summer Solstice (21st June, the right ascension and the declination of the sun being 90° and ω respectively), the *Autumn* commencing from the time, when the sun is at the Autumnal Equinox or the First Point of Libra (23rd September, right ascension and declination of the sun being 180° and 0° respectively) and the *Winter* commencing from the time, when the sun is at the Winter Solstice (22nd December, the right ascension and declination of the sun being 270° and $-\omega$ respectively).

The temperature conditions prevailing in any place during the different seasons of the year, depend upon many things, such as the meridian altitude of the sun, the length of the day, the distance of the sun, the height above sea level, nearness to the sea, and the presence of ocean currents. We shall consider the effect of the first

of these, in detail. The meridian altitude of the sun at any place, depends upon the variations in the declination of the sun from $-\omega$ to $+\omega$ during the course of the year, and the surface of the earth is conveniently divided into five zones according to the variations in the meridian zenith distances of the sun in these different regions.

39. The different zones of the earth.

The *torrid zone* is the region of the earth covered by two parallels to the earth's equator in north and south latitude ω ($23°\text{-}27\frac{1}{2}'$ nearly). The northern boundary is called the *tropic of cancer* and the southern boundary is called the *tropic of capricorn.* At any place within this belt the sun will pass through the zenith twice a year. Also, the sun's meridian zenith distances at any place of latitude φ during the course of the year would not undergo great variations, the value lying between $\omega+\varphi$ and $\omega-\varphi$. At all places on the tropic of cancer, where the latitude is $23°.\ 27\frac{1}{2}'$, the sun crosses the meridian at the zenith on the 21st June, when the sun is having the maximum north declination. *The north temperate zone* is the belt of the earth lying between the tropic of cancer and a parallel of north latitude $90°-\omega$, called the *arctic circle.* There is a corresponding belt called the *south temperate zone,*

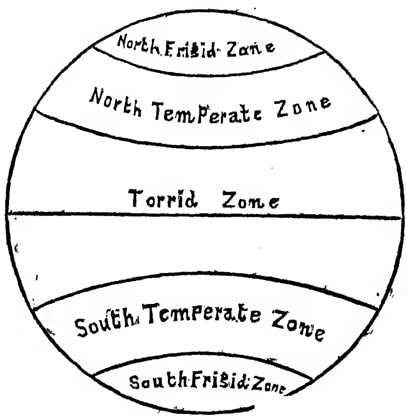


Fig. 27.

lying between the tropic of capricorn and the *antarctic* circle which is a parallel of south latitude $90° - \omega$. At a place lying between these belts, the sun would never come to the zenith. At any place here, the zenith distances of the sun during the year undergo greater variations than in the previous case. The arctic and the antarctic zones are the two polar caps of the earth, each of angular radius ω. The diurnal paths of the sun at different seasons and at different places can be indicated on the celestial sphere, if the latitude of the place and the different declinations of the sun are known. The following figures indicate roughly the apparent diurnal paths of the sun for the different places on the earth in the course of a year, when the declination changes from $-\omega$ to $+\omega$. Actually, the path of the sun on the celestial sphere is a continuous spiral curve.

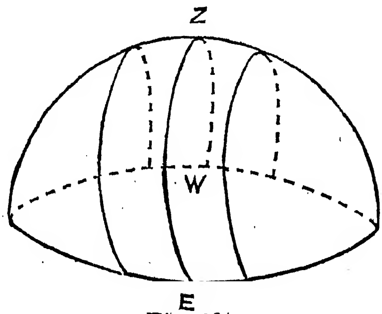


Fig. 28.
A place on the equator.

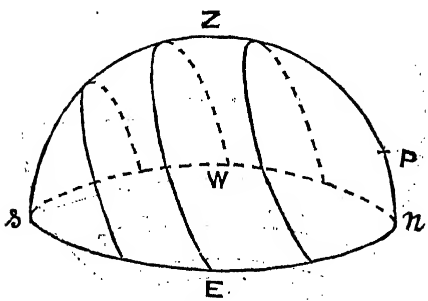


Fig. 29.
A place in the torrid zone.

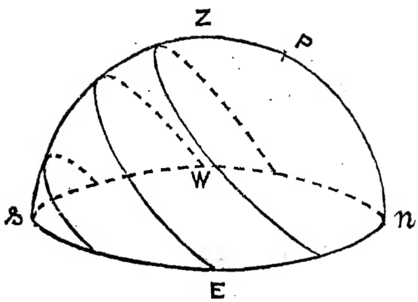


Fig. 30.

A place in the temperate zone.

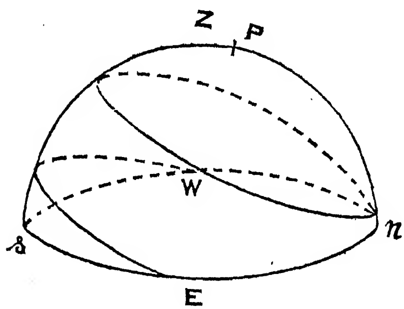


Fig. 31.

A place in the arctic circle.

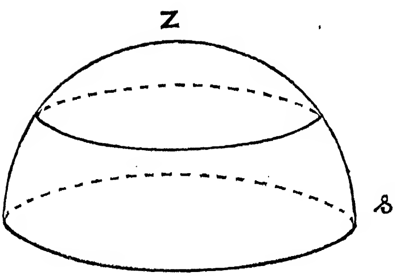


Fig. 32.

A place at the poles.

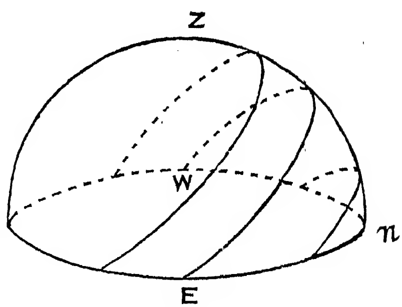


Fig. 33.

A place in the southern hemisphere.

40. The variation in the lengths of the day.

Such variations can be known from the hour angle of the sun at the time of sunset.

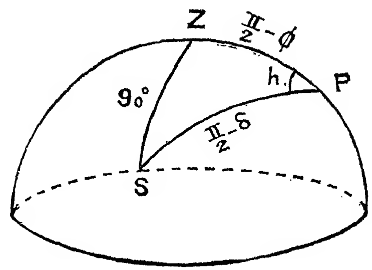


Fig. 34.

From the $\triangle$ Z PS, $\cos h = -\tan\delta \tan\varphi$.

The duration of daylight at a time when the sun's declination is $\delta = \frac{2}{15}\cos^{-1}(-\tan\delta\tan\varphi)$ hours, assuming that the sun's declination remains constant throughout that day. From the above equation, we get the length of the day at any place on any date, since the declination of the sun for the day could be known from the Nautical Almanac.

Note: Owing to the phenomena of dip and refraction, the sunrise takes place earlier than the time, when the sun comes to the true horizon and so the zenith distance of the sun at the time of sunrise is $(90+e)$ where e is a small quantity. Hence the duration of daylight is obtained from the equation.

$$\cos(90+e) = -\sin e = \sin\varphi\sin\delta + \cos\varphi\cos\delta\cos h.$$

$$\cos h = \frac{-(\sin e + \sin\varphi\sin\delta)}{\cos\varphi\cos\delta}$$

Duration of daylight $= \frac{2}{15}\cos^{-1}\left(-\frac{\sin e + \sin\varphi\sin\delta}{\cos\varphi\cos\delta}\right)$ hrs.

41. Perpetual day and night.

It is also clear that if $\tan\delta\tan\varphi$ is numerically equal to or greater than unity on a day for any place, the sun never sets at that place on that day. The sun's declination varies between $-\omega$ and $+\omega$ and therefore at a place of latitude φ, $(\varphi > 90 - \omega)$, perpetual day begins

from the time when the sun's declination attains the value $(90-\varphi)^\circ$ to the time when it reaches the same value after attaining the maximum declination ω. Similarly perpetual night reigns in the same place after the sun's declination has reached the value $-(90-\varphi)^\circ$ till it gets back to the same value after attaining its maximum south declination. In the arctic circle, on June 21st, the sun never sets, and on December 22nd, the sun never rises. At either of the poles, darkness prevails for nearly six months and perpetual daylight prevails for the remaining six months in the course of the year. (The effects of refraction and dip are not considered here).

42. The temperature effects experienced in different parts of the earth in the course of the year.

The temperature at any place depends upon the amount of heat received there. During daytime, the place gets heated by the sun's rays and therefore the temperature increases with the length of the day. Besides this cause, the variation in the sun's zenith distance considerably affects the heat of a place, and the quantity of heat received is proportional to the cosine of the sun's zenith distance. It should also be noted that part of the heat coming from the sun gets absorbed by the atmosphere round the earth and this absorption is least when the sun is directly overhead. All these combine to produce in the torrid zone high temperatures having small variations in the course of the year. When the sun's zenith distance increases at a place in the torrid zone, the heat received does not diminish to any appreciable extent, with the decrease in the length of the day.

In the north temperate zone, the days get longer and longer when the sun's declination increases. This accounts for the spring and the summer being warmer than the autumn and the winter when the days are

much shorter. As the earth is gaining maximum heat about the time of summer solstice and as it continues to do so for some more time the summer is warmer than the spring. For a similar reason, the winter is colder than the autumn.

As one goes further north, the variations in the length of the day become larger so that, larger fluctuations in temperature conditions between summer and winter are observed. The sun's change of altitude at noon every day also contributes to bring about the same effects.

In the arctic circle, where at some part of the year, the sun is continuously visible for more than 24 hours in spring and summer, and where the sun is not visible at all for a corresponding period of time in autumn and winter, there is a mild temperature during the former period and an extremely cold temperature during the latter. It is the low altitude of the sun, that does not give intense heat even when the sun is above the horizon in these places. It is not difficult to see that the seasons corresponding to spring, summer, autumn and winter of the northern hemisphere will begin for the southern hemisphere on September 23rd, December 23rd, March 21st and June 21st respectively.

Two other causes affecting the temperature conditions of a place on the earth are the distance of the earth from the sun which varies throughout the year and the local conditions, such as the existence of a large area of land or water, height of the place above sea level etc. Considering the first cause, it is found that the earth is nearest the sun on 31st December, a few days after winter solstice and that it is most distant on July 1st, a few days after summer solstice. The effect of this, is to make the winter milder and the summer cooler for the northern hemisphere and to make the temperature conditions more

acute during the corresponding seasons in the southern hemisphere.

The effects of the second cause are more important and observable. The fact that the southern hemisphere has a greater area of water and the northern hemisphere a greater area of land would account for a more or less equable climate in the southern hemisphere and also for the less marked character of the different seasons there.

43. Other phenomena connected with the sun and the moon.

(i) *The time occupied by the sun or the moon in rising at a place of latitude* φ.

The sun or the moon takes some time to rise at a place, since either of them has a disc of finite size. The time taken for the rise is the interval from the moment when the top of the disc comes to the horizon to the time when the bottom of the disc just grazes the horizon.

Let the diameter of the disc be D″. Then the problem is the same as the one in finding the acceleration in the time of rise due to dip, where the time required for a point at declination δ to go a vertical height of D″ due to diurnal motion has been found. If t'' is the time required for rising, it is easily found to be equal to $\frac{1}{15}\frac{D''}{\sqrt{(\cos^2\varphi-\sin^2\delta)}}$. From the formula, it is clear that the time is greatest during the two solstices, when the denominator of the expression becomes the smallest.

(ii) *Azimuth of the sun.*

The azimuth of the sun or the moon when either of them rises is obtained from the $\triangle$ P Sn. (Fig. 35)

$$\sin\delta = \cos\varphi\cos A$$

$$\therefore A = \cos^{-1}(\sin\delta\sec\varphi)$$

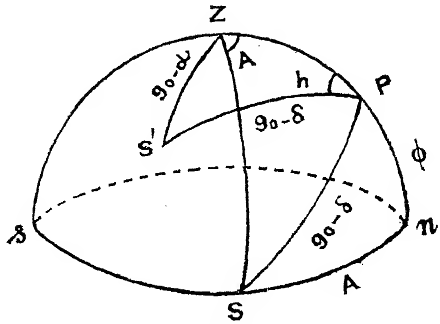


Fig. 35.

The sun's azimuth at any other time can also be calculated by an observation of its altitude. If α is the altitude observed when the sun is at S′ we have from the $\triangle$ P Z S′

$$\sin\delta = \sin\alpha \sin\varphi + \cos\alpha \cos\varphi \cos PZS'$$

$$\therefore \text{ The azimuth required} = \cos^{-1}\left(\frac{\sin\delta - \sin\alpha\sin\varphi}{\cos\alpha\cos\varphi}\right)$$

If the hour angle of the sun or the apparent time be known, the azimuth of the sun can be calculated from the formula $\cot A = \dfrac{\tan\delta\cos\varphi - \sin\varphi\cos h}{\sin h}$ (where h = hour angle). This method is often used when the sun's altitude cannot be observed conveniently, and has also the advantage of being free from errors of atmospheric refraction which are incidental to altitude observation. [See effects of refraction].

(iii) *Twilight.*

It is a matter of every day observation that in early morning, day light is found to creep in, even when the sun is below the horizon, and in the evenings, darkness does not come all on a sudden even when the sun has disappeared below the horizon. This phenomenon is known as 'twilight'. This is due to the fact, that the rays of the sun strike the extremely light particles

of matter that are suspended in the atmosphere, and are reflected in all directions at the place. The intensity of this reflected light gets diminished as the sun sinks further and further below the horizon. It has been found that this phenomenon lasts till the sun sinks up to 18° below the horizon. Therefore, the duration of twilight at any place depends upon the latitude of the place, and the sun's declination, for if h' is the hour angle of the sun when twilight ends, we have,

$\cos 108° = \sin \delta \sin \varphi + \cos \delta \cos \varphi \cos h'$ and if h is the hour angle of the sun at sunset,

$$\cos h = -\tan \varphi \tan \delta,$$

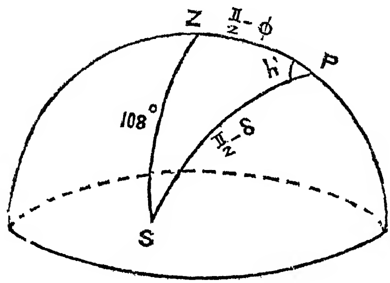


Fig. 36.

Therefore $\dfrac{h'-h}{15}$ hours will be the duration of twilight at any place of latitude φ on a day when the sun's declination is δ. Near the equator, twilight lasts for over an hour and in higher latitudes the duration of twilight is much longer.

If the sun at midnight in a place of latitude φ is not 18° below the horizon, then there will be no darkness at the place. For this, the polar distance of the sun should be less than $(\varphi + 18°)$ or $\delta > 72° - \varphi$, where δ is the sun's declination. From this, we can find the number of days, when there will be no real night at all for that place.

(iv) *To find when twilight is shortest at a place of latitude* φ.

If Z is the zenith of a place and S the sun, twilight lasts during the time when ZS changes from 90° to 108°. This change of zenith distance takes place as a result of the earth rotating along with the observer's zenith and the meridian.

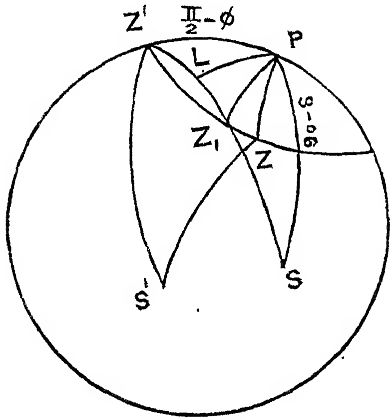


Fig. 37.

If S′ is the sun and PZ and PZ′ are the two positions of the observer's meridian at the beginning and at the end of twilight, ZPZ′ gives a measure of the duration of twilight. We have now to find when this will be shortest. In the △ ZS′Z′, ZZ′ cannot be less than (Z′S′ − ZS′) i. e. less than 18°.

∴ For the period of twilight to be shortest, Z′Z and S′ should be on the same great circle so that Z′Z is 18°. Let Z′, Z_1, and S be the new positions of the three points in this case. Draw PL ⊥ʳ to Z′Z_1.

From the △ PLS, $\sin\delta = \cos PL \cos SL$

$= \cos PL \cos 99° = -\cos PL \sin 9°$

From the △ PLZ′, $\sin\varphi = \cos PL \cos 9°$

∴ $\sin\delta = -\sin\varphi \tan 9°$, giving the time of the year, when twilight is shortest for the place.

The duration of the shortest twilight is given by $\frac{2}{15}\sin^{-1}\left(\frac{\sin 9^\circ}{\cos\varphi}\right)$ from the $\triangle$ PLZ$'$.

(v) *The height of the atmosphere.*

From the fact, that twilight lasts in a place till the sun sinks down to 18° below the horizon, we can get an

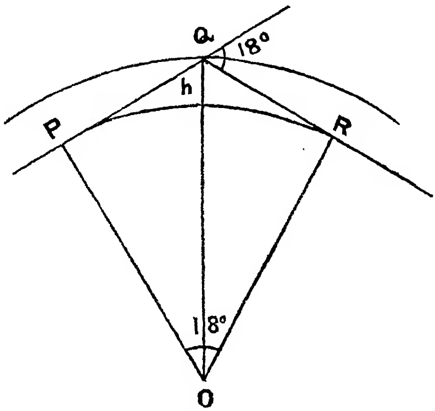


Fig. 38.

estimate of the height of the atmosphere round the earth, at least up to the extent, beyond which, no particle could be held in suspension.

Let a be the radius of the earth assumed spherical and if h be the height of the atmosphere, it is possible for P to get light from the particles at Q, which are illuminated by the sun at the last moment of twilight. The angle $POR = 18^\circ$, which is also the angle the sun has gone down below the horizon of P at the end of twilight.

$$\therefore P\hat{O}Q = 9^\circ$$

$\therefore h = a(\sec 9^\circ - 1)$, which is found to be about 50 miles. This value should not be taken to be accurate, as PQ, QR etc. are not actually straight lines, owing to the effect

of atmospheric refraction and as there may be very rare atmosphere at greater heights than Q.

(vi) *The sidereal time of sunrise or sunset at a place of latitude φ, given the sun's longitude.*

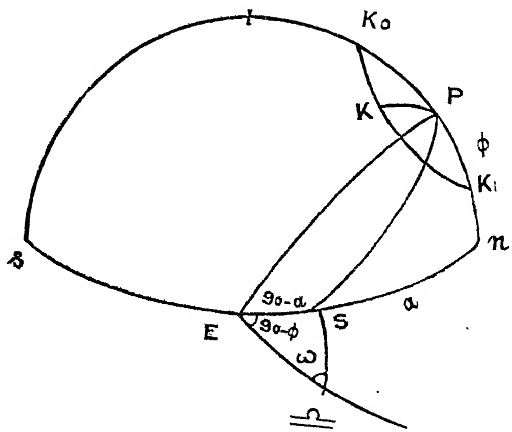


Fig. 39.

Let S be the sun at sunrise, when its longitude is λ and its azimuth a, ≏ the first point of libra, so that the portion of the ecliptic S ≏ $= 180^\circ - \lambda$.

E≏ is the equator. If T is the sidereal time of sunrise, ♈E $= T + 90^\circ$.

$\therefore$ E≏ $= (90 - T)$ Also, ES $= 90 - a$.

The angle SE≏ $= (90 - \varphi)$ since, it is equal to PZ.

$\therefore$ From the Δ ES≏, we have

$$\sin T \cos \omega = \cos T \cot (180 - \lambda) - \sin \omega \tan \varphi.$$

i.e. $\sin T \cos \omega + \cos T \cot \lambda + \sin \omega \tan \varphi = 0 \ldots\ldots (1)$ giving the sidereal time of sunrise or sunset when λ is known.

Let $l \cos \omega = \cos A$ and

$l \cot \lambda = \sin A$ and $\sin B = -l \sin \omega \tan \varphi$.

$$\therefore \tan A = \frac{\cot \lambda}{\cos \omega}$$

Then (1) can be written as

$$\sin (T + A) = \sin B.$$

$$\therefore T + A = B \text{ or } 180 - B.$$

$\therefore T = B - A$ or $180° - B - A$, one of which being the sidereal time of sunrise, and the other the sidereal time of sunset. From the sidereal time it is possible to get the mean time of sunrise and sunset. From equation (1) it is also possible to find the point of the ecliptic that rises at any given instant of time, for if T is supposed to be known, the equation gives λ. (This problem is very important in casting the horoscope of an individual in Astrology)

(vii) *The daily retardation in the time of sunrise.*

Owing to the daily apparent motion of the sun, and the consequent change in longitude, the sunrise takes place later and later every day. The daily change of longitude and the corresponding retardation in sidereal time of sunrise should be connected.

Differentiating equation (1), of (vi) keeping φ and ω as constants, we have,

$$(\cos T \cos \omega - \sin T \cot \lambda) \triangle T = \cos T \operatorname{cosec}^2 \lambda . \triangle \lambda .$$

$$\text{i. e. } \sin \lambda \ (\sin \lambda \cos T \cos \omega - \sin T \cos \lambda) \triangle T = \cos T \triangle \lambda .$$

Also, from triangle E S ≏, $\sin a = - \sin T \cos \lambda + \cos T \sin \lambda \cos \omega .$

$$\therefore \sin \lambda \sin a . \triangle T = \cos T \triangle \lambda .$$

If angle N S ≏ $= n$, we have from the $\triangle$ E S ≏

$$\frac{\sin (90 - T)}{\sin n} = \frac{\sin \lambda}{\sin (90 - \varphi)}$$

$$\text{i. e. } \cos T \operatorname{cosec} \lambda = \sin n \sec \varphi .$$

Also, from the $\triangle$ S P n.

$$\sin \delta = \cos a \cos \varphi .$$

$$\because \quad \triangle T = \frac{\cos T \triangle \lambda}{\sin \lambda \sin a} = \frac{\sin n . \triangle \lambda}{\cos \varphi \sin a}$$

$$= \frac{\sin n}{\cos \varphi} \triangle \lambda \frac{1}{\left(1 - \frac{\sin^2 \delta}{\cos^2 \varphi}\right)}$$

$$= \sin n \left(\cos^2 \varphi - \sin^2 \delta\right)^{-\frac{1}{2}} \triangle \lambda,$$

which is the daily retardation in the time of sunrise.

(viii) *Harvest Moon.*

The above equation will also explain certain phenomena connected with the rising of the moon, assuming that it moves on the plane of the ecliptic. The pole of the ecliptic describes a small circle of radius w round the celestial pole and the inclination of the ecliptic to the horizon is ZK where K is the pole of the ecliptic at any time. (See Fig. 38). This angle is least and greatest when K is at K_o and K_1 respectively. Also, if ≏ S is the ecliptic, ES = $P\hat{Z}K$, since E is the pole of ZP and S is the pole of ZK.

In the course of a sidereal day, the point of intersection of the ecliptic and the horizon varies on either side of E, and ω from zero to a maximum value $\sin^{-1}\left(\frac{\sin \omega}{\cos \varphi}\right)$ given by the angle between ZP and the vertical from Z tangential to the locus of K.

Applying the equation for diurnal retardation of the hour of rising to the moon, we get, when the moon is in Aries $\frac{\triangle T}{\triangle \lambda} = \frac{\sin n}{(1 - \sin^2 \varphi)^{\frac{1}{2}}}$ which is the smallest value in the time of retardation in the time of moon's rise if n is also smallest then.

If at this time, the sun be at ≏, the moon will be full. Hence the full moon at autumnal equinox for some nights rises at nearly the same time. This is called the *harvest moon*.

CHAPTER V.

FIRST POINT OF ARIES AND OBLIQUITY OF THE ECLIPTIC.

44. The setting of the sidereal clock.

The sidereal time at any instant is the hour angle of the first point of Aries and it is only after adjusting the clock to show this that we can directly get the R. A. of stars from the sidereal times of their transits. Now it is possible by observations of the sun separated by an interval of about six months to set the astronomical clock so that it will at any time indicate the hour angle of ♈. This method is due to Flamsteed, the first Royal Astronomer of Great Britain and the principle of the method is briefly as follows :—

(i) *Flamsteed's method of determining the R. A. of a star by solar observations and thereby fixing the position of ♈.*

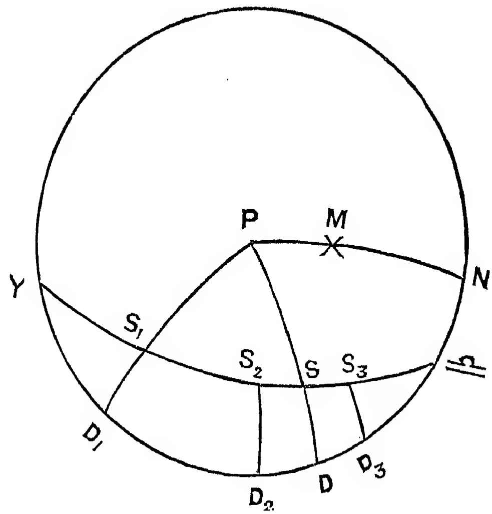


Fig. 40.

Let S_1 be the position of the sun on the ecliptic a few days after the vernal equinox and D_1 the foot of the

declination circle through S_1 so that $S_1 D_1$ is the declination of the sun at that position. Let S be the position of the sun before the autumnal equinox when it would have the same declination as at S_1.

Suppose that the sun is observed by the Transit Circle when it is at S_1 and the declination and the difference in the sidereal times of transit of the sun and a star M are noted. Let these quantities be d_1 and t_1. Now we have to get the difference in the times of transit of the sun and the same star when the sun is at the position S a few days before the autumnal equinox. But it may so happen that the sun may not transit when its declination is the same as on the first date of observation. But it would always be possible to find two consecutive days on which the declinations of the sun at noon are a little greater and smaller than d_1; Let these values be d_2 and d_3 and also the corresponding differences in the times of transit of the sun and the star M be t_2 and t_3.

We may assume that in the interval between S_2 and S_3, the sun's declination and R. A. both vary uniformly and therefore the change in declination is proportional to the corresponding change in R. A.

$$\therefore \frac{D_2 D}{D_2 D_3} = \frac{S_2 D_2 - S D}{S_2 D_2 - S_3 D_3} = \frac{d_2 - d_1}{d_2 - d_3}$$

$$\therefore D_2 D = \frac{d_2 - d_1}{d_2 - d_3} . D_2 D_3 = \frac{d_2 - d_1}{d_2 - d_3} (t_2 - t_3)$$

$$\text{Now } D N = D_2 N - D_2 D = t_2 - \frac{d_2 - d_1}{d_2 - d_3} (t_2 - t_3)$$

Also, since $S_1 D_1 = SD$, $\Upsilon D_1 = D \simeq$

$\therefore$ If O is the midpoint of the semicircle $\Upsilon \simeq$ we get $D_1 N + D N = 2 \text{ O N}$.

$$\therefore t_1 + t_2 - \frac{d_2 - d_1}{d_2 - d_3} . (t_2 - t_3) = 2 \text{ O N}$$

$$\therefore \text{O N} = \tfrac{1}{2}\left(t_1+t_2-\frac{d_2-d_1}{d_2-d_3}.(t_2-t_3)\right)$$

The R. A. of the star M $= \Upsilon\text{N} = \Upsilon\text{O} + \text{ON}$

$$= 6 \text{ hrs} + \tfrac{1}{2}\left(t_1 + t_2 - \frac{d_2 - d_1}{d_2 - d_3}.(t_2 - t_3)\right)$$

Here the R. A. of the star is determined in terms of the declination of the sun on the three days of observation. The Transit Circle gives directly the zenith distances of the sun on these days and so the R. A. of the star can also be given in terms of these.

If l is the latitude of the place and z_1, z_2, z_3 are the zenith distances of the sun on these dates, we have

$z_1 = l - d_1$; $z_2 = l - d_2$ and $z_3 = l - d_3$ (It is assumed that the sun's decl. is north and it transits south of the zenith)

$$\therefore \text{The star's R. A.} = 6 \text{ hrs} + \tfrac{1}{2}\left(t_1+t_2 - \frac{z_1-z_2}{z_3-z_2}(t_2-t_3)\right)$$

It would be seen from the chapter on Precession that Υ the origin from which R. A. of stars is measured is not a fixed point and it has a very small retrograde motion of 50″. 26 along the ecliptic. The effect of this is to change the R. A. and decl. of the stars. Therefore in such an observation as the one considered above, the R. A. of the star cannot strictly be the same during the two observations.

What we get as the R. A. of the star finally, is the R. A. of the star at the time midway between the two observations. Otherwise we are fixing the position of the first point of Aries at the time midway between the two observations and if we know its motion due to Precession, its actual position at any other time can be fixed. The setting of the Astronomical Clock consists in making it indicate this calculated R. A. of the star when that star

transits. Using the clock so set up and knowing its rate and error, it is possible to determine the R. A. of all other stars whose transits can be observed.

(ii) There is another method of determining ♈ by observing the sun and a star just before and after vernal equinox. At the time of vernal equinox the declination of the sun changes from a negative to a positive value. Let the sun's declinations observed on the noons previous and succeeding to the vernal equinox be $-\delta_1$ and $+\delta_2$ Let the sidereal hour intervals on these two days between the transit of the sun and a star be t_1 and t_2

Now the sun's increase in declination in one day is $\delta_1 + \delta_2$ and the sun's increase in R. A. is. $t_1 - t_2$, as the star has practically the same R. A. during the two observations. We can assume these two quantities to change uniformly during the interval of observation.

∴ When the sun is at ♈ i. e. when its declination is zero the increase in R. A. should be $\dfrac{\delta_1 (t_1 - t_2)}{\delta_1 + \delta_2}$.

This increase in R. A. of the sun brings it to ♈.

∴ The R. A. of the star = its distance from ♈ along the equator $= t_1 - \dfrac{\delta_1 (t_1 - t_2)}{\delta_1 + \delta_2}$.

When this star transits next time, set the sidereal clock to indicate the corresponding time and start it.

In this method the correction due to precession does not enter, as the observations are not separated by any long interval.

(iii) A third method of determining the fundamental R. A. of a star, that of the sun and the obliquity of the

ecliptic is based upon two observations of the sun's declination at any two transits.

Let α be the unknown R. A. of the star. Let the declinations of the sun on two noons be observed to be δ_1 and δ_2 and let the corresponding transits of the sun on these days precede that of the star by sidereal times t_1 and t_2 so that the R. A. of the sun on these two dates are $\alpha - t_1$ and $\alpha - t_2$

If ω be the obliquity of the ecliptic, we have

$$\tan \omega = \frac{\tan \delta_1}{\sin (\alpha - t_1)} \quad (1)$$

or $\sin (\alpha - t_1) = \tan \delta_1 \cot \omega$.

Similarly $\sin (\alpha - t_2) = \tan \delta_2 \cot \omega$.

$$\frac{\sin (\alpha - t_1 + t_1 - t_2)}{\sin (\alpha - t_1)} = \frac{\tan \delta_2}{\tan \delta_1}.$$

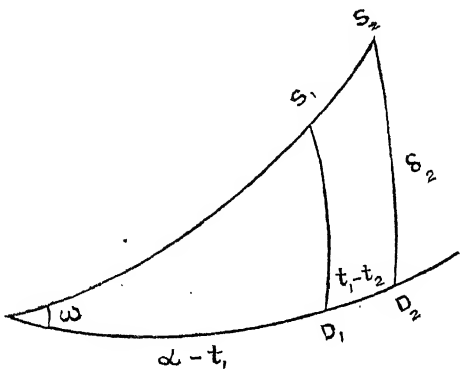


Fig. 41.

i. e. $\cos (t_1 - t_2) + \sin (t_1 - t_2) \cot (\alpha - t_1) = \dfrac{\tan \delta_2}{\tan \delta_1}$

i. e. $\cot (\alpha - t_1) = \tan \delta_2 \cot \delta_1 \operatorname{cosec} (t_1 - t_2) - \cot (t_1 - t_2)$, which gives the R. A. of the sun at the time of first observation. From this, α, the R. A. of the star is also known. After getting the R. A. of the sun on the first day of observation, the obliquity is calculated from (1).

45. The effect on the R. A. of the sun, of a small error in the assumed value of (i) obliquity or (ii) declination.

(i) How far an error in obliquity affects the R. A. of the sun can be seen by differentiating the formula.

$\tan \omega = \dfrac{\tan \delta}{\sin \alpha}$ which is the one used in getting the R. A. of the sun.

$$\sec^2 \omega.\ d\omega = -\frac{\tan \delta}{\sin^2 \alpha} \cos \alpha.\ d\alpha.$$

$$\text{or } d\alpha = -\frac{\sec^2.\ \omega.\ \sin^2 \alpha}{\tan \delta \cos \alpha} \cdot d\omega$$

$$= -\frac{\tan \alpha.\ d\omega}{\sin \omega \cos \omega} = -\frac{2 \tan \alpha}{\sin 2\omega} \cdot d\omega.$$

$\therefore d\alpha$, the error in α is greater, the greater the distance of the sun from the equinox.

$\therefore$ If $d\omega$ is to have the smallest effect we should observe the sun as near the equinox as possible. It is easy to show that in order that the R. A. should be correct to 0·1 sec. the sun should be observed within 20 degrees on either side of the equinox assuming an error of 1″ in ω.

(ii) We shall next consider the effect of an error in the observed declination on the R. A. of the sun.

Differentiating the equation

$\sin \alpha = \cot \omega \tan \delta$, and keeping ω constant,

we obtain $\cos \alpha.\ d\alpha = \cot \omega \sec^2 \delta.\ d\delta$

$\therefore \delta\alpha = \sec \alpha\ (1 + \tan^2 \delta) \cot \omega.\ d\delta.$

$= \sec \alpha\ (1 + \sin^2 \alpha \tan^2 \omega) \cot \omega.\ d\delta.$

Here also the error in α will be least if the sun is observed nearest the equinox.

46. Other methods of determining the obliquity.

By meridian observations of the sun at the solstices or very near the solstices

If the meridian zenith distances of the sun at the solstices be z_1 and z_2 then $\omega = \frac{1}{2}(z_2 - z_1)$ and the latitude of the place is given by $\varphi = \frac{1}{2}(z_1 + z_2)$.

But it must be noted that the sun is not likely to be exactly 90° from ♈ when it crosses the meridian; when it crosses the meridian it may have R. A. either $90+\alpha$ or $90-\alpha$ where α is a small quantity. δ may then be equal to $\omega - x$ where x is small. Therefore from the meridian observations of the sun near the solstice, the obliquity can be determined as follows :—

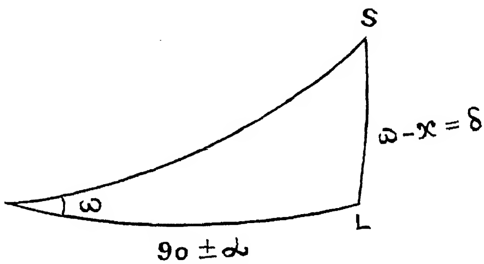


Fig. 42.

$$\tan \omega = \frac{\tan \delta}{\sin(90 \pm \alpha)}$$

$$= \frac{\tan \delta}{\cos \alpha}$$

$$\text{i. e. } \cos \alpha = \frac{\tan \delta}{\tan \omega} : \frac{\tan \delta}{\tan(\delta + x)}$$

$$\frac{1 - \cos \alpha}{1 + \cos \alpha} = \frac{\tan(\delta + x) - \tan \delta}{\tan(\delta + x) + \tan \delta}$$

$$\tan^2 \frac{\alpha}{2} = \frac{\sin x}{\sin(2\delta + x)}$$

$$\sin x \qquad \tan^2 \frac{\alpha}{} \qquad \sin(x + 2\delta)$$

$$x = \tan^2 \frac{\alpha}{2}. \quad \sin 2\delta + \frac{1}{2} \tan^4 \frac{\alpha}{2} \sin 4\delta + .$$

Note. The above is based upon the following result in Trigonometry.

If $\sin x = n \sin (x + \alpha)$

$$x = n \sin \alpha + \frac{n^2}{2} \sin 2\alpha + \frac{n^3}{3} \sin 3\alpha + \ldots\ldots$$

Proof:—

$$\sin x = n \sin (x + \alpha)$$
$$= n (\sin \alpha \cos x + \cos \alpha \sin x)$$

$$\therefore \sin x (1 - n \cos \alpha) = n \sin \alpha \cos x$$

$$\therefore \tan x = \frac{n \sin \alpha}{1 - n \cos \alpha} \qquad \frac{e^{xi} - e^{-xi}}{\left(e^{xi} + e^{-xi}\right) i}$$

$$\therefore \frac{e^{xi} - e^{-xi}}{e^{xi} + e^{-xi}} \qquad \frac{n i \sin \alpha}{1 - n \cos \alpha}$$

$$\frac{e^{xi}}{e^{-xi}} = \frac{1 - n \cos \alpha + n i \sin \alpha}{1 - n \cos \alpha - n i \sin \alpha} = \frac{1 - n e^{-\alpha i}}{1 - n e^{\alpha i}}$$

$$\therefore 2 x i = \log \left(1 - n e^{-\alpha i}\right) - \log \left(1 - n e^{\alpha i}\right)$$

$$= n \left(e^{\alpha i} - e^{-\alpha i}\right) + \tfrac{1}{2} n^2 \left(e^{2\alpha i} - e^{-2\alpha i}\right)$$
$$+ \tfrac{1}{3} n^3 \left(e^{3\alpha i} - e^{-3\alpha i}\right) + .$$

$$\therefore x = n \sin \alpha + \tfrac{1}{2} n^2 \sin 2\alpha + \tfrac{1}{3} n^3 \sin 3\alpha + \ldots\ldots\ldots$$

If φ be the latitude of the place and z the observed zenith distance of the sun,

$$\delta = \varphi - z \text{ and}$$
$$\omega = \delta + x$$
$$= \varphi - z + x,$$ known from observation of meridian zenith distance of the sun near one solstice only.

If there is any error in the value of φ it will be affecting δ ($\because \delta = \varphi - z$) and through it ω. But the value of x will not be affected so much.

$\therefore z - x$ may be supposed to be known accurately.

Now $z - x = \varphi - \omega$ .... (1).

$\therefore z + (\omega - x) = z + \delta = \varphi$

Another observation near the 2nd solstice gives $z' - x' = \varphi + \omega$ ...... (2)

From (1) and (2),

$(z' - x') - (z - x) = 2\,\omega$.

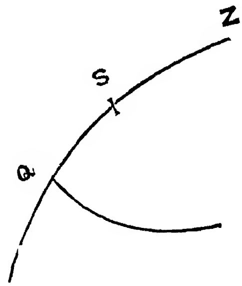


Fig. 43.

47. Transformation of co-ordinates referred to equator and ecliptic.

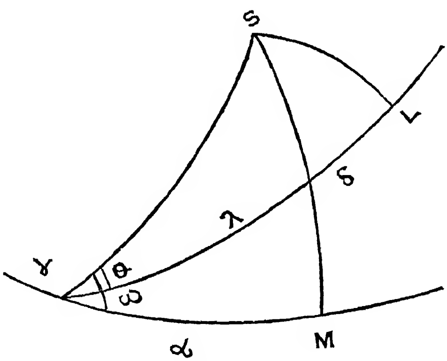


Fig. 44.

Given the R. A. and declination of S, to get λ the longitude and β the latitude of the body, ω being a known quantity.

$\cos \Upsilon S = \cos \delta . \cos \alpha$ $\qquad \therefore \Upsilon S$ is known.

$$\tan(\omega + \theta) = \frac{\tan \delta}{\sin \alpha}$$

$\therefore \omega + \theta$ is known $\qquad \therefore \theta$ is known.

Then $\sin \theta = \dfrac{\sin \beta}{\sin \Upsilon S}$ gives β

$$\cos\theta = \frac{\tan\lambda}{\tan \Upsilon S} \text{ gives } \lambda$$

Similarly if λ and β are given, $\cos \Upsilon S = \cos\lambda \cos\beta$ and this gives ΥS

$$\tan\theta = \frac{\tan\beta}{\sin\lambda} \text{ gives } \theta$$

Then, $\sin(\theta+\omega) = \dfrac{\sin\delta}{\sin \Upsilon S}$ gives δ and

$$\cos(\theta+\omega) = \frac{\tan\alpha}{\tan \Upsilon S} \text{ gives } \alpha$$

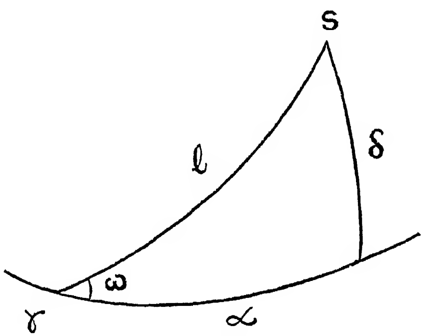


Fig. 45.

Since the sun is always on the ecliptic, the longitude of the sun at any time is l, and we have the following relations between l, ω, δ and α.

1. $\cos l = \cos\alpha \cos\delta$, connecting right ascension and declination of the sun with longitude.

2. $\dfrac{\sin\delta}{\sin l} = \sin\omega$, connecting declination and longitude.

3. $\tan\omega = \dfrac{\tan\delta}{\sin\alpha}$ connecting right ascension and declination of the sun.

4. $\cos\omega = \dfrac{\tan\alpha}{\tan l}$ connecting right ascension and longitude of the sun.

From (4) by differentiation,

$$\cos\omega \sec^2 l \frac{dl}{dt} = \sec^2 \alpha . \frac{d\alpha}{dt}$$

$$\text{or } \frac{d\alpha}{dt} \Big/ \frac{dl}{dt} = \frac{\cos\omega \cos^2\alpha}{\cos^2 l} = \frac{\cos\omega \cos^2\alpha}{\cos^2\alpha \cos^2\delta}$$

$$= \frac{\cos\omega}{\cos^2\delta}$$ which gives the ratio of the rates of changes of right ascension and longitude.

48. Position of the ecliptic at a given instant.

The *ascending point* (O) is the point of the ecliptic where it crosses the eastern horizon. The point where it crosses the western horizon is called the *descending point. The culminating point* (C) is the point where it cuts the meridian. The *nonagesimal* is the point (N) of the ecliptic 90° from the ascending point. This is the highest point of the ecliptic. To fix the position of the ecliptic at any instant, we have to get the ascending point or the culminating point and the nonagesimal.

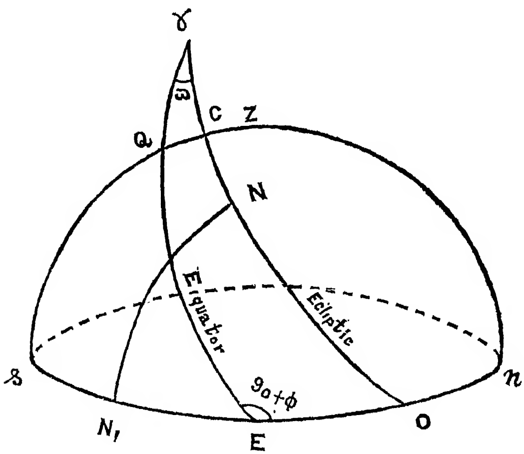


Fig. 46.

In the figure $\Upsilon Q = T$ (15 × sidereal time at any instant)

$\Upsilon E = 90° + T$ and

$\Upsilon EO = (90° + \varphi)$

$\therefore \cos E. \cos \Upsilon E = \sin \Upsilon E \cot EO - \sin E \cot \omega$

$\cos T \cot EO = \sin \varphi \sin T + \cos \varphi \cot \omega$

$\cot EO = \sin \varphi \tan T + \cos \varphi \cot \omega \sec T$, which gives EO.

Also $\dfrac{\sin O}{\cos T} = \dfrac{\sin \omega}{\sin EO}$. This gives O or NN′ the altitude of the nonagesimal.

Longitude of (N), the nonagesimal.

$\Upsilon N = \Upsilon O - 90°$. From $\Delta \Upsilon EO$,

$\cos \Upsilon E \cos \omega = \sin \Upsilon E \cot \Upsilon O - \sin \omega \cot E$

i. e. $-\sin T \cos \omega = -\cos T \tan \Upsilon N + \sin \omega \tan \varphi$

$\therefore \tan \Upsilon N = \tan T \cos \omega + \sin \omega \tan \varphi \sec T$.

Another method of getting ΥN is from

$\cos \Upsilon O = \cos \Upsilon E \cos EO + \sin \Upsilon E \sin EO \cos E$.

If the culminating point C is wanted, we have $\tan \omega \quad \dfrac{\tan QC}{\sin T}$ from which QC can be known.

$ZC = \varphi - QC =$ zenith distance of the culminating point.

CHAPTER VI.

KEPLER'S LAWS.

49. First and second laws of Kepler.

The apparent orbit of the sun round the earth or that of the earth round the sun lies in one plane and Kepler proved by observation that the earth's orbit round the sun is an ellipse at whose focus, the sun is situated. This is the first law of Kepler in planetary motion. Kepler also showed by observation that the radius joining the earth and the sun describes equal areas in equal intervals of time. This is the second law of Kepler. Both these laws could be deduced, theoretically, as was done by Newton later, from the law of gravitation which governs the motions of bodies in the universe.

50. To deduce Kepler's first law from observation.

The sun's disc appears largest on 31st December and smallest on July 1st, showing that the sun is nearest the earth on the former date and farthest from the earth on the latter date. Since the sun's distance is inversely pro-

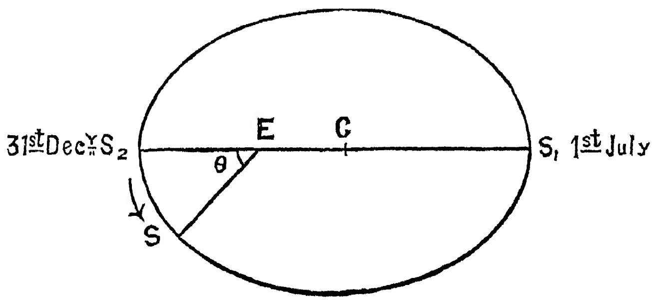


Fig. 47.

portional to the observed angular diameter of the disc, the ratio of the sun's distances from the earth on these dates could be known. Let these distances be represented by ES_2 and ES_1. If C is the middle point of $S_1 S_2$, and if $\frac{EC}{CS_1} = e$, e is found to be equal to $\cdot 0167$.

Next observe the sun's angular diameter on any other date when the longitude is θ° more than that on 31st December. The angular diameter on that date will be found to be proportional to $(1 + e \cos \theta)$ where $e = \cdot 0167$, or if r is the sun's distance on this date, $\frac{1}{r}$ is proportional to $(1 + e \cos \theta)$. Now it is known that a curve for which $\frac{1}{r}$ is proportional to $(1 + e \cos \theta)$, is an ellipse, if e is less than unity.

Another method of verifying Kepler's first law.

By observations with the transit circle the sun's R. A. and declination can be determined every noon, and from these the sun's longitude can be calculated for every day, using the formula, $\cos l = \cos \alpha \cos \delta$. This only gives us the direction of the sun in space. It is not possible to determine the actual distances of the sun by this method, but quantities proportional to the distances of the sun are known from the measure of the angular diameters of the solar disc on different days of the year. This is done by any form of micrometer attached to a telescope or by means of the heliometer (*See Chapter on Astronomical Instruments*). Since the distance of the sun is inversely proportional to the angular diameter, it is now possible to construct the apparent orbit of the sun, by taking along the different directions in which the sun is situated, lengths proportional to $\frac{1}{\text{angular diameter}}$, and then drawing the curve. When this is done, it will be found to be an ellipse with the earth at one of its foci.

51. Verification of the second law of Kepler.

It is found by observation that the daily increase in the longitude of the sun $\left(\frac{dl}{dt}\right)$ is directly proportional to

the square of the angular diameter and consequently inversely proportional to the square of the sun's distance. Therefore it is possible to infer that the radius vector to the sun describes equal areas in equal times, for, $\frac{d\,l}{d\,t} = \frac{k}{r^2}$ or $r^2 \frac{d\,l}{d\,t} = k$.

If r and $r + d\,r$ be the sun's distances from the earth on two days, and if the increase in sun's longitude be $\frac{d\,l}{d\,t}$, the area swept by the radius vector daily is given by $\frac{1}{2}\, r\,(r + d\,r) \sin \frac{d\,l}{d\,t}$ and it is equal to $\frac{1}{2}\, r^2 \frac{d\,l}{d\,t}$. This is a constant, since $r^2 \frac{d\,l}{d\,t} = k$.

Now the apparent motion of the sun described above is due to the motion of the earth round the sun in an opposite direction, so that it is definitely known that the earth is moving in an ellipse with the sun at one of its foci. It is also known that the other planets of the solar system move round the sun in elliptic orbits obeying the laws of Kepler.

52. Geocentric and Heliocentric Latitudes and Longitudes.

When we were considering the apparent annual motion of the sun, we have been taking the directions in space as seen from the earth. But now that we know that it is the earth which moves round the sun, as the other celestial bodies in the solar system also do, it is convenient to define the position of the bodies, as they would be seen from the sun. These directions are called *heliocentric directions*. The *heliocentric longitude and latitude* of a celestial body are the longitude and latitude measured in the usual way, where reference is made to the body relative to the sun as origin. The *geocentric*

longitude and *latitude* are the corresponding quantities measured from the earth as origin. In giving the co-ordinates of stars there is little difference between their

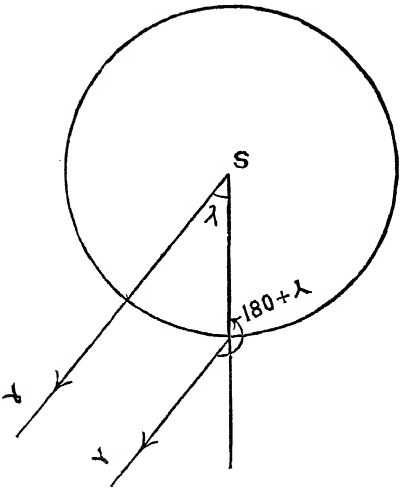


Fig. 48.

geocentric and heliocentric co-ordinates. This is due to their enormous distances from the earth. There are however slight differences in these two sets of co-ordinates in the case of some of the nearest stars; but these are negligible. This is not the case for the moon and the planets which are comparatively near bodies. One thing to be noted in this connection is that the sun is seen from the earth in the direction opposite to that in which the earth is seen from the sun. Therefore the heliocentric longitude of the earth differs from the geocentric longitude of the sun by 180°.

53. The eccentricity of the elliptic orbit described by the earth round the sun.

This can be found by observations of the greatest and the least angular diameters of the sun's disc. On these occasions, the earth is at the least and the greatest distance respectively from the sun. The corresponding dates are December 31st, and July 1st when the earth is said to be

at *Perihelion* and at *Aphelion* respectively. When reference is made to the sun as moving body, the corresponding positions of the sun are called *Perigee* and *Apogee*. If e is the eccentricity of the ellipse described by the earth, we have from the property of the ellipse,

$$\frac{\text{Distance of the earth at aphelion}}{\text{Distance of the earth at perihelion}} = \frac{1+e}{1-e}$$

$$= \frac{\text{Greatest angular diameter of the sun}}{\text{Least angular diameter of the sun.}}$$

$$\frac{32' - 36''}{31' - \cdot 32''} \text{ (by observations)}$$

$$\frac{489}{473}$$

$$\therefore e = \frac{8}{481} \quad \frac{1}{60} \text{ nearly} \left(\begin{array}{c} \text{correct value of } e \\ = \cdot 0167 \end{array} \right)$$

The line joining the perigee and apogee is called the *Apse Line* and this is the same as the major axis of the elliptic orbit. If the direction of this line is known, the orientation of the elliptic orbit is known. The longitude of perigee is given by the arithmetic mean of the longitudes of the sun when the angular diameters are equal before and after perigee. This method gives a more accurate value than the one which is directly calculated from the time when the sun's angular diameter is greatest, for, it is very difficult to find the exact time when the sun's angular diameter is maximum or minimum, as it remains apparently the same for some time near the maximum or minimum value. From the longitude of perigee, that of apogee is obtained by adding 180°. Therefore the direction of the apse line of the earth's orbit is known.

54. Forward motion of the apse line.

The determination of the longitude of perigee for consecutive years shows that the apse line is not fixed in

direction, but that it has a forward motion of about 11″·25 a year. This would mean that the elliptic orbit described by the earth round the sun does not get closed annually, but that it is going on continuously describing different ellipses with their apse lines inclined to one another at an angle of 11″·25 in a progressive manner.

55. Kepler's laws and Newton's deductions therefrom.

From Kepler's laws, Newton was led to the discovery of his law of gravitation. He showed that from the second law, it is possible to prove that there is a central force exerted by the sun on a planet.

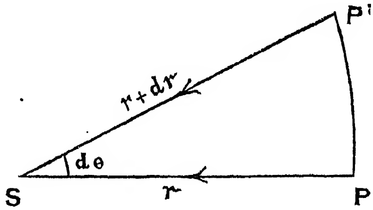


Fig. 49.

The second law of Kepler gives $r^2 \dfrac{d\theta}{dt} = h =$ constant.

Since $\frac{1}{2} r (r + dr) \sin d\theta =$ area described by radius vector in time dt, $\dfrac{\left(d\, r^2 . \dfrac{d\theta}{dt}\right)}{dt} = 0$. ·(1)

Now, if a body P describes a curve round S under the above law, then its acceleration perpendicular to SP is $\dfrac{1}{r} \dfrac{d}{dt}\left(r^2 \dfrac{d\theta}{dt}\right)$ and this is equal to zero by (1). Hence, there is no acceleration for the body in the direction perpendicular to SP at any time and therefore no force also in that direction. Therefore the whole force exerted on P is in the direction of PS and the acceleration is central only.

From Kepler's first law and the previous result Newton showed that the force exerted by the sun on the planet at any time is inversely proportional to the square of the distance of the planet from the sun.

When a body moves in an ellipse under a central force, we have the following equations :—

The polar equation to the ellipse is given by

$$r = \frac{l}{1 + e\cos\theta} \quad \text{.....(1) where } l \text{ is the semi-latus rectum.}$$

$$\frac{d^2 r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = -F = \text{acceleration away from the sun....(2)}$$

$$r^2 \frac{d\theta}{dt} = h \text{ ....(3)}$$

Differentiating (1), we have,

$$\frac{dr}{dt} = \frac{l\,e\sin\theta}{(1 + e\cos\theta)^2} \cdot \frac{d\theta}{dt} = \frac{e\sin\theta}{l} \cdot r^2 \frac{d\theta}{dt} = \frac{e\sin\theta}{l} \cdot h$$

$$\frac{d^2 r}{dt^2} = \frac{h\,e\cos\theta}{l}\,\frac{d\theta}{dt} = \frac{h\,e\cos\theta}{l}\,\frac{h}{r^2}$$

$$\frac{d^2 r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = \frac{h^2}{r^2}\left(\frac{e\cos\theta}{l} - \frac{1}{r}\right)$$

$$= \frac{h^2}{r^2}\left(\frac{e\cos\theta}{l} - \frac{1 + e\cos\theta}{l}\right) = -\frac{h^2}{l\,r^2}$$

$$F \quad \frac{h^2}{l\,r^2} = \frac{\mu}{r^2} \text{ where } \mu = \frac{h^2}{l}$$

∴ The acceleration and hence the force exerted on the planet is proportional to $\frac{1}{r^2}$. It was Kepler's third law which enabled Newton to show that the constant μ is the same for all planets.

Now $h = r^2 \frac{d\theta}{dt}$ = twice the area described in unit time $= \frac{2\pi ab}{T}$ where a and b are the semi-axes of

the ellipse and T is the periodic time of describing it.

$$\mu = \frac{h^2}{l} = \frac{4\,\pi^2\,a^2\,b^2}{\mathrm{T}^2\,l} = \frac{4\,\pi^2}{\mathrm{T}^2}\,a^3,\text{ since } \frac{b^2}{l} = a$$

But Kepler's third law establishes the fact that the sidereal period and the mean distance of a planet from the sun, are such that $\frac{a^3}{\mathrm{T}^2}$ is the same for all planets (say k.)

Then $\mu = 4\,\pi^2\,k$ = constant throughout the solar system.

Assuming Newton's law of gravitation to hold good between the sun and another body, it is also possible to show that the latter would always describe round the former an ellipse, parabola or hyperbola depending upon certain initial conditions, and in the case of an elliptic orbit the periodic time is proportional to $a^{\frac{3}{2}}$ where a is the semi-major axis of the orbit.

56. To find the actual position of a planet in its elliptic orbit round the sun at any time after perihelion passage.

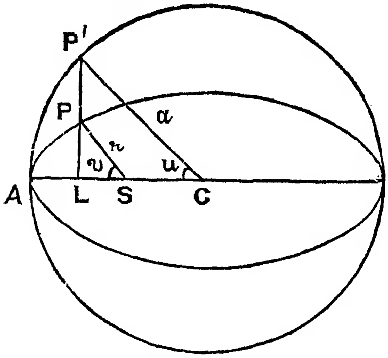


Fig. 50.

Let P be a planet moving round S in an elliptic orbit whose semi-major axis and eccentricity are a and e and

whose centre is C. Let the time for describing the elliptic arc AP be t. Let the auxiliary circle AP′ be drawn, corresponding to the elliptic orbit. Let the angle ASP, called the *true anomaly* be v and angle ACP′, called the *eccentric anomaly* be u.

The problem before us is to find values for SP (r) and v in terms of t; but it is not possible to get these values in finite terms. The usual procedure adopted is to get a series for these, after expressing them in terms of the eccentric anomaly.

(i) *r and v in terms of u.*

From the properties of the ellipse, we have,

$$r \cos v = a \cos u - ae \ldots\ldots\ldots\ldots\ldots(1) \text{ and}$$

$$r \sin v = \frac{b}{a} \cdot a \sin u = b \sin u \ldots\ldots(2)$$

$$\therefore\ r = a\,(1 - e \cos u) \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

From (1) and (3)

$$r\,(1 - \cos v) = a\,(1 + e)\,(1 - \cos u) \text{ and}$$

$$r\,(1 + \cos v) = a\,(1 - e)\,(1 + \cos u)$$

$$\therefore\ \tan\frac{v}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{u}{2} \ldots\ldots\ldots\ldots(4)$$

(ii) *u in terms of t.*

It is also possible to express t in terms of u. Let n be the circular measure of the mean angular velocity of the radius vector from the sun to a planet, so that if T be the periodic time of the planet,

$$n = \frac{2\pi}{T}.$$

A point moving round S with constant angular velocity n would describe an angle nt called the *mean anomaly* and represented by m.

$$\frac{\text{area of ASP}}{\text{area of ellipse}} = \frac{t}{T} \ldots\ldots\ldots(5) \text{ (By Kepler's 2nd law).}$$

But $\dfrac{\text{area of ASP}}{\text{area of ASP}'} = \dfrac{b}{a}$, by projection

Now $\quad \text{area ASP}' = \frac{1}{2}(a^2 u - a^2 e \sin u)$

$$= \tfrac{1}{2} a^2 (u - e \sin u)$$

$$\frac{\text{Area ASP}}{\text{Area of ellipse}} = \frac{b}{a} \cdot \frac{a^2}{2} \frac{(u - e\sin u)}{\pi ab.}$$

$$= \frac{u - e \sin u}{2\pi} = \frac{t}{\text{T}} \text{ from (5).}$$

$\therefore \; nt = m = u - e \sin u$............(6) (since $n\text{T} = 2\pi$).

Equation (6) gives the expression for t in terms of u.

The next procedure is to get from (6) u in terms of m by the method of approximation.

First putting $u = m$, in $e \sin u$

we get $u = m + e \sin m$..........(7) [First approximation for u]

Secondly, putting this value of u in equation (6) we get omitting terms containing e^3 and higher powers of e

$u = m + e \sin (m + e \sin m)$

$= m + e \sin m + \dfrac{e^2}{2} \sin 2m$...(8) [2nd approximation for u]

$$\therefore \sin u = \sin m. \cos\left\{e \sin m + \frac{e^2}{2} \sin 2m\right\}$$

$$+ \cos m \sin\left\{e \sin m + \frac{e^2}{2} \sin 2m\right\}.$$

$$= \sin m\left(1 - \frac{e^2}{2} \sin^2 m\right) + \cos m\left(e \sin m + \frac{e^2}{2} \sin 2m\right)$$

$$= \sin m - \frac{e^2}{2} \sin^3 m + \frac{e}{2} \sin 2m + e^2 \sin m. \cos^2 m.$$

$$= (1 + e^2) \sin m + \frac{e}{2} \sin 2m - \frac{3}{2} e^2 \sin^3 m$$

$$= (1 + e^2) \sin m + \frac{e}{2} \sin 2m - \frac{3e^2}{8}(3 \sin m - \sin 3m)$$

$$=\left\{1-\frac{e^2}{8}\right\}\sin m+\frac{e}{2}\sin 2m+\frac{3}{8}e^2\sin 3m\ldots\ldots\ldots\ (9)$$

Substituting this in (6), we get as a third approximation,

$$u=m+\left\{e-\frac{e^3}{8}\right\}\sin m+\frac{e^2}{2}\sin 2m$$

$$+\frac{3}{8}e^3\sin 3m\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

(iii) *v in terms of m.*

In order to get v in terms of m we have to eliminate u between (4) & (6) or (4) & (10)

To do this, we have to express v in terms of u

Putting $e=\sin\varphi$ in (4),

$$\tan\frac{v}{2}=\frac{1+\tan\frac{\varphi}{2}}{1-\tan\frac{\varphi}{2}}\cdot\tan\frac{u}{2}$$

Writing their exponential values for v and u, we have,

$$e^{iv}=\frac{e^{iu}\left\{1-\tan\frac{\varphi}{2}\cdot e^{-iu}\right\}}{\left\{1-\tan\frac{\varphi}{2}\cdot e^{iu}\right\}}$$

$$\therefore\quad iv=iu+\log\left\{1-\tan\frac{\varphi}{2}e^{-iu}\right\}-\log\left\{1-\tan\frac{\varphi}{2}e^{iu}\right\}$$

$$=iu+\tan\frac{\varphi}{2}\left\{e^{iu}-e^{-iu}\right\}+\tfrac{1}{2}\tan^2\frac{\varphi}{2}\left\{e^{2iu}-e^{-2iu}\right\}$$

$$\therefore\quad v=u+2\left\{\sin u\tan\frac{\varphi}{2}+\frac{1}{2}\tan^2\frac{\varphi}{2}\sin 2u+\ldots\ldots\ldots\ldots\right\}$$

$$\text{Now, }\tan\frac{\varphi}{2}=\frac{1-\sqrt{1-e^2}}{e}=\frac{e}{2}+\frac{e^3}{8}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\text{Since } e=\sin\varphi=\frac{2\tan\frac{\varphi}{2}}{1+\tan^2\frac{\varphi}{2}}$$

$$\therefore \quad v=u+2\sin u\left\{\frac{e}{2}+\frac{e^3}{8}\right\}+2\cdot\frac{e^2}{4}\sin 2u$$

$$+\,2.\ \frac{e^3}{8}\cdot\frac{\sin 3u}{3}\ \ldots\ldots\ldots$$

$$=u+\left\{e+\frac{e^3}{4}\right\}\sin u+\frac{e^2}{4}\sin 2u+\frac{e^3}{12}\sin 3u \ldots \quad (11)$$

(correct to the 3rd power of e)

Now to express v in terms of m, in equation (11) substitute for u from (10), for $e\sin u$ from (9), for $\frac{e^2}{4}\sin 2u$ from (7) and put $u=m$ in $\frac{e^3}{12}\sin 3u$ and $\frac{e^3}{4}\sin u$.

$$\therefore \quad v=m+(2e-\tfrac{1}{4}e^3)\sin m+\frac{5}{4}e^2\sin 2m+\frac{13}{12}e^3\sin 3m \quad (12)$$

Equation (12) gives the true anomaly in terms of the mean anomaly, correct up to the 3rd power of e. The third power of e is too small to require consideration in ordinary calculations and may therefore be neglected. The expression then becomes

$v=m+2e\sin m+\frac{5}{4}e^2\sin 2m$, which can also be directly got from $v=u+e\sin u+\frac{e^2}{4}\sin 2u$.

$$\because \quad v=m+e\sin m+\frac{e^2}{2}\sin 2m$$

$$+e\sin(m+e\sin m)+\frac{e^2}{4}\sin 2m$$

$$=m+e\sin m+\frac{3}{4}e^2\sin 2m+e\sin m+\frac{e^2}{2}\sin 2m$$

$$=m+2e\sin m+\frac{5}{4}e^2\sin 2m.$$

Alternative Method.

There is an easy method of getting the value of the true anomaly in terms of the mean anomaly without using the eccentric anomaly at all. Here, we first get the mean anomaly in terms of the true anomaly and then derive the required solution by reversal of the series as follows:—

If dm be the increase in the mean anomaly in time dt we have $\frac{dm}{2\pi} = \frac{dt}{T} = \frac{\frac{1}{2}r^2\,dv.}{\pi\;ab.}$

$$\therefore \quad \frac{dm}{dv} \quad ab - \frac{l^2}{ab\,(1+e\cos v)^2}$$

$$= \frac{(1-e^2)}{(1+e\cos v)^2}$$

$$\therefore \quad m = (1-e^2)^{3/2}\int_0^v (1-2e\cos v + 3e^2\cos^2 v - 4e^3\cos^3 v + \ldots\ldots\ldots\ldots)\,dv$$

$$= \left\{1-\frac{3}{2}e^2\ldots\right\}\left\{v - 2e\sin v + \frac{3}{2}e^2\left(v+\frac{\sin 2v}{2}\right) - e^3\left(\frac{1}{3}\sin 3v + 3\sin v\right)\ldots\ldots\ldots\ldots\right\}$$

$$= v - 2e\sin v + \frac{3}{4}e^2\sin 2v - \frac{e^3}{3}\sin 3v \ldots\ldots (A)$$

Since $m = nt$, (A) gives also the time required for a planet from perihelion to attain a given anomaly v. There is however a finite expression for t in terms of v whenever the orbit is any conic section. It is not necessary to derive it here (See Elementary Treatise on Dynamics of a Particle by Loney: Page 91).

Reversing the above series we get as a first approximation.

$$v = m \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$m = v - 2e\sin m$$

or $v = m + 2e\sin m \ldots\ldots\ldots\ldots (2)$ 2nd approximation,

Using this value of v in (A), we have

$$m=v-2e\sin(m+2e\sin m)+\frac{3}{4}e^2\sin 2m$$

$$=v-2e(\sin m+2e\sin m\cos m)+\frac{3}{4}e^2\sin 2m$$

$$=v-2e\sin m-\frac{5}{4}e^2\sin 2m.$$

$\therefore\ v=m+2e\sin m+\frac{5}{4}e^2\sin 2m$.........(3) 3rd approximation.

For all practical purposes, this would be sufficient. If the next higher approximation is required, substitute in (A), for $\sin v$ from (3), for $\sin 2v$ from (2) and for $\sin 3v$ from (1) and we get the value of v correct to the third power of e as follows :—

$$m=v-2e\sin\left(m+2e\sin m+\frac{5}{4}e^2\sin 2m\right)$$

$$+\frac{3}{4}e^2\sin(2m+4e\sin m)-\frac{e^3}{3}\sin 3m,$$ keeping terms up to e^3 only.

$$\therefore\quad v=m+2e\sin m\left(1-\frac{4e^2\sin^2 m}{2}\right)$$

$$+2e\cos m\left(2e\sin m+\frac{5}{4}e^2\sin 2m\right)$$

$$-\frac{3}{4}e^2\left(\sin 2m+4e\sin m\ \cos 2m\right)+\frac{e^3}{3}\sin 3m.$$

$$=m+2e\sin m-4e^3\sin^3 m+2e^2\sin 2m$$

$$+\frac{5}{4}e^3\left(\sin 3m+\sin m\right)-\frac{3}{4}e^2\sin 2m$$

$$-\frac{3}{2}e^3\left(\sin 3m-\sin m\right)+\frac{e^3}{3}\sin 3m.$$

$$=m+2e\ \sin m+\frac{5}{4}e^2\sin 2m+e^3\left\{\left(\sin 3m-3\sin m\right)\right.$$

$$+\frac{5}{4}\left(\sin.3m+\sin m\right)-\frac{3}{2}\left(\sin 3m-\sin m\right)$$

$$\left.+\frac{1}{3}\sin 3m\right\}$$

$$=m+2e\sin m+\frac{5}{4}e^2\sin 2m+\frac{13}{12}e^3\sin 3m-\tfrac{1}{4}e^3\sin m.$$

57. A graphical method to determine v, u, and r when m is given.

(*See Monthly Notices R. A. S. Vol. 66 Page 519*)

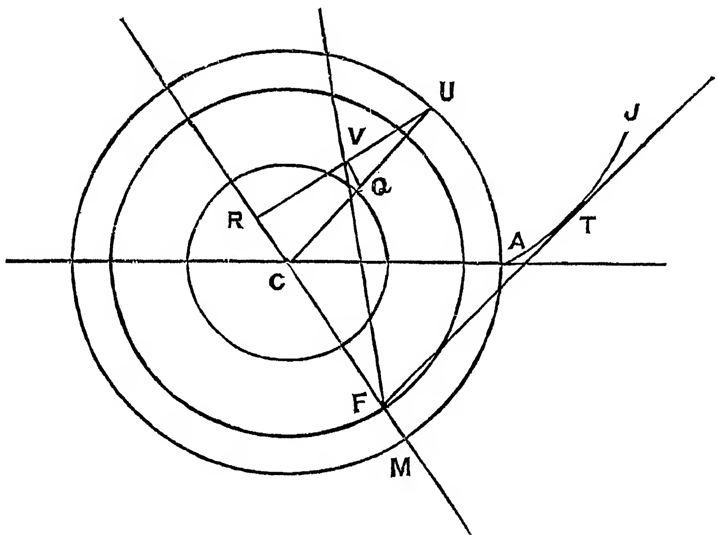


Fig. 51.

Three concentric circles are taken of radii b, ae and a, called the minor circle, focal circle, and major circle respectively. The involute to the major circle is drawn, starting from any point A (say).

[A circle is the locus of centres of curvature of the involute of the circle]

Let A C M be the given mean anomaly (m), and let C M cut the focal circle at F.

The normal at any point T to the involute of the major circle is a tangent to it, say at U. From F draw the tangent F T to the involute. Then CU is parallel to FT. Draw QV perpendicular to RU which is perpendicular to C F M.

$$\left.\begin{array}{l} \mathrm{UT} = ae \sin \mathrm{UCR}. \\ \quad = ae \sin \mathrm{FCU}. \end{array}\right\} \left\{\begin{array}{l}\text{Since the perpendicular from} \\ \text{C to FT is equal to UT.}\end{array}\right\}$$

But $\mathrm{UT} = \mathrm{AU} = a.\ \mathrm{ACU} = a\ (\mathrm{FCU} - \mathrm{FCA})$

If $\mathrm{FCU} = u$, we have

$$ae \sin u = a\ (u - m)$$

$$\therefore\ u = m + e \sin u$$

Join FV. Let $\mathrm{MFV} = v$ and $\mathrm{FV} = r$.

Then $r \cos v = \mathrm{FR} = \mathrm{CV} \cos u - \mathrm{CF} = a \cos u - ae$......(1)

and $r \sin v = \mathrm{RV} = b \sin u$..................................(2)

(1) and (2) clearly show that $v = \mathrm{MFV}$ and $r = \mathrm{FV}$.

Hence we get the eccentric anomaly, the true anomaly and the distance of the sun, from the mean anomaly and a, e and b which are known.

CHAPTER VII

TIME

58. Mean solar time.

In the previous chapters reference was made to two ways of reckoning time. One of these is the sidereal time and it is defined to be the hour angle of the first point of Aries. The other is called the apparent solar time and it is denoted by the hour angle of the sun. But both these measures of time are not suitable for business purposes in this world. If all people agree to adopt the sidereal time, there would be no relation between the hours giving such a time and the phenomenon of day and night, which are important factors in guiding man's activities. It would be seen that apparent noon or mid-day would occur approximately at 0 hr. on March 21st, at 6th hr. on June 21st, at 12th hr. on September 23rd and at 18th hr. on December 22nd. If we adopt the apparent solar time, there is no difficulty of the kind mentioned above, but such a time could not be kept by a clock of uniform rate; for, the apparent position of the sun depends both upon the rotation of the earth as well as the revolution of the earth round the sun. Now this latter motion of the earth is not uniform as the earth moves on an elliptic orbit round the sun. In fact we have already seen that the length of a sidereal day is constant, whereas the length of a solar day differs from that of the sidereal day by a quantity which is equal to the sun's daily motion in right ascension.

Now, with a view to get over the inherent disadvantages of these two times, *Mean Time* has been introduced. It can be indicated by a uniformly moving clock and it will never be far different from the apparent time given by the actual sun. It is taken to be the hour angle of the

mean sun which is only a point moving uniformly round the equator at such a rate that its R. A. at any time will not be far different from that of the true sun. Thus the mean time clock follows a point which moves with uniform change of R. A., and when this point passes the meridian at a place, the mean time clock should record the time of mean noon.

59. Equation of time.

As has been mentioned already, mean time would not be much different from the apparent time as indicated by the true sun. The amount of time which has to be added to the apparent time to get mean time is called the *equation of time.* The equation of time arises from the two causes which contribute to the sun's nonuniform motion. They are :—(1) The unequal motion of the sun on the ecliptic itself, and (2) the obliquity of the ecliptic. The equation of time could then be divided into two parts. The first part of it is contributed by what is called *the equation of the centre* and the second part by what is called *the reduction to the equator*.

60. Equation of the centre.

Let a uniformly moving point called the *dynamical mean sun* describe the orbit of the sun in the same time as the latter, coinciding with the sun at perigee and at apogee. Soon after perigee, the true sun moves much faster than the dynamical mean sun. Therefore the R. A. of the true sun is in excess of that of the dynamical mean sun, or in otherwords, the hour angle of the dynamical mean sun is greater than that of the true sun. Thus after perigee, the time given by the dynamical mean sun is greater than that given by the true sun. This excess is called the equation of the centre (E_1). E_1 is thus positive from the time when the true sun is at perigee (31st December) to the time when it is at apogee (1st July).

E_1 becomes negative during the remaining half of the year.

61. Analytical expression for the equation of the centre.

Let the dynamical mean sun have at any time the longitude L, which is the mean longitude of the true sun, and let K be the longitude of perigee. Then (L—K) is the angular distance of the dynamical mean sun from the apse line. This corresponds to the mean anomaly and from the expression for the true anomaly which we have already obtained in terms of the mean anomaly in Page 92, it could be seen that, if S be the true longitude of the sun

$$S=L+2\,e\sin(L-K)+\tfrac{5}{4}\,e^2\sin(2L-2K)\ldots\ldots\ldots\ldots(1)$$

$$\therefore\; S-L=2e\sin(L-K)+\tfrac{5}{4}\,e^2\sin(2L-2K)\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

which gives the equation of the centre.

62. The lengths of the seasons.

The sun's mean and true longitudes are connected by the equation,

$$L=S-2e\sin(S-K)+\tfrac{3}{4}\,e^2\sin(2S-2K)$$

(neglecting terms containing e^3 and higher powers of e)

The above equation can be obtained either directly or by reversing the equation (1).

Taking only the first two terms of the right-hand side, we get,

$$L=S-2e\sin(S-K)$$

$\therefore$ The mean longitude of the sun is obtained by adding to the true longitude, the quantity, $-2e\sin(S-K)$ or $-115'.\sin(S-K)$. [taking 1 radian$=3438'$ and $e=\cdot0168$].

It is also seen that the maximum value of E_1 is about $7\frac{2}{3}$ minutes which is the value of $115'$ converted into its time equivalent.

Let L_0 be the mean longitude when the sun is at the first point of Aries.

Then $L_0 = 2e \sin K$, since $S = 0$.

If L_1, L_2, L_3, and L_4 are the mean longitudes of the sun at summer solstice, autumnal equinox, winter solstice and vernal equinox respectively, we have,

$$L_1 = \tfrac{1}{2}\pi - 2e \cos K$$
$$L_2 = \pi - 2e \sin K$$
$$L_3 = \tfrac{3}{2}\pi + 2e \cos K$$
$$L_4 = 2\pi + 2e \sin K \text{ (same as } L_0\text{)}$$

The differences between these consecutive mean longitudes, *viz.* L_0, L_1, L_2, L_3, L_4 multiplied by the factor $\frac{365{\cdot}24}{2\pi}$ give the lengths of the seasons as follows:—

$$\text{No. of days in spring} = \frac{365.24}{2\pi}\left(\frac{\pi}{2} - 2e\cos K - 2e\sin K\right)$$

$$= \begin{array}{ccc} \text{days} & & \text{hours} \\ 92 & — & 20{\cdot}2 \end{array}$$

$$\text{No. of days in summer} = \frac{365.24}{2\pi}\left(\frac{\pi}{2} - 2e\sin K + 2e\cos K\right)$$

$$= \begin{array}{ccc} \text{days} & & \text{hours} \\ 93 & — & 14{\cdot}4 \end{array}$$

$$\text{No. of days in autumn} = \frac{365.24}{2\pi}\left(\frac{\pi}{2} + 2e\cos K + 2e\sin K\right)$$

$$= \begin{array}{ccc} \text{days} & & \text{hours} \\ 89 & — & 18.7 \end{array}$$

$$\text{No. of days in winter} = \frac{365.24}{2\pi}\left(\frac{\pi}{2} + 2e\sin K - 2e\cos K\right)$$

$$= \begin{array}{ccc} \text{days} & & \text{hours} \\ 89 & — & 0{\cdot}5 \end{array}$$

In the above calculations, e is taken to be $0{\cdot}01675$ and K to be $281° - 13'$

It should be noted that these quantities are liable to very small variations due to the perturbations caused by the other planets of the solar system.

63. The reduction to the equator.

The second part of the equation of time is due to the fact that the sun is moving on the ecliptic and not on the equator. The excess of the R. A. of a point on the ecliptic, over its longitude is called the *reduction to the equator* (E_2).

The mean time is the hour angle of the mean sun whose R. A. is the same as the longitude of the dynamical mean sun moving uniformly along the ecliptic. It is

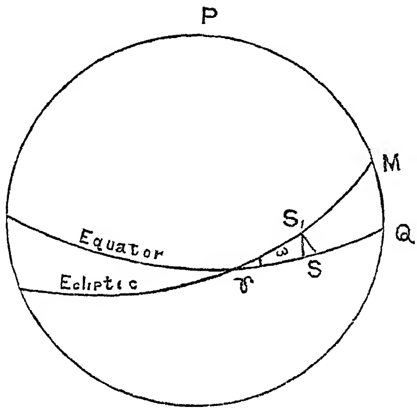


Fig. 52.

seen that when the dynamical mean sun is between ♈, the first point of Aries and M, the summer solstice on the ecliptic, its longitude is greater than its R. A. Therefore, during this period the R. A. of the mean sun is greater than that of the dynamical mean sun (S_1). Hence the hour angle of S is less than that of S_1. In other words E_2 is negative during spring. It could be similarly seen that E_2 is positive during summer, negative during autumn and again positive during winter. E_2 is zero only when the sun's mean longitudes are 0°, 90°, 180° and 270°. On these occasions the true sun will be very near the equinoxes and the solstices. Therefore E_2 may be taken to be zero on the equinoctial and solstitial dates.

64. Analytical expression for the reduction to the equator.

If S is the longitude of the sun, α its R. A and δ its declination,

$$\cos\omega = \frac{\tan\alpha}{\tan S} \quad \text{or} \quad \frac{1-\cos\omega}{1+\cos\omega} = \frac{\tan S - \tan\alpha}{\tan\alpha + \tan S}$$

$$\text{or} \quad \frac{\sin(\alpha - S)}{\sin(\alpha + S)} = -\tan^2\frac{\omega}{2}$$

$$\therefore\ \alpha - S = -\tan^2\frac{\omega}{2}\sin 2S + \tfrac{1}{2}\tan^4\frac{\omega}{2}\sin 4S$$

$$-\tfrac{1}{3}\tan^6\frac{\omega}{2}\sin 6S + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

(See Loney's Trigonometry Part II.)

Equation (3) gives the reduction to the equator expressed in radians.

Now, 1 Radian$=\dfrac{86400}{2\pi}=13751$ seconds of time and

$\omega = 23° - 27' - 4''$.

$\therefore$ $\alpha - S = -592''{\cdot}38 \sin 2S + 12''{\cdot}76 \sin 4S, -0{\cdot}36 \sin 6S$ showing that the maximum value of the reduction to the equator is nearly 10 minutes.

65. Expression for the equation of time.

When S is eliminated between the equations

$$\alpha = S - \tan^2\frac{\omega}{2}\sin 2S + \tfrac{1}{2}\tan^4\frac{\omega}{2}\sin 4S,$$

and $S = L + 2e\sin(L-K) + \frac{5}{4}e^2\sin(2L-2K)$ we get,

$$\alpha = L + 2e\sin(L-K) + \tfrac{5}{4}e^2\sin(2L-2K)$$

$$-\tan^2\frac{\omega}{2}\left\{\sin 2L + 4e\sin(L-K)\cos 2L\right\}$$

$$+\tfrac{1}{2}\tan^4\frac{\omega}{2}\sin 4L,$$

Now, the equation of time is given by,

$$\alpha - \mathrm{L} = 2e \sin\ (\mathrm{L} - \mathrm{K}) - \tan^2 \frac{\omega}{2} \sin 2\mathrm{L}, \qquad \text{omitting } e^2,$$

$$e \tan^2 \frac{\omega}{2} \text{ and smaller terms.}$$

or when expressed in hours

$$\alpha - \mathrm{L} = \frac{12}{\pi} \left\{ 2e \sin\ (\mathrm{L} - \mathrm{K}) - \tan^2 \frac{\omega}{2} \sin 2\mathrm{L} \right\} \ldots\ldots\ldots(4)$$

The first term of the equation of time is the equation of the centre and the second term is the reduction to the equator for the dynamical mean sun.

Substituting for $e = {\cdot}0168$ and $\mathrm{K} = 281^\circ{\cdot}2$, we get

$\alpha - \mathrm{L} = 90^s \sin \mathrm{L} + 452^s \cos \mathrm{L} - 592^s \sin 2\mathrm{L}$.................

If T is the sidereal time and α, the R. A. of sun,

$(\mathrm{T} - \alpha)$ is the apparent solar time, and

$(\mathrm{T} - \mathrm{L})$ is the mean solar time.

Now, $(\mathrm{T} - \mathrm{L}) - (\mathrm{T} - \alpha) = \alpha - \mathrm{L} =$ equation of time

Thus the equation of time is the correction to be applied to the apparent solar time to get the mean solar time.

66. Graphical representation of the equation of time.

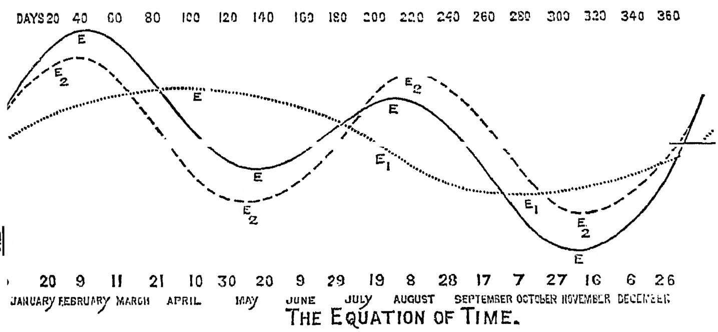


Fig. 53.

The variations in both E_1 and E_2 could be represented graphically by means of curved lines, by plotting the points, taking the time of the year along the x axis and the values of E_1 and E_2 on the various dates along the y axis.

The curve representing the variation of E_1 will cut the x axis on July 1st and December 31st and the curve representing E_2 will cut the x axis in four points corresponding to the dates, March 21st, June 21st, September 23rd and December 22nd. The ordinates of these two curves for the same date could be added, and a separate curve drawn with these ordinates. The curve thus drawn would represent the variation of E, the equation of time. This resultant curve representing the variation of the equation of time will be found to cut the x axis at four points in the course of the year. The corresponding dates are April 16, June 14, September 1 and December 25. This shows that E vanishes four times a year. From the curve it could also be seen that the maximum values of the equation of time occur on February 12th ($14\frac{1}{2}$ minutes nearly) and on July 27th ($6\frac{1}{4}$ mins.) and the minimum values occur on May 15th ($3\frac{3}{4}$ mins.) and on November 3rd ($-16\frac{1}{3}$ mins.)

67. Vanishing of the equation of time.

That the equation of time vanishes not less than four times a year could be seen from the fact that the expression (4) for E changes alternately from positive to negative when the values, 45°, 90°, 180° and 360° are substituted for L. This shows that the curve representing E should cut the x axis in four places. The values of L for which $E=0$, could be seen to be approximately 23°, 83°, 159° and 272°, in the course of one year.

There is also another way of proving the fact that the equation of time vanishes four times a year. As the greatest value (9.9 mins.) of the equation of time due to obliquity (E_2) is greater than the greatest value (7.7 mins.) of the equation of time due to eccentricity (E_1) we can see that $E_1 + E_2$ has the same sign as E_2 at the time when the maximum of E_2 occurs, irrespective of the sign of E_1. Now the maxima of E_2 occur in the months

of February, May, August and November and their values are $+9.9$ mins. -9.9 mins, $+9.9$ mins. and -9.9 mins. respectively. Hence the signs of $E=(E_1+E_2)$ are $+,-,+,-,+,$ in February, May, August, November and February respectively. Therefore E should be equal to zero at least once between February and May, once between May and August and so on. Thus we find that the equation of time vanishes at least four times in the year.

68. Stationary equation of time.

From the graph representing the variation of E, we are able to see that E has turning values on February 12, May 15, July 27 and November 3.

The condition that E, the equation of time should be maximum or minimum is that,

$$dE=d\,(\alpha-L)=d\alpha-dL=0$$

$$\therefore \quad d\alpha=dL$$

If m and v are the mean and true anomalies and L and S, the mean and true longitudes of the sun, then,

$$L=m+K \quad \text{and}$$

$$S=v+K \quad \text{where } K=\text{longitude of perigee.}$$

$$\therefore \ dm=dL=d\alpha \text{..............................(1)}$$

Also, $\tan\alpha=\tan S\cos\omega$, where ω is the obliquity.

$$\therefore \ \sec^2\alpha.\,d\alpha=\sec^2 S.\,dS\cos\omega \text{.........(2)}$$

If r is the radius vector drawn from the earth to the sun and a, b, the semi-axes of the elliptic orbit of the earth round the sun, it follows, that,

$$\frac{\frac{1}{2}r^2\,dv}{\pi ab}=\frac{dm}{2\pi}$$

or $\quad r^2\,dv=ab.\ dm$..(3)

Multiplying (1), (2) and (3) we get,

$$dm.\sec^2\alpha.\,d\alpha.\,r^2\,dv=d\alpha\sec^2 S\,dS\cos\omega\,ab\,dm$$

$i.\,e.\quad r^2\sec^2\alpha=ab\sec^2 S\cos\omega$..........................(4)

(since $dS=dv$)

or $r^2\,(1+\tan^2 S\cos^2\omega)=ab\sec^2 S.\cos\omega$

$r^2\,(\cos^2 S+\sin^2 S\cos^2\omega)=ab\cos\omega$ ....(5)

If l is the semi-latus-rectum of the elliptic orbit of the earth and e, the eccentricity,

$$\frac{l}{r}=1+e\cos(S-K)$$

Equation (5) reduces to,

$$\frac{l^2}{\left\{1+e\cos(S-K)\right\}^2}\cdot(\cos^2 S+\sin^2 S\cos^2\omega)=a^2(1-e^2)^{\frac{1}{2}}\cos\omega.$$

or $\quad (1-e^2)^{\frac{3}{2}}(\cos^2 S+\cos^2\omega\sin^2 S)$

$$=\cos\omega\left\{1+e\cos(S-K)\right\}^2 \text{ ....(6)}$$

From equation (6) it is possible to calculate S, when the equation of time has a turning value, and from the values of S, the dates corresponding to these values of S also could be found out. From the dates so found, the maximum or minimum value of E is obtained by the substitution of the values of the sun's mean longitude in the expression for E.

69. Conversion of sidereal time to mean time and vice versa.

As the local mean time for any station has to be determined by star observations and as the latter could give only local sidereal time, it is necessary to establish the relations that exist between sidereal and mean times for a place.

The tropical year consists of $365\frac{1}{4}$ mean solar days and the mean sun increases in R. A by 360° during this interval. Therefore, the number of transits that ♈ makes across a certain meridian is one more than the transits that the mean sun makes in the same place in the course of one tropical year.

Hence $365\frac{1}{4}$ mean solar days $= 366\frac{1}{4}$ sidereal days. From this,

(a) $$1 \text{ mean solar day} = \left(1 + \frac{1}{365\frac{1}{4}}\right) \text{ of a sidereal day}$$
$$= 1{\cdot}00274 \text{ sidereal days}$$
$$= 1 \text{ sid. day} + 4 \text{ min.} - 4 \text{ seconds.}$$

$\therefore$ 1 mean solar hour $=$ 1 sid. hour $+$ 10 sec $- \frac{1}{6}$ second.

And 6 mean solar minutes $=$ 6 sid. minutes $+$ 1 second.

Therefore, to reduce a given interval of mean time to sidereal interval, add 10 seconds for every hour and 1 second for every 6 minutes of mean time and finally subtract 1 second for every minute so added.

or (b) $$1 \text{ sidereal day} = \left(1 - \frac{1}{366\frac{1}{4}}\right) \text{ mean solar days}$$
$$= (1 - {\cdot}00273) \text{ of a solar day.}$$

1 sidereal day $=$ 1 mean solar day $-$ 4 min. $+$ 4 sec.

1 sidereal hour $=$ 1 mean solar hour $-$ 10 sec $+ \frac{1}{6}$sec.

6 sidereal minutes $=$ 6 mean solar minutes $-$ 1 second.

Therefore to reduce a given interval of sidereal time to mean time, subtract 10 seconds for every hour and 1 second for every 6 minutes in the given interval and finally add 1 second for every minute so subtracted.

The Nautical Almanac gives the sidereal time of mean noon for Greenwich on every day of the year, and also the mean time of sidereal noon every day for the same station. From these data and the above formulae of conversion, it is easy to find (1) the mean solar time corresponding to a given sidereal time and (2) the sidereal time corresponding to a given mean time at Greenwich. The method is given below:

(1) Let the given sidereal time be s on any day, and let s_0 be the sidereal time of mean noon on the same day as given in the Nautical Almanac.

The sidereal interval after mean noon $= s - s_0$

The mean time interval after mean noon

$$= (s - s_0)\,(1 - k') \text{ where } k' = \frac{1}{366\frac{1}{4}}$$

$$= \text{mean time.}$$

If in the above, one takes from the Nautical Almanac, m_0, the mean time of previous sidereal noon, the following procedure may be adopted.

Mean time interval corresponding to $s = s(1-k')$

$\therefore$ Mean time required $= m_0 + s\,(1-k')$

(2) Let the given mean time be m, and let s_0 be the sidereal time of mean moon on the same day as given in the Nautical Almanac.

The sidereal interval corresponding to m mean hours

$$= m\,(1+k) \text{ where } k = \frac{1}{365\frac{1}{4}}$$

$\therefore$ sidereal time required $= s_0 + m\,(1+k)$.

If this quantity is greater than 24, subtract 24.

Note.

In order to find the sidereal time corresponding to mean time or mean time corresponding to sidereal time at any other place, not lying on the Greenwich meridian, the sidereal time at mean noon, or the mean time at sidereal noon at the particular meridian should be derived first and then the method given above should be followed. If the place of observation is not on the Greenwich meridian, the mean time of transit of the first point of Aries which is given for Greenwich, must be corrected by subtraction of 9·8296 secs. or $\left\{\dfrac{3 \text{ min. } 55{\cdot}909 \text{ sec.}}{24}\right\}$ for each hour of longitude, if the place is west and by addition if it is to the east. Similarly the sidereal time at mean noon should be corrected

by the addition of 9·8565 seconds or $\left\{\frac{3 \text{ min. } 56{\cdot}555 \text{ sec.}}{24}\right\}$ for each hour of longitude if the place is west and by subtraction if it is east. [9·8296 is the excess of one mean solar hour over one sidereal hour, expressed in mean time while 9·8565 is the same excess expressed in sidereal time. For, in one mean solar day a point on the equator rotates through $360° + 59' - 8''.33$, while in one sidereal day, the same point rotates only through 360°. The mean time interval and the sidereal interval corresponding to this excess of $59' - 8''.33$, are 3 mins. 55·909 secs. and 3 mins. 56·555 secs. respectively.]

70. Different kinds of years.

The year is ordinarily understood to be the period of the apparent motion of the sun round the ecliptic, but some of the points such as the vernal equinox, from which the sun's daily motion is measured, are not fixed points on the ecliptic. Therefore, in defining the year, the origin from which the sun's motion is reckoned has to be specified. The period of time that elapses between two consecutive returns of the sun to the vernal equinox is called a *Tropical year*. Its duration in mean solar units is 365·2422 days ($365\frac{1}{4}$ days nearly). The period of time taken by the sun to go round the ecliptic from any point on it is called a *Sidereal year*. Its length is 365·2564 mean solar days. The tropical year is of shorter duration than the sidereal year, as the first point of Aries moves annually $50''.26$ along the ecliptic in the retrograde direction. The interval between successive passages of the sun through perigee is called an *Anomalistic year*. Its length is 365·2596 mean solar days. This is even longer than the sidereal year, as the apse line of the earth's orbit has an annual forward motion of about $11''.25$ in the plane of the ecliptic. It is easy to compare the lengths of these three kinds of years as follows:—

Tropical year :	Sidereal year :	Anomalistic year.
$=359^\circ-59'-9''{\cdot}74$:	360° :	$360^\circ-0'-11''{\cdot}25$.

The sidereal year is longer than the tropical year by about 20 minutes and the anomalistic year is longer than the sidereal year by about $4\frac{1}{2}$ minutes.

None of the above kinds of years could be conveniently taken for ordinary purposes, as they involve fractions of a day and as they do not mark the recurrence of the seasons. Therefore, the *Civil year* has been introduced, which has as its basis, the tropical year, and according to which the seasons fall at fixed months of the year. The civil year contains 365 mean solar days for three consecutive years and 366 days for the fourth year. Thus four civil years exceed four tropical years by about 44 mins.-56.08 secs. The fourth year is called a *Leap year*. This system of reckoning time is due to Julius Cæsar and hence his name is associated with this mode of constructing the calendar. Ordinarily the leap years are chosen to be those years whose numbers are multiples of four.

Now, as 4 civil years exceed 4 tropical years by 44 mins. 56·08 seconds, 400 civil years exceed 400 tropical years by 3 days-2 hours-53·3 secs. Unless this difference is also compensated, the civil year in course of time would not mark the recurrence of the seasons. For this, Pope Gregory XIII suggested a correction, by which 3 days were to be dropped in 400 years of the Julian calendar. Thus by the Gregorian correction to the Julian calendar, any year which is a multiple of hundred is not taken to be a leap year, unless the number denoting the century is a multiple of four. The year 1900 A. D. is not a leap year while 2000 A. D. is a leap year. Even after this correction, the Gregorian system of reckoning makes 4000 civil years exceed 4000 tropical years by 1 day 4 hrs, and 55 minutes.

The civil day at any place is reckoned from midnight to midnight, and any event occuring at 12 noon on January 1st is said to occur on January 1·5, as January 1st began on the previous midnight, whereas an event that occurs at 12 noon on December 31st can either be said to occur on December 31·5 or on January 0·5

The length of the tropical year has been defined, but not its mode of reckoning. It is reckoned in mean solar days given by the mean sun, and therefore it is convenient to fix the beginning of the tropical year with reference to some position of the mean sun on the ecliptic. Therefore the instant when the mean sun's R. A. or the sun's mean longitude is 280° was chosen as the beginning of the solar or the tropical year, by Bessel of Germany. The beginning of the *Besselian year* 1931 is 1931 January 1·322 days, Greenwich civil time; 1932 Besselian year begins at January 1·564 Greenwich civil time, obtained by adding 365·242 days to the previous date. The Besselian year is very useful in calculations and observations relating to the heavenly bodies.

A period of 12 lunar months is called a *Synodic year*. Its length is nearly 354½ days, a lunar month being the interval between two consecutive new moons (29 days 12 hrs. 44 mins. 2·8 sec.)

For observations relating to certain heavenly bodies such as the variable stars, it is usual to express the time that has elapsed after a fixed epoch chosen long ago. The epoch of mean noon of January 1st of 4713 B. C. is one such and at any instant the number of mean solar days that has elapsed after this, is called the *Julian date* of the instant. The time of an observation on January 13th at 18 hours Greenwich civil time is denoted by 2426355·25, as the Julian date is measured from the mean noon of January 13, whereas Greenwich civil time is measured from midnight to midnight.

CHAPTER VIII.

LATITUDE, LONGITUDE AND TIME OF A PLACE.

71. Introduction.

The determination of the exact position of a point on the surface of the earth is of primary importance and for this, the two co-ordinates of the point viz the latitude and longitude have to be known. An accurate knowledge of the latitude and longitude of an observatory is very essential, as other determinations such as the co-ordinates of stars, local time etc. are all dependant upon them. The mariners have to know at any time the exact place where they are sailing, as they have always to be certain that they are not far out of the route laid out for the voyage. In the case of determinations of latitude in the sea, one cannot expect the same degree of accuracy as is possible in land observations. This is due to the fact, that as the ship is always moving, and that in an unsteady steamer, observations by means of very delicate instruments that are used in land are not possible. At sea, a well-determined latitude can often be relied on only to within a quarter of a mile, whereas in land it is possible to depend upon it up to within ten yards. [A second of latitude corresponds to a distance of about 34 yards].

According to the convenience of the observer, there are different methods of determining the latitude of a place. These methods fall into two groups:—

(1) Meridian observations,

(2) Ex-meridian observations.

72. Latitude by a single meridian altitude.

This is the simplest and the most accurate method available at a fixed station.

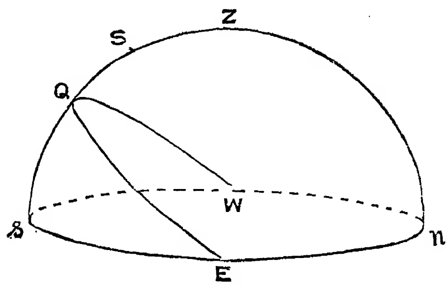


Fig. 54.

Let the meridian altitude s S of a star S be a so that its zenith distance z is $90^\circ - a$. If l be the latitude of the place, $l = d + z = d + 90^\circ - a$, d being the north declination of the star which is transitting south of the zenith. (The declination of the star can be obtained from the Nautical Almanac)

The formula gets slightly modified in the signs of d and z if the star transits north of the zenith, or if the declination of the star is south.

In a fixed observatory, the meridian altitude of a star or any other heavenly body, is always very accurately found by means of the transit circle (See chapter on Astronomical Instruments). The latitude of the place is got from a number of values obtained from observations of different stars, either by taking their mean value or by applying the least square solution to them. In any other station on land, where a meridian instrument, like the transit circle, could not be had, the meridian altitude is found by means of a sextant using an artificial horizon (See chapter on Astronomical Instruments).

73. Latitude by observation of circumpolar stars.

In this method also, the observation is made, when the star is on the meridian. If the star is circumpolar, find the zenith distances or altitudes at the upper and lower transits. Let a_1, a_2 be the two altitudes, corrected for

refraction, dip etc., Then l, the latitude of the place is given by $l = \frac{a_1 + a_2}{2}$.

As this expression does not involve the declination of the star, this method of determining the latitude of a place is fundamental, and is the one usually adopted in fixed observatories. From a large number of observations of circumpolar stars by means of a transit circle, the best value for the latitude of a place is chosen. It is from this known latitude and observed zenith distances of stars, that the declinations of the stars are obtained. Therefore, it has to be clearly understood, that when a star's declination is used in finding the latitude of a place, as in the previous method, this declination should have been arrived at by observations from another place of known latitude.

74. Latitude by observations of meridian altitudes of two stars.

A third method involving meridian observations to find the latitude of a place is to choose two stars of known declination that transit at nearly the same altitude, on opposite sides of the zenith, and to observe their altitudes.

Let a_1 and a_2 be the observed meridian altitudes of the stars and d_1, d_2 be their declinations, one of them transiting south of the zenith and the other, north. Let u_1 and u_2 be the corrections for refraction (atmospheric refraction increases the true altitude of a body), D the correction for dip, and e the correction for instrumental errors. Taking z_1 and z_2 to be the actual zenith distances of the two stars culminating south and north of the zenith respectively, we have,

$$90° - z_1 = a_1 - u_1 - D + e \qquad (1)$$

and $$90° - z_2 = a_2 - u_2 - D + e \qquad (2)$$

Now, the latitude of the place is given by,

$$l = z_1 + d_1 = d_2 - z_2 = \frac{z_1 - z_2 + d_1 + d_2}{2} \quad .(3)$$

From (1) and (2)

$$z_1 - z_2 = (a_2 - a_1) + u_1 - u_2$$

$$\therefore \quad l = \frac{(a_2 - a_1) + (u_1 - u_2) + (d_1 + d_2)}{2}.$$

This result is evidently independant of the instrumental errors and correction due to dip. The only correction that is to be considered in this expression for the latitude is that due to refraction and this too can be neglected as the stars chosen have nearly the same altitude.

75. Latitude by simultaneous observations of the altitudes of two known stars.

In this method two stars that are outside the meridian of a place are observed simultaneously.

Let z and z' be the zenith distances of two stars A and B, whose altitudes have been observed and corrected for refraction, dip, etc. Let their polar distances be x

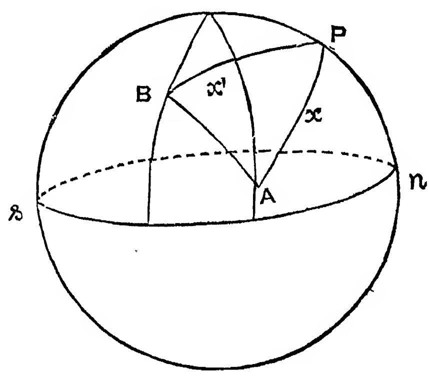


Fig. 55.

and x', and let h be the difference between their right-ascensions. Then, from the triangle APB, AB is known, since, $\cos AB = \cos x \cos x' + \sin x \sin x' \cos h$............(1)

Also, angle PAB is known from

$$\frac{\sin \text{PAB}}{\sin x'}=\frac{\sin h}{\sin \text{AB}} \quad \dots\dots (2)$$

Again, from triangle AZB,

$$\sin\frac{\text{ZAB}}{2}=\sqrt{\frac{\sin \frac{1}{2}(z+z'-\text{AB}) \sin \frac{1}{2}(z'+\text{AB}-z)}{\sin z \sin \text{AB}}} \quad \dots\dots (3)$$

From (2) and (3) angle ZAP is known.

The latitude is therefore deduced from the triangle ZAP, since,

$$\sin \varphi=\cos \text{ZP}=\cos x \cos z+\sin x \sin z. \cos \text{ZAP}.$$

When the horizon is well-defined, this method is very useful, as we can accurately find out the altitudes of a pair of bright stars, and so long as the same pair of stars are used for observation in a number of places it is possible to calculate before-hand the values of AB and PAB and use them in the final steps of the calculation.

If simultaneous observations of two stars A and B are not possible, take the altitude of A at time t_1, the altitude of B at time t_2, and the altitude of A again at time t_3. Now from the observations of A at times t_1 and t_3, we get the change in the altitude of A, during the interval (t_3-t_1) and from this, assuming that the altitude of A changes uniformly in the interval, the altitude of A at time t_2 can be easily calculated. Use this with the altitude of B at time t_2 and make the calculations as before.

76. Latitude by observations on a single star or on the sun.

It is also possible to calculate the latitude of a place by observing the altitudes of a single star at sidereal hours t_1 and t_2. In this case, the angle APB (Fig. 55) $=15\,(t_2-t_1)^\circ$ and PA=PB.

$$\sin\frac{(t_2-t_1)\,15^\circ}{2}=\frac{\sin\frac{AB}{2}}{\sin PA},\ \text{which gives the value of } \frac{AB}{2}.$$

Also, $\cos PAB=\dfrac{\tan\frac{AB}{2}}{\tan PA}$, which gives angle PAB.

The rest of the procedure is the same as in the previous section.

In this method the observations can be made on the sun instead of on a star. But, here the declinations of the sun are not the same at both the observations and the sun has also a motion in R. A. during this interval. So when great accuracy is required, allowance should be made for these changes also.

In determining the latitude of a place at sea, mariners frequently make use of this method. Two observations are taken on the sun or on a star, separated by an interval of time and as the steamer is ordinarily in motion all the while, allowance has to be made for this motion also. The two places of observation

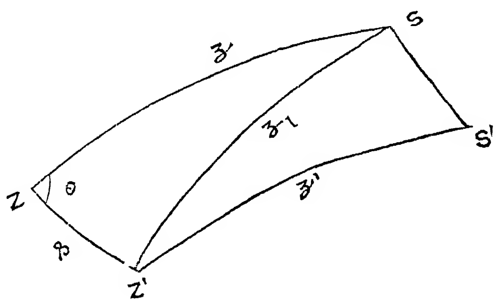


Fig. 56.

will have different zeniths and therefore the zenith distances of the sun or the star for the two places would be different. If Z and Z′ are the zeniths of the two

places, and z and z' are the zenith distances of the observed body S and S′ at times t and t', we have to calculate for the determination of latitude, the zenith distance z_1 of S at time t'. Now, this could be derived only if the direction of motion of the steamer is known.

If the course of the steamer makes an angle θ with the vertical through S,

we get $z_1 = z - s \cos \theta$ nearly,
where s = angular distance travelled by the steamer in the interval.

The two zenith distances to be used in the formula for the determination of the latitude are z_1 and z' and the latitude obtained is that of the place of second observation. The most favourable condition for getting the best result by this method, is deduced from the following considerations.

Generally all observations are likely to have some errors, and the best observation is that in which the errors affect the result least. In the above observation, the altitudes may not be correct and we have to see when these affect the latitude least.

Let S and S′ be the two objects observed and P, the pole. Let z and z' be the true zenith distances of S and

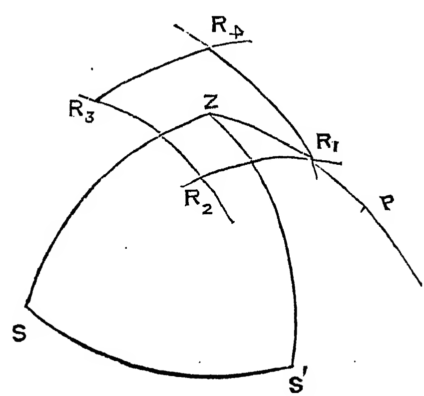


Fig. 57.

S′ and Z the true zenith.

Let θ and θ' be the errors in the observed altitudes and let us suppose that both the errors are positive. Then the apparent zenith which is observed will be a point R_2, where $SR_2 = z - \theta$, and $S' R_2 = z' - \theta'$. (If both the errors are negative the apparent zenith will be R_4 where $SR_4 = z + \theta$ and $S' R_4 = z' + \theta'$, and when one error is positive and the other negative the apparent zeniths are R_1 and R_3.) Now it is evident from the figure that the distance ZR_2 between the true and apparent zeniths is equal to the diagonal of a parallelogram of sides θ and θ' on a spherical surface. Now, this is least when the sides of the figure are inclined at right angles, i. e. when $SZS' = 90°$. The two bodies should therefore differ in azimuth by 90°. If the observations are made with the same instrument, the errors of observation are likely to be the same, and of the same sign, so that in this case, the erroneous zenith obtained will be either R_2 or R_4. Now, if P is the pole, the true co-latitude PZ will differ least from PR_4 or PR_2 when PZ is perpendicular to ZR_4 or ZR_2. Therefore ZS′ and ZS are to be on the same side of the meridian and are to be each inclined to the meridian at 45° at the time most suitable for observation.

77. Latitude by the Prime Vertical Instrument.

In this method, the star is observed when it is on the prime vertical by an instrument called the prime vertical instrument. This is an instrument similar to the transit circle, but it is so placed that the telescope describes the plane of the prime vertical. Now, observe the sidereal times of transit of a star of declination δ when it crosses the prime vertical, both east and west of the meridian. Half the time interval gives the hour angle of the star, converted into time, in the latter position.

If φ be the latitude, $\cos h = \dfrac{\tan PZ}{\tan PS}$

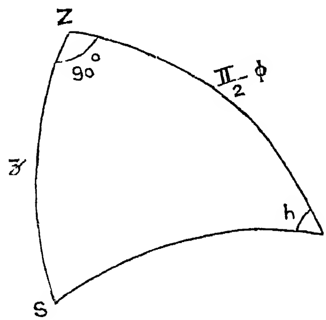


Fig. 58.

or $\cot\delta\cos h = \cot\varphi$, from which φ can be calculated.

The principal advantage of the above method consists in not using the altitude of the star to determine the latitude and therefore, there is no correction required for refraction, dip etc. If it is not possible to observe both the transits, take the value of h as the difference between the sidereal time of one of the transits across the prime vertical and the known R. A. of the star.

78. Latitude from observed altitude near the meridian.

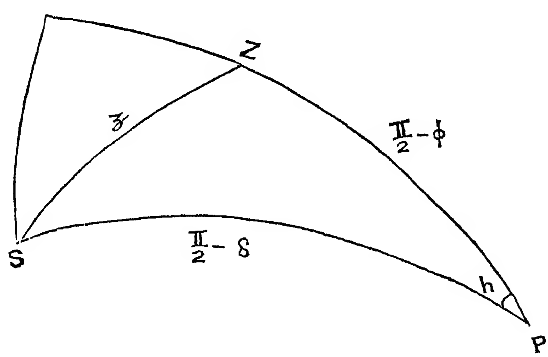


Fig. 59.

Let z be the observed zenith distance of a body S, when it is very near the meridian, and let h be its hour angle then. Let S′ be the point of the meridian where S

would transit. Then ZS′ and ZS will differ only by a small quantity.

Let $\quad ZS' = z - x$ (where x is small)

$$= PS' - PZ.$$

$$= \left(\frac{\pi}{2} - \delta\right) - \left(\frac{\pi}{2} - \varphi\right)$$

$$= \varphi - \delta \qquad \ldots \qquad \ldots \qquad (1)$$

From the triangle ZPS,

$$\cos z = \sin\varphi \sin\delta + \cos\varphi \cos\delta . \cos h$$

$$= \sin\varphi \sin\delta + \cos\varphi \cos\delta \left(1 - 2\sin^2\frac{h}{2}\right)$$

$$= \cos(\varphi - \delta) - 2\cos\varphi \cos\delta \sin^2\frac{h}{2}$$

i. e. $\cos z - \cos(z - x) = -2\cos\varphi \cos\delta \sin^2\frac{h}{2}$

$$2\sin\frac{x}{2}\sin\left(z - \frac{x}{2}\right) = 2\cos\varphi\cos\delta\sin^2\frac{h}{2}$$

i. e. $\sin\dfrac{x}{2} = \dfrac{\cos\varphi\cos\delta\sin^2\frac{h}{2}}{\sin\left(z - \frac{x}{2}\right)}$

Since x is small,

$$x = \frac{2\cos\varphi . \cos\delta\sin^2\frac{h}{2}}{\sin 1'' . \sin z} \text{ seconds of arc} \qquad (2)$$

From equation (2), x can be calculated and by substituting it in (1), we can get the latitude of the place. It should be noted that (2) gives x in terms of φ which we have to find out. But here we can take the value $(z+\delta)$ for φ. Equation (2) can be written as

$$x\sin 1'' = \frac{2\cos\varphi\cos\delta}{\sin z}\left(\frac{h\sin 15'}{2}\right)^2$$

where h is small and is expressed in minutes of time.

$$\text{or}\quad x = \frac{\cos\varphi\cos\delta\sin^2 15'}{2\sin(\varphi-\delta)\sin 1''}\,.\,h^2 \text{ nearly}$$

$= C.\ h^2$, where C is a constant, depending upon the latitude of the place and the declination of the body observed.

For convenience, C could be found out separately and tabulated for different stars. This method is specially useful when the meridian observation of a body is prevented by clouds; but when the meridian observation is also possible, the value of the latitude derived from it could be checked by the one derived from such an ex-meridian observation.

79. Latitude from the altitude of the pole star.

The pole star is about 1°-30′ from the pole and so observations on it could be made at any time, when that part of the sky is clear. This is a useful method both at sea and at any land station. Moreover, the slow motion of the star about the pole is advantageous in as much as any small error in the hour angle of the star does not affect the result.

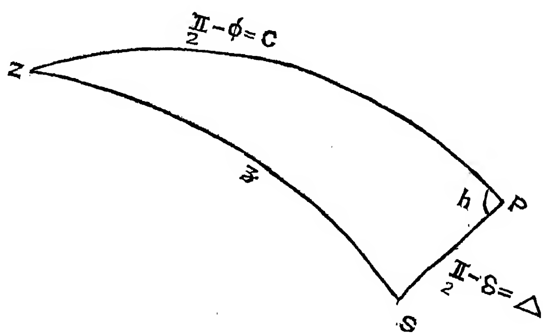


Fig 60.

Let z be the zenith distance of the pole star after allowing for refraction, dip etc., and let the corresponding hour angle be h. Let c be the co-latitude of the place.

Now $c=z+x$ where x is small.

$\cos z=\cos c\cos\Delta+\sin c\sin\Delta\cos h$

$$= \cos(z+x)\cos\Delta + \sin(z+x)\sin\Delta\cos h$$

$$= (\cos z - x\sin z - \tfrac{1}{2}x^2\cos z + \tfrac{1}{6}x^3\sin z + \ldots\ldots)$$

$$(1-\tfrac{1}{2}\Delta^2+\ldots\ldots) + (\sin z + x\cos z - \tfrac{1}{2}x^2\sin z + \ldots\ldots\ldots\ldots)$$

$$(\Delta - \tfrac{1}{6}\Delta^3 + \ldots\ldots\ldots)\cos h \text{ (since } x \text{ and } \Delta \text{ are small)}$$

Arranging the above in ascending powers of x and Δ and omitting small quantities of order higher than the 3rd we get,

$$\cos z = \cos z - x\sin z + \Delta\sin z\cos h - \tfrac{1}{2}x^2\cos z$$
$$-\tfrac{1}{2}\Delta^2\cos z + \Delta\, x\cos z\cos h + \tfrac{1}{6}x^3\sin z$$
$$+\tfrac{1}{2}x.\,\Delta^2\sin z - \tfrac{1}{2}x^2\,\Delta\sin z\cos h - \tfrac{1}{6}\Delta^3\sin z\cos h$$

$$\therefore\ x = \Delta\cos h - \tfrac{1}{2}\cot z\,(x^2+\Delta^2-2\,\Delta\,x\cos h)$$
$$+\tfrac{1}{6}(x^3+3\,x\,\Delta^2-3\,x^2.\,\Delta.\cos h - \Delta^3\cos h)\ldots\ldots\ldots\ldots(1)$$

(dividing by $\sin z$)

This equation has to be solved and this is done approximately as follows:—

$x=\Delta\cos h$ (as a first approximation). Then substituting for x^2 and neglecting terms of the 3rd order of smallness, we get as a second approximation from (1)

$$x = \Delta\cos h - \tfrac{1}{2}\cot z\,(\Delta^2\cos^2 h + \Delta^2 - 2\,\Delta^2\cos^2 h)$$
$$= \Delta\cos h - \tfrac{1}{2}\Delta^2\cot z\sin^2 h\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Again substituting this value of x from (2) in the terms of the second order, and $x=\Delta\cos h$ in the terms of the third order in (1) we get,

$$x = \Delta\cos h - \tfrac{1}{2}\Delta^2\cot z\sin^2 h + \tfrac{1}{3}\Delta^3\cos h\sin^2 h$$
$$= c - z = 90^\circ - \varphi - z.$$

$$\therefore\ \varphi = 90^\circ - z - \Delta\cos h + \tfrac{1}{2}\Delta^2\cot z\sin^2 h\sin 1''$$
$$-\tfrac{1}{3}\Delta^3\cos h\sin^2 h\sin^2 1''$$ where Δ is expressed in seconds of arc

If accuracy up to half a second alone is expected in the determination of φ, the last term in the above equation can be omitted.

80. Application of differential formulae.

If all the six elements of a spherical triangle be given small increments, the altered elements will not in general be the parts of another spherical triangle. This is due to the fact that any three sides and any three angles could not form a spherical triangle. As a matter of fact in any spherical triangle, given any three elements, the others could be generally determined and therefore if six independant quantities are chosen as parts of a spherical triangle, there must also be three equations which these quantities should satisfy. In most cases, where the variations of the elements of a spherical triangle take place, two of the elements remain constant and the relative variations of two other elements are to be found out. In such a case, mere differentiation of an equation involving these elements would give the required relation.

We shall apply this, in finding out the error that will arise in the determination of the latitude of a place by the observation of the altitude of a star, whose hour angle h and declination δ are known accurately.

Let z be the complement of the altitude observed and a the azimuth of the star measured west from north. Let n be the north point of the horizon.

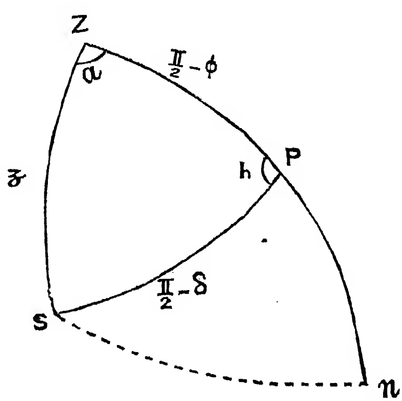


Fig. 61.

From the triangle ZPS, we have,

$\cos z = \sin \delta \sin \varphi + \cos \delta \cos \varphi \cos h$.........(1)

Differentiating the above, and treating h and δ as constants, we get

$$-\sin z.\, d\,z = (\sin \delta \cos \varphi - \cos \delta \sin \varphi \cos h)\, d\,\varphi.$$

From the triangles PSn and nZS, we have

$$\cos \mathrm{S}\,n = \sin \delta \cos \varphi - \cos \delta \sin \varphi \cos h$$
$$= \sin z \cos a$$

$\therefore \sin z.\, d\,z = -\sin z \cos a.\, d\,\varphi$

or $d\,\varphi = -\sec a.\, d\,z.$

Hence if $d\,z$ be the error in the zenith distance of the star, the error in the latitude would be $-\sec a$ times that error.

The least value for this is when $a = o$ or π and therefore any error in the zenith distance of a star affects the latitude least when the star is on the meridian.

Similarly, the effect of a small error in the observed altitude of a body on the hour angle can be calculated as follows:—The zenith distance used in the above formula (1) is erroneous. Differentiating this, regarding φ and δ as constants, we obtain,

$-\sin z\, dz = -\cos \varphi \cos \delta \sin h.\, d\,h$......................(2)

Also, if a is the azimuth of the body measured from the north point through west, south and east, we get from the triangle PZS,

$$\frac{\sin z}{\sin h} = \frac{\cos \delta}{\sin a} \text{ ....(3)}$$

substituting in (2), we have,

$d\,h = \sec \varphi \operatorname{cosec} a.\, d\,z$..............................(4)

From (4) it is found that any error in the observed altitude would least affect the calculated time, if the observation is made on or near the prime vertical.

Again, the effect of an error in the declination of a body on its hour angle can be estimated as follows;—

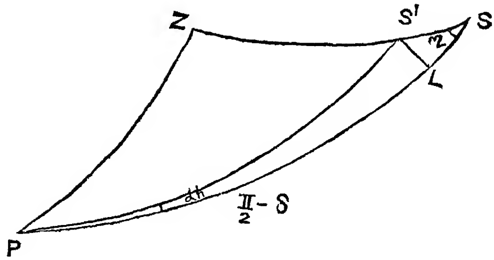


Fig. 62.

The parallactic angle η of a body is the angle between the vertical and the declination circle through the body.

Let S′ be the apparent position due to the error in declination and S, the true position of the body.

Draw S′ L perpendicular to P S

Considering S′ S L as a plane triangle, we have

$$S'L = SL \tan \eta$$

or $\quad d\,h \cos \delta = d\,\delta \tan \eta$

or $\quad d\,h = \tan \eta \sec \delta\, d\,\delta$

If in the above observation, there is a small error in the latitude of the place, its effect on the hour angle is determined as follows:—

Differentiating (1) regarding z and δ as constants,

$$0 = (\cos \varphi \sin \delta - \sin \varphi \cos \delta \cos h)\, d\,\varphi$$
$$- \cos \varphi \cos \delta \sin h\, d\,h.$$

or $\sin z \cos a\, d\,\varphi = \cos \varphi \cos \delta \sin h\, d\,h$

$= \sin z \sin a \cos \varphi\, \mathrm{d}\,h$, where a is the azimuth.

$\therefore\ d\,h = \cot a \sec \varphi\, d\,\varphi.$

Now $d\,h$ is least when a is $\frac{\pi}{2}$ or $\frac{3\pi}{2}$ and therefore the best time for observation is when the body is on the prime vertical.

81. Local time by meridian observation.

The determination of the time at any place is important in itself. Besides this, it is also essential in the determination of the longitude of a place, as we shall see later on. We have already seen that in a place where the meridian has been fixed and where there is a transit circle, the sidereal time at any instant when a star is observed to transit is just equal to the right ascension of the star. The setting of the mean time clock can be done either by converting this time into corresponding mean time or by observing the instant of the transit of the centre of the sun's disc, and making the clock read at that time the time of meridian passage of the sun on that day, which is known from the equation of time for the day. In a place where the meridian has not been previously determined the instant of meridian passage of a star or the sun is the instant when the body attains the greatest altitude. This can always be ascertained by a sextant or a theodolite. (See chapter on Astronomical Instruments).

82. Local time by a single ex-meridian altitude.

If the latitude of a place is known a single altitude observation would give the local time. If P is the pole, Z the zenith, and S, the body observed, the three sides of the triangle Z P S are known.

$$\therefore \quad \cos z = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos h \text{.....................} (1)$$

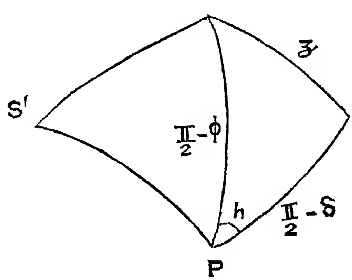


Fig. 63

This gives h, the hour angle of the body.

If S is the sun, what we get will be the apparent time, and if the equation of time is added to this, we shall obtain the mean time. If S be a star of known R. A., the hour angle of S together with its R. A., gives the sidereal time at the moment.

83. Determination of the time and meridian by the method of equal altitudes.

The method of equal altitudes not only serves to determine the local time, but is also useful in determining the meridian line as the azimuths of a star are equal when its zenith distances are the same before and after culmination. (Here the azimuth is measured from the north or south point in opposite directions.) Therefore the direction bisecting these two directions of the body at equal altitudes gives the meridian line. Let the readings of the horizontal circle of a theodolite or an altazimuth be taken when the body has equal altitudes. Now the reading of the horizontal circle when the telescope describes the meridian plane is the one corresponding to the arithmetic mean of these two readings. To ensure the telescope describing the meridian plane, one has merely to set it so as to make the horizontal circle give the above reading.

If the instants when a celestial body has the same zenith distance before and after culmination are known, the local time at the instant when the body transits is easily calculated, provided the declination of the body does not undergo any change during the interval between the two observations. (This is true in the case of a star)

Let S′ and S be the positions of the body when it has the same altitude, east and west of the meridian of a place.

Then the triangles Z P S′ and Z P S are congruent.

$\therefore\ Z\hat{P}S' = Z\hat{P}S$

$\therefore$ If t_1 and t_2 be the times when the body is at S′ and S

respectively $\left(\frac{t_2-t_1}{2}\right)$ would be the interval after t_1 that the body would take to culminate.

$\therefore\ t_1+\frac{t_2-t_1}{2}=\frac{t_1+t_2}{2}$ is the time at the moment of culmination of the body. Either a sextant or a theodolite can be used in this observation. If the sun is the body observed, PS and PS′ will not be equal as the sun's declination changes and so a small correction has to be applied. The chief advantages of this method consist in the fact that observations are made on the body when the circumstances are the same and therefore corrections due to instrumental errors, refraction, dip etc. do not affect the result.

84. Other methods of determining the meridian.

If observation is made on the sun, a small rod (gnomon) fixed on to a horizontal plane and a small circle with the foot of the rod as centre would enable us to get both the time of apparent noon and also the meridian of the place. All that we have to do, is to observe the times and points when and where the extremity of the shadow cast by the rod will touch the circle. At these two instants of time, the sun has the same altitude and azimuth east and west of the meridian. The bisector of the angle subtended by the two points at the centre of the circle gives the meridian line.

At sea, the north and south line can be determined from the magnetic north and south line as shown by the compass needle, if the deviation of the compass needle for the place be known. These deviations are different for different places on the earth and have been tabulated for reference.

Another method of determining the north and south line at sea would be to observe the directions where the sun rises and sets. The north and south line bisects the

angle between these two directions. The small change in the declination of the sun between the two observations is neglected.

By two observations of the pole star when its azimuth readings are farthest from the meridian, it is possible to get the direction of the meridian line, which is merely the direction given by the reading midway between the two readings.

When the latitude of the place is known, a single observation of the altitude of a known body gives the direction of the meridian as can be seen from the following.

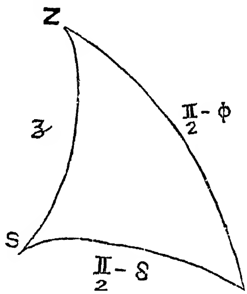


Fig. 64.

In the triangle P Z S, all the sides are known.

$$\tan\frac{PZS}{2}=\sqrt{\frac{\sin(s-PZ)\sin(s-z)}{\sin s\sin(s-SP)}}$$

Where $2s=PZ+ZS+SP$.

This gives the azimuth of S and therefore the direction of the meridian.

If the hour angle of the body be known, then the latitude need not be known; for, if h be the hour angle,

$$\frac{\sin h}{\sin z}=\frac{\sin PZS}{\cos\delta}$$

$\therefore$ PZS is known.

85. Determination of Longitude.

1. We have already seen that the difference between the local times of two stations A and B is $\frac{1}{15}\times$ the longitude

difference of the two places. Therefore if one could carry an accurate time recorder, say a chronometer indicating the time of a place like Greenwich, then all that is necessary to determine the longitude of a place east or west of Greenwich is to calculate the local time by any of the methods already indicated and see how far the calculated time is ahead or behind the Greenwich time. This value multiplied by 15 gives the longitude of the place east or west of Greenwich. Here we consider the meridian of Greenwich to be the initial line from which longitudes are measured east and west.

If the chronometer has an error and an unknown rate then allowance has to be made for these. Suppose we wish to find the longitude of a station P. For this a chronometer is compared with the standard clock at Greenwich and its error E_1 is known. It is then compared with the standard clock at P and its error E_2 on that clock also is noted. Again it is brought back to Greenwich and compared a second time with the standard clock there and the error E_3 is known. Let T be the interval in hours between the two comparisons at Greenwich. The rate of the chronometer is given by $\frac{E_3 - E_1}{T}$. From this the Greenwich time that should be indicated by the chronometer when the comparison took place at P is known. The difference between this and the correct local time gives the longitude of the place.

2. Another method of arriving at the longitude difference between two places A and B will be to arrange two observers stationed in these places to see some phenomenon visible to both and note the corresponding local times. From the difference of the times, the longitude difference is deduced. In the case of most of the celestial phenomena whose occurrence could be predicted beforehand, such as the eclipses of the satellites of Jupiter, the

beginning or end of a lunar eclipse or occultations of stars by the moon, their times of occurrence are given in the Nautical Almanac for the meridian of Greenwich. Therefore if the local time of the occurrence of any of these phenomena is noted by an observer he can deduce his longitude from that. This method is not capable of giving accurate values as the times of the observations of these phenomena could not themselves be estimated accurately. For instance, the disappearance of a satellite of Jupiter is gradual and not sudden and again the boundary of the earth's shadow on the moon during a lunar eclipse is so undefined that even the best observers differ by a few minutes in their estimation of the beginning or the end of an eclipse.

A flash of light from some source such as a rocket fired at a station or the bursting of a meteor could also be used in determining the longitude difference between two stations A and B, from where these signals could be seen and the local times of occurrence of these signals could be recorded.

3. If two places A and B are electrically connected the following method helps us to find their longitude difference.

Let a signal sent from A the more easterly station at time T_1 of A be received at B at time T_2 of B. Then provided the transmission of the signal is instantaneous, the longitude difference in hours would be $T_1 - T_2$. If x be the time of transmission of the signal then the longitude difference $= T_1 + x - T_2$

If another signal is also sent from B to A at time T_2' of B and received at A at T_1' of A, we have the same longitude difference given by $T_1' - (T_2' + x)$

$\therefore$ Longitude difference between A and B $= \dfrac{T_1 - T_2 + T_1' - T_2'}{2}$

In this we have not taken into consideration the personal equation of the observers at A and B. Allowance could be made for this if we could take another pair of observations interchanging the observers at the stations A and B.

If E_1 is the correction for personal equation of the observer at A and E_2, that of the observer at B in the first pair of observations, we have,

the longitude difference $L = \dfrac{T_1 - T_2 + T_1' - T_2'}{2} + E_1 - E_2.$

If the observations be now repeated and the corresponding times be denoted by small letters, we have

$L = \dfrac{t_1 - t_2 + t_1' - t_2'}{2} + E_2 - E_1$ since the observer at A has now a personal equation E_2 and the observer at B has a personal equation E_1.

$$\therefore \qquad L = \tfrac{1}{4}\ (T_1 - T_2 + T_1' - T_2' + t_1 - t_2 + t_1' - t_2')$$

86. Longitude by lunar distances.

The moon moves through 360° relative to the stars in $27\frac{1}{3}$ days and this is equal to 1″ in every two seconds of time. Now this motion of the moon could easily be found as there are many bright stars which lie on the path of the moon and the angular distances of the moon's centre from some of the important bright stars could be derived from the hourly places of the moon, given in the Nautical Almanac. Therefore when the angular distance observed is any quantity, the corresponding Greenwich time could be computed by proportional parts, assuming that the angular distance changes uniformly in a short period. Now if the local time at the instant of observation of the angular distance between any star and the moon be known, then we get the longitude of the place from the difference between this time and the calculated Greenwich time.

In measuring the angular distance between a star and the moon, the instrument often used is a sextant and the reading given is the apparent distance between the two. Now the reading given in the Nautical Almanac leads to the geocentric distance between the two at different times. Therefore from the sextant observations we have to get the geocentric distance after applying the due corrections for parallax, refraction and dip. This operation is what is called the method of clearing the distance and is explained in the next paragraph. Besides one observer taking the sextant reading of the apparent angular distance of the star from the moon, there are usually two others also taking simultaneously the altitudes of the moon and the star. Any one of these latter readings would give the local time so that one need not refer to any standard clock for ascertaining the local time at the moment of observation.

87. Method of clearing the distance.

Let S and M be the geocentric positions of the star and the moon after applying the above corrections and S′ and M′ be their apparent positions. In the case of a

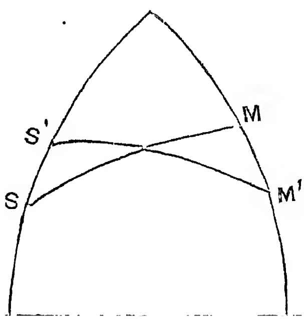


Fig. 65.

star the refraction effect is the only correction whereas for the moon, besides the refraction correction there is also the parallax correction which is much greater and is opposite in character to the refraction effect,

Let the apparent altitudes of the star and the moon be a' and b' and the apparent distance between them be d' and let the corresponding true quantities be a, b, d.
If the angle SZM be θ, we have

$$\cos d = \sin a \sin b + \cos a \cos b \cos \theta.$$

$$\cos d' = \sin a' \sin b' + \cos a' \cos b' \cos \theta.$$

$$\frac{\cos d - \sin a \sin b}{\cos a \cos b} = \frac{\cos d' - \sin a' \sin b'}{\cos a' \cos b'} \qquad (1).$$

Equation (1)gives d as a and b can be known from a' and b' from the laws of refraction and parallax. It can be transformed into another so as to suit logarithmic computation as follows:—

$$\frac{\cos d - \sin a \sin b}{\cos a \cos b} + 1 = \frac{\cos d' - \sin a' \sin b'}{\cos a' \cos b'} + 1$$

ie. $$\frac{\cos d + \cos (a+b)}{\cos a \cos b} = \frac{\cos d' + \cos (a'+b')}{\cos a' \cos b'}$$

or $$\frac{\left(1 - 2 \sin^2 \frac{d}{2}\right) + \left(2 \cos^2 \frac{a+b}{2} - 1\right)}{\cos a \cos b}$$

$$= \frac{2 \cos \frac{a'+b'+d'}{2} \cos \frac{a'+b'-d'}{2}}{\cos a' \cos b'}$$

or $$\sin^2 \frac{d}{2} = \cos^2 \frac{a+b}{2} - \frac{\cos a \cos b}{\cos a' \cos b'} \cos \frac{a'+b'+d'}{} \cos \frac{a'+b'-d'}{}$$

$$= \cos^2 \frac{a+b}{2} \left(1 - \sin^2 \theta\right)$$

$$= \cos^2 \frac{a+b}{} \cos^2 \theta.$$

or $$\sin \frac{d}{2} = \cos \frac{a+b}{2} \cos \theta \qquad (2)$$ where,

$$\sin^2 \theta = \frac{\cos a}{\cos a} \, \frac{\cos b}{\cos b'} \, \frac{\cos\frac{a'+b'+d'}{2} \cos\frac{a'+b'-d'}{2}}{\cos^2 a+b}$$

88. Sumner's method of determining a ship's position at sea.

Though any one of the methods given before would enable an observer to locate his position by knowing the longitude and latitude of his place, there is another way of locating one's position by taking two altitudes of the sun and noting the Greenwich times of observation, given by a chronometer (See Chapter on Instruments). This method is due to Captain Sumner and is often used by the mariners as an easy means of knowing their position on the globe.

Before determining the position of the observer, he has to locate the point on the earth at whose zenith the sun is situated at the moment of observation. The latitude and west longitude of such a point are given by the declination of the sun at the moment and the Greenwich apparent time. Now the Greenwich apparent time is obtained from the chronometer time by subtracting the equation of time from it. This point is called the *sub-solar point.*

The observed zenith distance of the sun is the same as the angular distance of the place of observation from the sub-solar point. Therefore the place of observation lies on a circle whose centre is the sub-solar point and whose angular radius is equal to the observed zenith distance of the sun. Such a circle is called the *circle of position.*

The two altitudes of the sun observed and the two chronometer times, give two circles of position, intersecting at two points, one of which is the observer's position. To

decide between these two points there are other things such as the azimuth of the sun that could be calculated and that would agree with only one of the two points.

The same method could be used on two stars for which the substellar points could be constructed. The latitudes and longitudes of these points are given by the declinations of the stars and their hour angles for Greenwich. The zenith distances observed give as before the circles of position, and from these the position of the observer can be ascertained.

CHAPTER IX.

PRECESSION AND NUTATION.

89. Introduction.

The phenomenon of precession was known to the Hindus many centuries before the Christian Era. Hipparchus, a Grecian Astronomer, by comparing the positions of the stars he observed, with the earlier observations on record, came to the conclusion that the longitude of all stars underwent an annual increase of 36″, while their latitudes remained the same. This was explained by the hypothesis, that the Vernal Equinox or the first point of Aries had an annual retrograde motion of the same magnitude. As there is no change in the latitude of the stars it follows that the ecliptic plane remains fixed. The pole of the equator is thus rotating round the pole of the ecliptic, thereby causing the retrograde motion of the first point of Aries. This retrograde motion is known as *precession* and its actual value is about 52.″26 per year.

90. Dynamical explanation of precession.

As the shape of the earth is spheroidal, and as the sun and the moon do not remain in the equatorial plane always, it follows that the resultant attraction of the sun or the moon on the earth, is not a single force through the centre of gravity of the earth. Therefore it affects the axis of rotation of the earth, by giving it a velocity of rotation round the axis of the ecliptic. A force acting through the centre of gravity of a rotating body will have no effect on the rotation of the body about its centre of gravity. The resultant direction of the force of attraction between the earth and the sun or the moon, does not lie in the equatorial plane PQ, but acts along a direction such as SL in the plane NSN′. If the earth was not a rotating

body this force would tend to make NN′ perpendicular to SL. But the rotation of the earth about its axis NN′, makes this axis rotate about an axis perpendicular to SL, just as the axis of a top which is in rotation about its axis

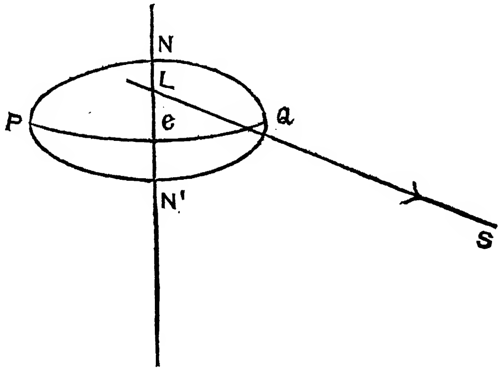


Fig. 66.

of symmetry, is itself seen rotating about the vertical. At each instant, the axis of the top is moving at right angles to the direction of gravity without falling, owing to the fact that its rotational velocity is much more than the conical motion of the axis itself. In the case of the earth, the diurnal rotation is much more rapid than the rotation of the axis about the normal to the plane of the ecliptic. This motion of the terrestrial axis about the axis of the ecliptic is called the *precession due to the sun.*

The moon also exerts a similar precessional action on the earth's axis. Although the moon's attractive force on the earth is much less than that of the sun, the precessional effect depends only on the difference between the attractions exerted by the moon on different parts of the earth. The proximity of the moon makes this difference nearly double of what the sun is able to produce.

By solar precession, the earth's axis takes about 26,000 years to make one revolution about the pole of the ecliptic; but by lunar precession the pole of the lunar

orbit makes a revolution round the pole of the ecliptic in 19 years. As the moon is very near the plane of the ecliptic the principal part of the moon's precessional force, supposing the latter to be on the ecliptic, and the precession due to the sun constitute what is termed *luni-solar precession.* By this, the first point of Aries, and the first point of Libra, move in the ecliptic in a retrograde direction at the rate of 50.″26 a year, keeping the inclination of the equator to the ecliptic at its constant value ω.

91. Nutation.

The influence of the remaining part of the attractive force exerted by the moon, after allowing for its effect if it were on the ecliptic plane consists in changing the value of ω periodically, and also in giving a small periodical displacement to ♈ from its mean position given by luni-solar precession. These changes are known by the term *nutation* and were discovered by the great Astronomer Bradley.

92. Planetary Precession.

In explaining lunisolar precession the ecliptic was assumed to be fixed and the equator was found to be moving in a retrograde direction keeping the same inclination to the ecliptic. But owing to the attraction between the other planets and the earth, the plane of the ecliptic itself is not fixed and this causes additional changes in the positions of the equinoctial points and also in the value of the obliquity. These changes are termed planetary precession.

93. The effects of luni-solar precession on the right ascension and declination of a star.

By lunisolar precession, the celestial pole rotates round the pole of the ecliptic annually, with a constant angular velocity K. (say). The effect of this on the stars could be seen by giving to the celestial sphere containing

the stars, a rotation K about the pole of the ecliptic in the opposite direction and it is easy to find how such a

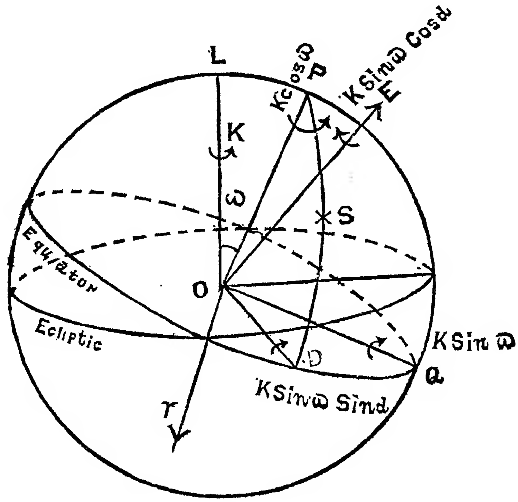


Fig. 67.

rotation affects the right ascension and declination of all the stars. Let K be the rotation about O L given to the star S of right ascension α and declination δ.

Resolving the rotation K about O P and O Q, P being the celestial pole, we get K cos ω about O P.........(a), and—K sin ω about O Q, negative sign denoting clockwise rotation. Resolving the rotation—K sin ω about O Q, about O D, and about O E perpendicular to O D, (where O D is the intersection of the equatorial plane and the plane of the declination circle through the star),

we get, $-$K sin ω sin α about O D..........................(b)

and $-$K sin ω cos α about O E..........................(c)

Now, the effects of precession are the effects of rotations (a), (b) and (c). Of these (a) and (b) affect right ascension only.

If $d\alpha'$ is the change in the R. A due to (b) alone

$$d\alpha' \cos\delta = +K \sin\omega \sin\alpha \sin\delta$$

$$\text{i. e. } d\alpha' = +K \sin\omega \sin\alpha \tan\delta$$

Hence change in R. A due to precession is given by

$$\Delta\alpha = K\cos\omega + K\sin\omega\sin\alpha\tan\delta \quad\text{..........(1)}$$

And change in declination is given by

$$\Delta\delta = K\sin\omega\cos\alpha \quad\text{..............................(2)}$$

94. Effects of Precession and Nutation.

If the co-ordinates of a star are subject to changes not only due to precession but also due to a change $\Delta\omega$ in ω due to nutation, we can get the same by considering the effect of a counterclockwise rotation of the celestial sphere $\Delta\omega$ about O♈. Now $\Delta\omega$ about O♈ can be resolved into $\Delta\omega\cos\alpha$ about O D and $-\Delta\omega\sin\alpha$ about O E, the former affecting right ascension only and the latter, declination only.

Hence the change in right ascension due to a counterclockwise rotation of $\Delta\omega\cos\alpha$ about O D

$$= -\Delta\omega\cos\alpha\tan\delta.$$

And the change in declination due to a rotation of $-\Delta\omega\sin\alpha$ about O E $= \Delta\omega\sin\alpha$.

Hence, the total change in R. A. due to precession K and nutation $\Delta\omega = K\cos\omega + K\sin\omega\sin\alpha\tan\delta$

$$-\Delta\omega\cos\alpha\tan\delta$$

$$= K\cos\omega + \tan\delta\,[K\sin\omega\sin\alpha - \Delta\omega\cos\alpha] \quad\text{......(3)}$$

The change in declination

$$= K\sin\omega\cos\alpha + \Delta\omega\sin\alpha \quad\text{..................(4)}$$

Formulae (3) and (4) can be put in the form

$$\alpha' - \alpha = f + \tfrac{1}{15}g\sin(G+\alpha)\tan\delta \quad\text{.........(5)}$$

$$\delta' - \delta = g\cos(G+\alpha)$$

where $f = \frac{1}{15}K\cos\omega$, $g\cos G = K\sin\omega$ and

$$g\sin G = -\Delta\omega.$$

Hence f, g, and G are independant of the star's co-ordinates and depend only on the day of the year. They are called *independant day numbers* and could be computed for each day of the year. These results can be

applied to all stars for getting their co-ordinates as affected by precession and nutation on any date.

95. Effect of precession and nutation on the length of the sidereal day.

The sidereal day is the interval between two consecutive transits of the first point of Aries. Since ♈ is not uniformly moving due to precession and nutation, the length of the sidereal day is not constant. It is convenient to take as the sidereal day, the average of all these apparent sidereal days. The actual position of ♈ round about its mean position, completes a cycle in the course of about $18\frac{1}{2}$ years and the variations of the actual sidereal days are found to range between 23 hrs. 59 m. 58.86 sec. and 24 hrs. 0 m. 1.14 sec. The errors in the clock due to the irregularities in the motion of ♈ are not usually taken into account in ordinary cases.

CHAPTER X.

THE ABERRATION OF LIGHT.

96. Introduction.

The rays of light proceeding to us from a star do not reach us instantaneously, though they travel at an enormously great speed. The velocity of light and that of the earth together cause an apparent shift in the position of the star and this is known as *aberration*. This quantity is indeed very small and though its presence was known even from the year 1680 when Picard announced such a phenomenon, it remained unexplained till the time of Bradley (1728). He explained how the true position of a star could be ascertained by applying to the apparent position, a correction for aberration.

97. Relative velocity and aberration.

Let P Q represent the velocity of the observer due to the motion of the earth, and R Q the velocity of light from a star. Now the relative velocity of light coming from a star is obtained by giving to the observer a velocity equal and opposite to the one possessed by him

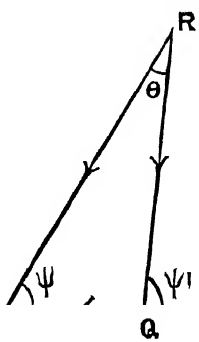


Fig. 68.

and combining it with the velocity of light. This process gives R P as the velocity of light relative to the observer.

Therefore the apparent direction of the star is PR whereas its true direction is QR. The angle PRQ (θ) is called the *aberration*. The point O towards which the observer is moving at any time is called the *apex*. Due to aberration the apparent position of a star is nearer to the apex than the true position.

Let v be the velocity of the observer, c the velocity of light and ψ the angle between the apparent direction of the star and that of the apex. Then if θ be the aberration, we have $\sin\theta = \frac{v}{c}\sin\psi$, which is the expression giving the amount of aberration of a star. As $\frac{v}{c}$ is a very small quantity we may use ψ' instead of ψ without any great error. Also, since θ is a small angle, we can replace $\sin\theta$ by the circular measure of the angle θ in the same equation. Thus the equation of aberration can be put as follows :—

Aberration of a star in seconds of arc $= a'' = k\sin\psi'$ where k is a constant which can be determined in many ways as described later.

98. Different kinds of aberration.

(i) *Annual aberration.*

The direction and amount of displacement of a star due to aberration depend upon the co-ordinates of the apex and also the velocity of the observer. All observers at rest on the earth have ordinarily two velocities and corresponding apices. The one is due to the revolution of the earth round the sun and the other is due to the rotation of the earth about its own axis. Due to the first motion, taking the earth's orbit to be a circle the apex of the observer is always a point on the ecliptic 90° behind the sun and such a point has a periodic motion round the ecliptic once in a year.

The following figure would perhaps be of use in illustrating the above fact.

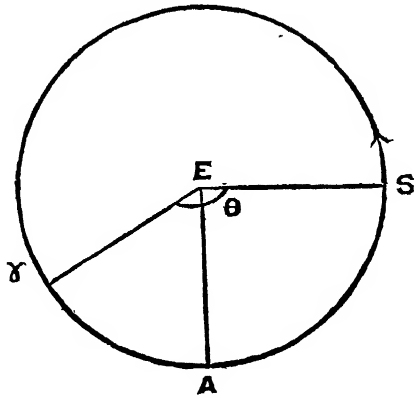


Fig. 69.

Let the sun move apparently round the ecliptic in the direction marked. Taking the ecliptic to be a great circle of the celestial sphere, the velocity of the earth at any time is towards A, 90° behind the sun on the ecliptic.

Thus the longitude of the apex A is $(\theta - 90°)$ where θ is the longitude of the sun at the moment. As the apex has here a periodic motion, the apparent place of a star also changes periodically due to this kind of aberration. This is called *annual aberration,* as it is due to the annual motion of the earth round the sun.

(ii) *Diurnal aberration.*

The second velocity of the observer is that due to the diurnal rotation of the earth and the apex of the observer due to this velocity is the east point of the horizon. The displacement of a star due to this velocity of the observer is called the *diurnal aberration.* The velocity of the observer due to the earth's rotation in latitude φ is found to be about 460 cos φ metres per second and the velocity of light from a star is 300,000 kilometres per second. Therefore the coefficient $\frac{v}{c}$ for diurnal aberration becomes

$0''.3 \cos \varphi$ nearly. This quantity is usually neglected, except when very great accuracy is required in defining the actual position of a star. For a star on the meridian, the time of transit is delayed by the diurnal aberration.

(iii) *Planetary aberration*

In the two kinds of aberration mentioned above, we assumed that the star which sent the light was stationary. If the star also is in motion, its apparent position would be quite different from what we got hitherto. Though this kind of aberration is not taken into account in calculating the true position of a star from its apparent position (as the stars do not appreciably change their relative positions in the course of a few years though they may have individually large velocities), such an aberration has to be considered in the case of the moon and the planets which are moving fast with respect to the fixed stars in the celestial sphere.

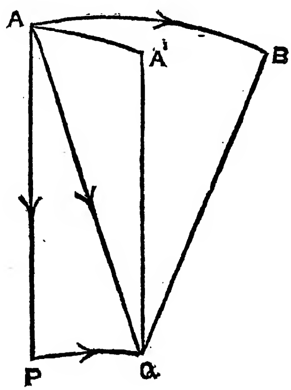


Fig. 70.

Let AB be part of the path of a planet, described by it in time t. Let the earth describe PQ in the corresponding time. Let the light that left the planet while it was at A reach the earth while the latter is at Q. Then AQ is the actual path of the light from the planet in time t. To get the relative velocity of the light from the planet

to the observer, we have to give the observer, a movement equal and opposite to P Q and compound it with A Q. Thus we get A P as the apparent direction of the planet as seen by the observer. The observer is at Q when the light from the planet reaches him and therefore he sees the planet in the direction Q A′ parallel to P A. But the actual position of the planet at the end of time t is B. Hence the planetary aberration is the angle B Q A′. To get the apparent place of a planet from the actual place due to aberration, the following procedure may therefore be adopted. Let t be the time that light takes to reach the earth from the present position of the planet. Then it is possible to know the exact places of the earth and the planet at time t before the present moment. Now the true direction of the planet from the earth at that time was the apparent direction of the planet now. The true position of the planet differs from this by the planetary aberration A′ Q B.

Of these kinds of aberration, the annual aberration is the most important. So we shall next consider some of its effects on the true position of a star.

99. The effect of annual aberration on the latitude and longitude of a star.

Let P be the true position of a star of latitude β and longitude λ, ♈ A D S, the ecliptic where S is the position of the sun at the moment considered and A the apex (so that A S $=90^\circ$) to which the earth is carried away, assuming the earth's orbit to be circular. On the great circle PA, P′ will be the apparent place of the star where, P P′ $= k \sin$ P A (k being the aberration constant)

Draw P′N perpendicular to P D.

Then $\delta\lambda$ (change in longitude) $=$ P′N $\sec\beta$.

$$= k \sin PA \sin APD \sec\beta$$

$$= k \sin PA \frac{\sin AD}{\sin PA} \sec\beta$$

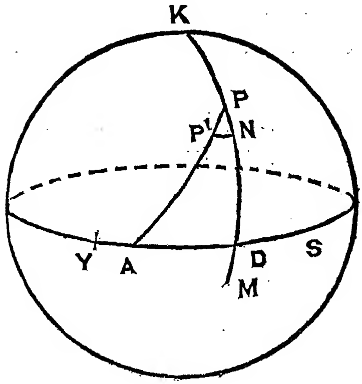


Fig. 71.

$$= k \sin AD \sec \beta$$

$$= k \sec \beta \sin [\lambda - (\theta - 90^\circ)]$$ where θ is the sun's longitude at the moment

$$= k \sec \beta \cos (\lambda - \theta) \ldots\ldots (1)$$

This is the quantity to be added to the apparent position in order to get the true position of the star.

Again, $\delta \beta$ (change in latitude) $= PN$

$$= k \sin PA \cos APD$$

$$= k \cos AM \quad (\text{if } PM = 90^\circ)$$

$$= k \cos AD \sin PD$$

$$= k \cos (\lambda - \theta + 90^\circ) \sin \beta$$

$$= k \sin (\theta - \lambda) \sin \beta \ldots\ldots (2)$$

This is the quantity to be added to the apparent latitude to get the true latitude of the star.

From the above formula, it is easily seen that the effect of aberration is to make a star move in a small ellipse round its true position.

For, assuming $P'N = x$ and $PN = y$ referred to P as origin and taking rectangular axes on the tangent plane at P to the celestial sphere, we have

$$\left(\frac{x}{k}\right)^2 + \left(\frac{y}{k \sin \beta}\right)^2 = \cos^2 (\theta - \lambda) + \sin^2 (\theta - \lambda) = 1$$

The major axis of the ellipse of aberration is parallel to the ecliptic and is equal to $2k$ and the minor axis is equal to $2k \sin \beta$.

100. To find the aberration of a star in right ascension and declination.

Let P be the true place of a star of R. A. (α) and declination (δ) and P′ its apparent place on the celestial

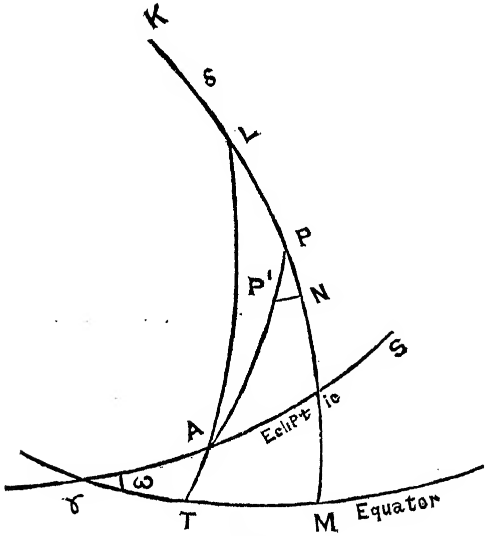


Fig. 72.

sphere. ♈ A S is the ecliptic and ♈ M the equator. L is the pole and $PLK = 90°$, measured on the declination circle through the star. S is the position of the sun on the ecliptic at the moment and A the apex 90° behind the sun.

Draw L A T the secondary through A to the equator.

Draw P′N perpendicular to P M.

If the aberration in R. A. be $\delta\alpha$,

we have, $\delta\alpha \cos\delta = P'N = k \sin PA \sin P'PN$

$$= k \sin ALP \sin LA = k \sin TM \cos AT$$

$$= k \cos AT \sin(\alpha - ♈T)$$

$$= k\,[\cos AT \cos ♈T \sin\alpha - \cos AT \sin ♈T \cos\alpha]$$

$$= k\,[\sin\alpha \cos ♈A - \cos\alpha \sin ♈A \cos\omega]$$

$\therefore\ \delta\alpha$ (aberration in R·A)

$$= k \sec\delta\,[\sin\alpha\sin\theta + \cos\alpha\cos\theta\cos\omega]$$

where θ is the sun's longitude at the moment.

This is the quantity to be added to the apparent right ascension to get the true right ascension of the star.

If the aberration in declination be $\triangle\,\delta$, we have

$$\begin{aligned}
\triangle\,\delta = PN = k \sin PA.\cos P'PN &\\
&= -k\cos AK \text{ (since } PK = 90^\circ) \\
&= -k\,[\cos AL\cos KL - \sin AL\sin KL \\
&\qquad\qquad \cos ALP] \\
&= -k\,[\sin AT\cos\delta - \cos AT\sin\delta\cos(\alpha - \Upsilon T)] \\
&= -k\,[\cos\delta\sin\omega\sin\Upsilon A - \sin\delta\,(\cos AT\cos\Upsilon T. \\
&\qquad \cos\alpha + \cos AT\sin\Upsilon T\sin\alpha)] \\
&= -k\,[\cos\delta\ \sin\omega\ \sin(\theta - 90) - \sin\delta\cos\alpha. \\
&\quad \cos(\theta - 90) - \sin\delta\sin\alpha\sin(\theta - 90)\cos\omega] \\
&= \quad k\,[\cos\delta\cos\theta\sin\omega + \sin\delta\sin\theta\cos\alpha \\
&\qquad - \sin\delta\sin\alpha\cos\theta\cos\omega]
\end{aligned}$$

This is the quantity to be added to the apparent declination to get the true declination of the star.

101. Bradley's discovery of aberration.

The phenomenon of aberration was discovered by the astronomer Bradley in the year 1728. He was observing the star Υ Draconis, whose R. A. was 270° nearly, and wanted to explain the periodic changes in its polar distance. This star always transited very near the zenith at the place of observation (London lat. 51°) and therefore the error due to refraction was negligible. The polar distance of the star was observed to be the greatest and the least at the end of March and September respectively, the difference between the two values being about 39·″5. The sun's position on these two dates being very near the equinoxes, it was possible to account for these changes on the assumption of an aberrational change in the star's

declination. Taking the sun's longitudes on the two dates to be 0° and 180°, the changes in the star's true declination due to aberration should be $-k \sin(\omega+\delta)$ and $+k \sin(\omega+\delta)$. The value of $(\omega+\delta)$ being nearly 75°, $2k \sin 75°$ should be the maximum change in the declination of the star. Substituting the value of k (20·″5) it is found to agree with the observed change viz., 39·″5. If k be treated as an unknown quantity, this observation gives us the value of k from the equation $2k \sin 75° = 39''\!.5$,

$$\therefore\ k = 20''\!.5 \text{ nearly.}$$

102. A rough method of determining the constant of aberration.

By comparing a number of observations of the eclipses of the satellites of jupiter, it is found that light takes about 8 minutes 18 seconds to come to the earth from the sun. Now, the earth travels in its orbit during this time a distance $\dfrac{498 \times 2\pi r}{365\frac{1}{4} \times 24 \times 60 \times 60}$ where r is the radius of the earth's orbit round the sun supposed to be circular.

$$\frac{v}{c} = \frac{498 \times 2\pi r}{365\frac{1}{4} \times 24 \times 60 \times 60 \times r}.$$

If the constant of aberration k is expressed in seconds of

$$\text{arc, } k = \frac{60 \times 60 \times 180}{\pi}\ \frac{v}{c}$$

$$= \frac{498 \times 2\pi \times 60 \times 60 \times 180}{365\frac{1}{4} \times 24 \times 60 \times 60 \times \pi}$$

$$= 20 \cdot 45.$$

Now, the aberration for a star is given by $a = 20 \cdot 45'' \sin \psi$ where ψ is the angle between the apex and the true direction of the star. The value of the constant is a varying quantity as the velocity of the earth is not constant as has been supposed.

103. Method of determining the aberration constant by the zenith telescope.

Two stars S_1, S_2 differing in R. A. by about 180° and culminating very near and on opposite sides of the zenith are chosen. These stars are observed at first when S_1 transits at about 6 A. M. and therefore S_2 transits at about 6 P. M. These stars are again observed six months later when S_2 will transit at 6 A. M. and S_1 at 6 P. M. Fulfillment of these conditions enables the constants to be derived with the best accuracy possible. (Ref. Ball's Astronomy, Edition 1908, pages 264 & 265).

Let α_1 and δ_1 be the mean values of R. A. and declination of S_1 at the beginning of the year of observation. Let an approximate value of aberration be supposed to be known by any of the previous methods discussed (say k). Then if $k+k_1$ be the more accurate value of the constant, then k_1 is to be found. From the mean values of the R. A. and declination at the beginning of the year, the apparent value of declination on the date of observation is obtained allowing for precession, nutation and also using the approximate aberration constant. Let p_1 be the correction for δ_1, thus obtained.

If φ is the latitude of the place and z_1, the zenith distance of the star S_1, corrected for refraction, we have

$$\begin{aligned}\varphi &= z_1+\delta_1+p_1+k_1\,[\sin\delta_1\sin\alpha_1\cos\omega\cos\theta \\ &\qquad -\cos\delta_1\sin\omega\cos\theta-\sin\delta_1\cos\alpha_1\sin\theta\,] \\ &= z_1+\delta_1+p_1+k_1\,c_1 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\quad (1)\end{aligned}$$

Observations of the star S_2, whose mean R. A. and declination are α_2 and δ_2 respectively on the same day 12 hours later give,

$$\varphi = -z_2+\delta_2+p_2+k_1\,c_2 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\quad (2)$$

On repeating the observations six months later and taking into account even the small change which the latitude of the place might have undergone, we get

$$\varphi' = z_1' + \delta_1 + p_1' + c_1' k_1 \qquad (3)$$

$$\varphi' = -z_2' + \delta_2 + p_2' + c_2' k_1 \qquad (4)$$

From (1) and (2),

$$0 = z_1 + z_2 + \delta_1 - \delta_2 + p_1 - p_2 + k_1 (c_1 - c_2) \qquad (5)$$

This would itself give k_1, if δ_1 and δ_2 are known accurately.

But from (3) and (4)

$$0 = z_1' + z_2' + \delta_1 - \delta_2 + p_1' - p_2' + k_1 (c_1' - c_2') \qquad (6)$$

Now (5) and (6) give on eliminating $(\delta_1 - \delta_2)$

$$k_1 (c_1' - c_2' - c_1 + c_2) = z_1 - z_1' + z_2 - z_2' + p_1 - p_2 - p_1' + p_2'$$

or $k_1 = \dfrac{z_1 - z_1' + z_2 - z_2' + p_1 - p_2 - p_1' + p_2'}{c_1' - c_2' - c_1 + c_2}$

In obtaining the value of k_1 it was the practice to observe z_1, z_2, z_1' and z_2' by the meridian circle, but at present, observations could be taken by a fixed zenith telescope having a camera attachment. Both the stars will leave their trails on the photographic plate and the distance between their trails gives the value of $z_1 + z_2$ and also $z_1' + z_2'$.

104. Effects of aberration on the spherical co-ordinates of a body.

Let η, ζ be the spherical co-ordinates of a point P on the celestial sphere.

Let r be the radius of the celestial sphere, and x, y, z be the rectangular co-ordinates of P, referred to the axes through the observer as centre and towards the three points of R. A. and declination; 0°, 0°; 90°, 0°; and 0°, 90°;

$$\therefore\ x = r \cos\zeta \cos\eta\ ;\ y = r \cos\zeta \sin\eta\ ;\ \&\ z = r \sin\zeta.$$

Let v_0 be the velocity of the observer moving towards a point (η_0, ζ_0) and μ be the true velocity of light from the point P (the true place of star) and let μ' be the apparent velocity of light from the apparent position of the star (η', ζ').

Then, we have the following equations by projection of the true and the apparent velocities along the three rectangular axes chosen.

$$\mu \cos \zeta \cos \eta = \mu' \cos \zeta' \cos \eta' - v_o \cos \zeta_o \cos \eta_o \text{.....(1)}$$
$$\mu \cos \zeta \sin \eta = \mu' \cos \zeta' \sin \eta' - v_o \cos \zeta_o \sin \eta_o \text{.....(2)}$$
$$\mu \sin \zeta = \mu' \sin \zeta' - v_o \sin \zeta_o \text{......................(3)}$$

Multiplying (1) by $\sin \eta'$, (2) by $\cos \eta'$ and subtracting, we get $\mu \cos \zeta \sin (\eta' - \eta) = - v_o \cos \zeta_o \sin (\eta' - \eta_o)$......(4)

Multiplying (1) by $\cos\left(\frac{\eta+\eta'}{2}\right)$, (2) by $\sin\left(\frac{\eta+\eta'}{2}\right)$ and adding, we get

$$\mu \cos \zeta \cos \left(\frac{\eta-\eta'}{2}\right) = \mu' \cos \zeta' \cos \left(\frac{\eta'-\eta}{2}\right)$$
$$- v_o \cos \zeta_o \times \cos \left(\eta_o - \frac{\eta+\eta'}{2}\right) \text{.......................... (5)}$$

Dividing the above by $\cos\left(\frac{\eta-\eta'}{2}\right)$, we have

$$\mu \cos \zeta = \mu' \cos \zeta' - v_o \cos \zeta_o \cos \left(\eta_o - \frac{\eta+\eta'}{2}\right) \sec \frac{\eta-\eta'}{2}$$
$$\text{............(6)}$$

Multiplying (3) by $\cos \zeta'$, (6) by $\sin \zeta'$ and subtracting

$$\mu \sin (\zeta' - \zeta) = - v_o \cos \zeta_o \sin \zeta' \cos \left(\eta_o - \frac{\eta+\eta'}{2}\right) \sec \frac{\eta-\eta'}{2}$$
$$+ v_0 \sin \zeta_0 \cos \zeta' \text{.........(7)}$$

Since $\eta' - \eta$ and $\zeta' - \zeta$ are small, we get from (4), and (7)

$$\eta' - \eta = - \frac{v_o}{\mu} \cos \zeta_o \sin (\eta - \eta_o) \sec \zeta,$$ where η is put for η' on the right side

and $$\zeta' - \zeta = - \frac{v_o}{\mu} \cos \zeta_o \sin \zeta \cos (\eta_o - \eta)$$

$+ \frac{v_o}{\mu} \sin \zeta_o \cos \zeta$, where ζ is put for ζ' on the right side.

From the above, it is easy to deduce the aberration of a star in any system of co-ordinates; for instance, to get aberration in R. A. and declination put $\eta=\alpha$ and $\zeta=\delta$ and we get,

$$\alpha'-\alpha=\frac{v_0}{\mu}\cos\delta_0\sec\delta\sin(\alpha_0-\alpha)\text{.........(A) and}$$

$$\delta'-\delta=\frac{v_0}{\mu}[\sin\delta_0\cos\delta-\cos\delta_0\sin\delta\cos(\alpha_0-\alpha)]\text{...(B)}$$

105. Effect of the eccentricity of the earth's orbit on aberration.

The aberrational effects so far considered were under the assumption, that the earth's orbit was circular. We have now to see, how they are modified when the actual orbit of the earth is considered.

Let S be the sun's geocentric longitude. Then the earth's heliocentric longitude is S+180°. Let K be the longitude of perihelion and θ, the true anomaly.

We have $S+180^\circ=K+\theta$.

$$\therefore\ \frac{dS}{dt}=\frac{d\theta}{dt}$$

Let r be the radius vector to the earth from the sun and let the velocity of the earth v be directed at any time towards the point (α_0, δ_0).

Now the components of this along three perpendicular axes, chosen in the usual manner are :—

$v\cos\alpha_0\cos\delta_0$, $v\cos\delta_0\sin\alpha_0$ and $v\sin\delta_0$ and these are respectively equal to $-\frac{d}{dt}(r\cos S)$,

$-\frac{d}{dt}(r\sin S\cos\omega)$ and $-\frac{d}{dt}(r\sin S\sin\omega)$, since the co-ordinates of the sun with respect to the earth as origin are $r\cos S$, $r\sin S\cos\omega$, and $r\sin S\sin\omega$.

Also, from the equation to the earth's orbit round the

sun, $\dfrac{l}{r} = 1 + e \cos \theta$

and by logarithmic differentiation,

$$\frac{dr}{dt} = \frac{r\, e \sin \theta}{1 + e \cos \theta} \frac{d\theta}{dt}$$

$$= \frac{r^2\, d\theta / dt}{r\,(1 + e \cos \theta)}\, e \sin \theta \quad = c\, e \sin \theta,$$

$$\text{where } c = \frac{r^2}{l} \frac{d\theta}{dt} = \frac{h}{l}$$

$$\text{Also } r \frac{d\theta}{dt} = c. \frac{l}{r} = c\,(1 + e \cos \theta).$$

$$\therefore \quad v \cos \delta_\circ \cos \alpha_\circ = -\frac{d}{dt}(r \cos \mathrm{S}) = c\,[-e \sin \theta \cos \mathrm{S} + \sin \mathrm{S}\,(1 + e \cos \theta)]$$

$$= c\,(\sin \mathrm{S} - e \sin \mathrm{K}) \quad \text{since } 180^\circ + \mathrm{S} = \mathrm{K} + \theta.$$

$$v \cos \delta_\circ \sin \alpha_\circ = c \cos \omega[-e \sin \theta \sin \mathrm{S} - \cos \mathrm{S}\,(1 + e \cos \theta)]$$

$$= c \cos \omega\,[-\cos \mathrm{S} + e \cos \mathrm{K}]$$

$$v \sin \delta_\circ = c \sin \omega\,[-e \sin \theta \sin \mathrm{S} - \cos \mathrm{S}\,(1 + e \cos \theta)]$$

$$= c \sin \omega\,[-\cos \mathrm{S} + e \cos \mathrm{K}]$$

Substituting these values in the equations (A) and (B) for the aberrations in R. A. and declination, we have

$$\alpha' - \alpha = \frac{c}{\mu} \sec \delta\,(-\sin \alpha \sin \mathrm{S} - \cos \alpha \cos \mathrm{S} \cos \omega)$$

$$+ e \frac{c}{\mu} \sec \delta\,[\sin \alpha \sin \mathrm{K} + \cos \alpha \cos \mathrm{K} \cos \omega]$$

$$\delta' - \delta = \frac{c}{\mu}\,[\cos \omega \sin \alpha \sin \delta \cos \mathrm{S} - \sin \omega \cos \delta \cos \mathrm{S} - \cos \alpha \sin \delta \sin \mathrm{S}]$$

$$+ e \frac{c}{\mu}\,[\sin \omega \cos \delta \cos \mathrm{K} + \cos \alpha \sin \delta \sin \mathrm{K} - \cos \omega \sin \alpha \sin \delta \cos \mathrm{K}]$$

The quantity $\frac{c}{\mu}$ is known as the constant of aberration. The second term in each of the above equations is independant of the sun's longitude and has only a small effect on aberration (as e is a factor in it) and this remains practically unaltered during some years for the same star. Unless a high degree of accuracy is expected, this term is omitted in ordinary calculations.

The net result of considering the earth's orbit as elliptic and not circular is thus to change the constant of aberration from $\frac{v}{\mu}$ to $\frac{c}{\mu} = \frac{h}{l\,\mu} = \frac{2\,\pi\,a\,b}{\mathrm{T}\,l\,\mu} = \frac{2\,\pi\,a}{\mu\,\mathrm{T}\,(1-e^2)^{\frac{1}{2}}}$ where a is the semi-major axis and e the eccentricity of the elliptic orbit of the earth and T the length of the year. The constant of aberration given above is in circular measure.

106. Analytical expression for planetary aberration.

Planetary aberration can be derived analytically by assuming the corpuscular theory of light, in the following manner.

Let x_0, y_0. z_0, and x, y, z, be the co-ordinates of the position of a planet P and that of the earth at time t_0; let the velocities of the planet and the earth remain unaltered during the interval (say t) that light takes to reach the earth from the planet.

Let (X, Y, Z), (X_0, Y_0, Z_0) and (x', y', z') be the components of the velocities of a ray of light, the planet and the earth respectively.

Then we have the following equations, connecting the velocities of the earth and the planet.

$$x_0 + (\mathrm{X} + \mathrm{X}_0)\,t = x + x'\,t$$
$$y_0 + (\mathrm{Y} + \mathrm{Y}_0)\,t = y + y'\,t$$
$$z_0 + (\mathrm{Z} + \mathrm{Z}_0)\,t = z + z'\,t$$

or
$$x_0 + Xt = x + (x' - X_0)t$$
$$y_0 + Yt = y + (y' - Y_0)t$$
$$z_0 + Zt = z + (z' - Z_0)t$$

The above equations show that planetary aberration could be obtained by compounding with the actual velocity of the earth (x', y', z') a velocity equal and opposite to that of the planet (X_0, Y_0, Z_0) and then regarding the planet at rest.

Consider the case of a planet V moving in a circular orbit in the plane of the ecliptic, the earth's orbit being also supposed to be circular.

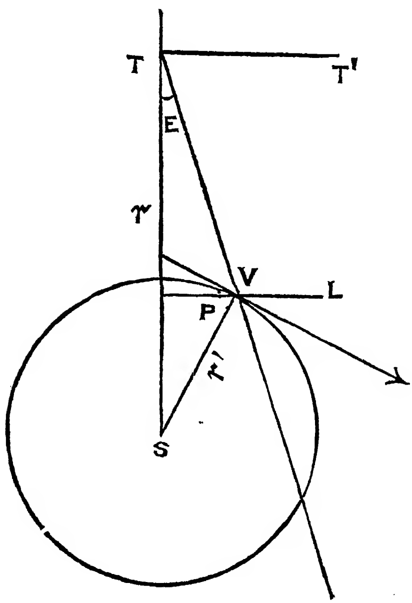


Fig. 73.

Let S represent the sun, T, the earth moving with velocity v along T T' and V, the planet. Let r and r' be the radii of their orbits. If θ and θ' be the angular velocities of the earth and the planet, $r\,\theta = v$ and $r'\,\theta' = v'$,

where v and v' are the velocities of the earth and the planet respectively and if $\frac{2\pi}{\theta} = \mathrm{T}$ and $\frac{2\pi}{\theta'} = \mathrm{T}'$, be their sidereal periods, we have from Kepler's 3rd law,

$$\frac{\mathrm{T}^2}{\mathrm{T}'^2} = \frac{r^3}{r'^3} = \frac{\theta'^2}{\theta^2} = \frac{\frac{v'^2}{r'^2}}{\frac{v^2}{r^2}} = \frac{v'^2\, r^2}{v^2\, r'^2}$$

$$\therefore \frac{v'}{v} = \sqrt{\frac{r}{r'}} \quad \therefore v' = v\left(\frac{r}{r'}\right)^{\frac{1}{2}}$$

Let E be the elongation of V as seen from T and P the elongation of T as seen from V

$\hat{\mathrm{S}} = \pi - (\mathrm{E} + \mathrm{P})$

Resolving along and perpendicular to T T′, we have after changing the sign of the velocity of the planet V and compounding it with the velocity of earth,

$$v + v' \cos(\mathrm{E}+\mathrm{P}) \text{ along T T}'$$
$$\text{and } -v' \sin(\mathrm{E}+\mathrm{P}) \text{ along T S.}$$

Taking T T′ as x axis and T S as y axis, we have, using the first two equations of the aberrational effects on the spherical co-ordinates of a body and putting in (1) and (2) of article 104, $\zeta = 0$, $\zeta' = 0$, $\zeta_0 = 0$. $\eta = 90° - \mathrm{E}$;

$$\eta' = 90° - \mathrm{E}',\ v \cos \eta_0 = v + v' \cos(\mathrm{E}+\mathrm{P})$$
$$\text{and } v \sin \eta_0 = -v' \sin(\mathrm{E}+\mathrm{P})$$

$$\therefore \mu \sin \mathrm{E} = \mu' \sin \mathrm{E}' - v - v' \cos(\mathrm{E}+\mathrm{P})$$
$$\mu \cos \mathrm{E} = \mu' \cos \mathrm{E}' + v' \sin(\mathrm{E}+\mathrm{P})$$

Eliminating μ' and remembering that $\mathrm{E}' - \mathrm{E}$ is small

$$\mu \sin(\mathrm{E}' - \mathrm{E}) = v \cos \mathrm{E} + v' \cos \mathrm{P}$$

If v_0 be the velocity of a planet of the solar system at the distance r_0,

$$v = \frac{v_0 \sqrt{r_0}}{\sqrt{r}}\ ; \text{ and } v' = \frac{v_0 \sqrt{r_0}}{\sqrt{r'}}$$

$$E' - E = \frac{v_0}{\mu} \sqrt{r_0} \left(\frac{\cos E}{\sqrt{r}} + \frac{\cos P}{\sqrt{r'}} \right)$$

$$= \text{planetary aberration.}$$

It is to be noted, that when the correction for planetary aberration is applied, what is really obtained is the true position of the planet, when the ray of light left it, i. e. the position of the planet t seconds earlier than the time of observation, where t is the time taken by light to travel between the earth and the planet.

CHAPTER XI.

REFRACTION

107. General refraction and the fundamental laws.

When a ray of light passes from one medium to another of different density, it undergoes a change in direction. This phenomenon is called *refraction* and follows certain well-known laws, which are :—

(1) The incident ray, the refracted ray and the normal at the point of incidence, to the boundary of the two media lie in the same plane.

(2) The ratio of the sine of the angle of incidence to that of the angle of refraction is a constant for the same two media. This constant is called the *index of refraction.* The index of refraction depends also on the nature of light. When light passes from one medium to a denser medium, the index of refraction is greater than unity; hence the ray gets bent towards the normal, on entering the denser medium.

108. Media bounded by parallel planes.

Consider a medium B bounded by two parallel planes lying within a medium A. (see fig. 74) Let P Q R S be the path of a ray of light in the two media. Draw K Q K′ and L′ R L normal to the boundary planes. Let $\angle KQP = i$, $\angle K'QR = \angle L'RQ = r$ and $\angle LRS = i'$.

If μ be the index of refraction,

$$\frac{\sin i}{\sin r} = \mu \text{ and } \frac{\sin i'}{\sin r} = \mu.$$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{\sin i'}{\sin r};$$

$$\therefore \quad \angle i = \angle i';$$

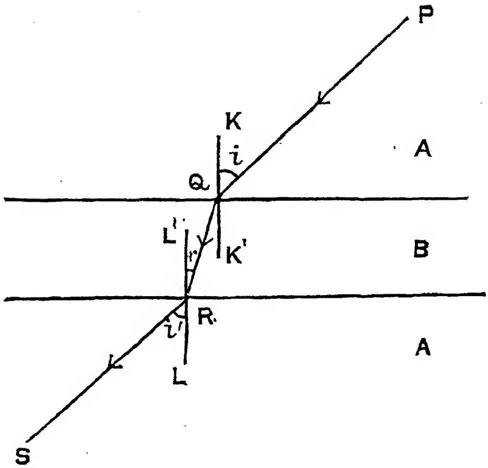


Fig. 74.

Since the two normals are parallel, PQ is parallel to RS. Thus the direction of a ray in A remains the same, whether a medium B or several media like B be inserted in A, or not. Also, when there are media of different densities separated by parallel planes lying within a medium A, the path of a ray of light in A will get refracted in each of these media exactly as if the ray passes from A to the individual medium directly. In other words, if $A_1, A_2, A_3 \ldots\ldots A_n$ are n media bounded by parallel planes lying within a medium A, and if $PP_1\ P_2 \ldots Pn \ldots P'$ be the path of a ray, the paths $P_2\ P_3$, $P_3\ P_4$ etc. are those of rays $Q_2\ P_2$, $Q_3\ P_3$ etc. parallel to PP_1. (Fig. 75)

Let $\mu_1, \mu_2, \mu_3 \ldots \mu n$ be the indices of refraction for a ray of light from A to A_1, A_2 etc. to An respectively, and let ${}_1\mu_2, {}_2\mu_3, \ldots\ldots$ be the indices for a ray from A_1 to A_2, A_2 to A_3 etc. The latter can be expressed in terms of the former.

$$\frac{\sin i}{\sin r_1} = \mu_1 ; \qquad \ldots\ldots (1)$$

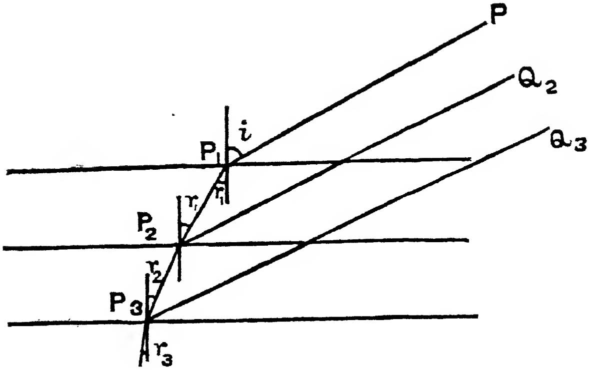


Fig. 75.

$$\frac{\sin i}{\sin r_2} = \mu_2 ;. \qquad .(2)$$

$$\frac{\sin i}{\sin r_3} = \mu_3 ;. \qquad .(3)$$

etc.

$\sin i = \mu_1 \sin r_1 = \mu_2 \sin r_2 = \mu_n \sin r_n$. Again if i' is the angle that the ray $P_n P'$ makes with the normal,

$$\frac{\sin i'}{\sin r_n} = \mu_n = \frac{\sin i}{\sin r_n} :$$

$\therefore \angle i = \angle i'$;

Hence $P P_1$ and $P_n P'$ are parallel:

Further $\frac{\mu_1}{\mu_2} = \frac{\sin r_2}{\sin r_1}$

But $\frac{\sin r_1}{\sin r_2} = {}_1\mu_2$

${}_1\mu_2 = \frac{\mu_2}{\mu_1}$

Similarly ${}_2\mu_3 = \frac{\mu_3}{\mu_2}$

${}_3\mu_4 = \frac{\mu_4}{\mu_3}$ etc.

109. Atmospheric refraction.

The earth's atmosphere acts as a refracting medium in all astronomical observations. Hence the direction in which a star is seen, is altered and the observed co-ordinates of the star have to be corrected for this error due to refraction, to get the star's true position.

The earth's atmosphere extends to about hundred miles and is of varying density. Barometric pressure, temperature, height above the surface of the earth and humidity are some of the factors that change the density of the atmosphere from point to point.

But for astronomical purposes, it is convenient to replace this non-homogeneous atmosphere by a series of thin spherical shells concentric with the earth's surface (assumed spherical) throughout each of which the density is uniform. It may be assumed that the index of refraction for each layer is constant, the change from layer to layer being infinitesimal and the indices of refraction gradually decreasing from its value at the earth's surface to unity at the outermost layer. Hence a ray of light proceeding from a star gets refracted towards the zenith as it enters each of these layers (see Fig. 78). In other words for the observer the star is shifted towards the zenith. It will be observed that the entire path of the ray lies in the vertical plane passing through the star.

An approximate formula for correction due to refraction for bodies near the zenith may be obtained by neglecting the curvature of the earth's surface and the several layers of the atmosphere. In this case, the problem reduces to that of a ray of light from a star X passing through a series of a media bounded by parallel planes. If the number of layers be $n+1$ with indices of refraction $\mu_0, \mu_1, \mu_2, \ldots \mu_n$, and if the angles of incidence and refraction at the r^{th} medium be z_{r-1} and z_r respectively, $\mu_0 \sin z_0 = \mu_1 \sin z_1 = \mu_2 \sin z_2 = \ldots \quad \ldots \mu_n \sin z_n$. If the

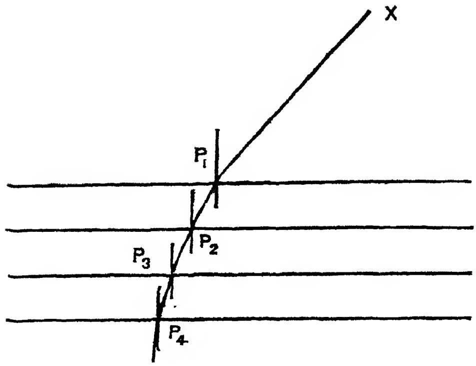


Fig. 76.

$(n+1)^{th}$ layer be vacuum extending to the star, $\mu_n=1$ and z_n is the true zenith distance of the star. If ρ is the effect of refraction expressed in radian measure, $z_n=z_0+\rho$:

$$\therefore \quad \mu_0 \sin z_0 = \mu_n \sin z_n$$
$$= \sin (z_0+\rho).$$

where μ_0 is the index of refraction at the surface of the earth, its value being 1·00029. Expanding $\sin(z_0+\rho)$ and substituting $\cos\rho=1$ and $\sin\rho=\rho$, since ρ is small.

$$\mu_0 \sin z_0 = \sin z_0 + \rho \cos z_0.$$
$$\therefore \quad \rho \cos z_0 = \sin z_0\,(\mu_0-1).$$
$$\therefore \quad \rho = (\mu_0-1)\tan z_0. \text{ where } \rho \text{ is in circular measure}$$
$$= \frac{(\mu_0-1)\,180\times60\times60}{\pi}\tan z_0 \text{ seconds of arc.}$$
$$= 206265\,(\mu_0-1)\tan z_0 \text{ seconds.}$$

This formula does not give reliable values for large zenith distances. This is because, when rays of light traverse the earth's atmosphere from low altitudes, they pass through an extensive region of the earth's atmosphere in such a way that the curvature of the several strata cannot be neglected.

110. Cassini's formula of refraction.

A formula for refraction based upon the hypothesis of a homogeneous atmospheric shell surrounding the earth was derived by Cassini. Though this is not representing the actual condition of the atmosphere, the formula gives fairly satisfactory results for large zenith distances except very near the horizon.

Let a ray of light from a star X enter the homogeneous atmosphere at A and reach the observer at B along A B. Join O A and O B, O being the centre of the

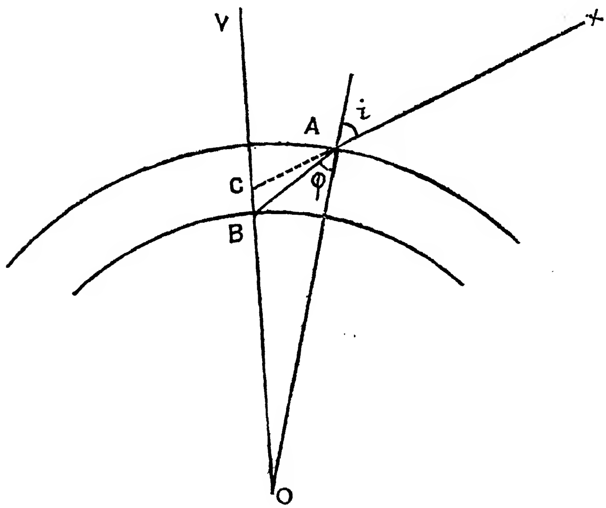


Fig. 77.

earth. Produce X A to cut O B at C. Let the observed zenith distance V B A be z_0 and the real zenith distance V C A be z. If ρ be the amount of refraction,

$$\rho = z - z_0 = \angle BAC$$

$= i - \varphi$, where i is the angle of incidence and φ the angle of refraction at A.

$$\therefore \quad i = \varphi + \rho$$

Now $\sin i = \mu \sin \varphi$.

i. e. $\sin(\varphi + \rho) = \mu \sin \varphi$.

This gives $\rho = (\mu - 1) \tan \varphi$

From triangle OAB, $\frac{\sin \varphi}{\sin z_0} = \frac{a}{a+h}$, a being the earth's radius and h being the height of the atmosphere.

$$\therefore \tan \varphi = \frac{a \sin z_0}{\sqrt{(a+h)^2 - a^2 \sin^2 z_0}} .$$

Since h is small compared to a, higher powers of $\frac{h}{a}$ than the first, may be neglected. Then,

$$\tan \varphi = \frac{\sin z_0}{\left(\cos^2 z_0 + \frac{2h}{a}\right)^{\frac{1}{2}}}$$

$$= \tan z_0 \left(1 + \frac{2h}{a} \sec^2 z_0\right)^{-\frac{1}{2}}$$

$$= \tan z_0 \left(1 - \frac{h}{a} \sec^2 z_0\right)$$

$$= \left(1 - \frac{h}{a}\right) \tan z_0 - \frac{h}{a} \tan^3 z_0.$$

$$\therefore \rho = (\mu - 1) \left\{ \left(1 - \frac{h}{a}\right) \tan z_0 - \frac{h}{a} \tan^3 z_0 \right\}$$

Thus the value of ρ may be expressed as $A \tan z_0 + B \tan^3 z_0$, A and B being numerical constants.

111. Differential formula for atmospheric refraction

A detailed study of atmospheric refraction may be made by deriving the differential equation to the path of a ray of light travelling in a non-homogeneous medium. Consider two consecutive spherical layers of the earth's atmosphere with refractive indices μ and μ' and radii r and r' respectively. Let P Q T be the path of a ray of light. Join OP and OQ, O, being the centre of the earth.

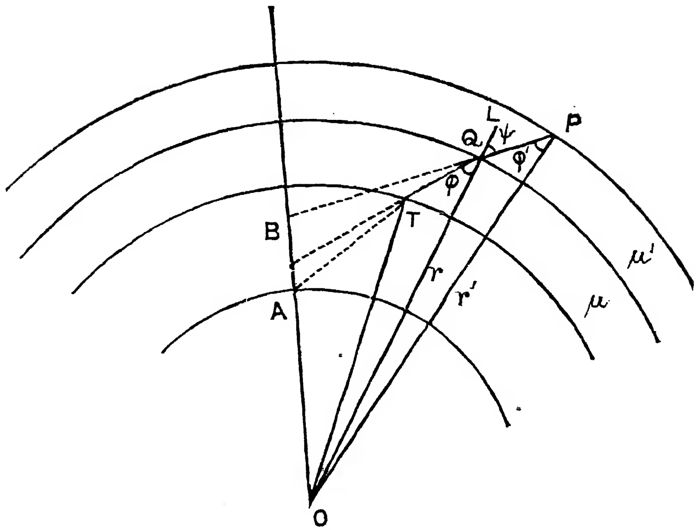


Fig. 78.

Taking angles as marked in the figure, the change of direction of the ray is $\psi-\varphi$ and let this be denoted by $\triangle\rho$.

Now $\dfrac{\sin\psi}{\sin\varphi}=\dfrac{\mu}{\mu'}$, $\therefore$ $\sin\psi=\dfrac{\mu}{\mu'}\sin\varphi$.

But $\psi=\varphi+\triangle\rho$; and $\mu'=\mu-\triangle\mu$.

$$\therefore \quad \sin(\varphi+\triangle\rho)=\frac{\mu}{\mu-\triangle\mu}\sin\varphi.$$

Expanding both sides and keeping only first order of infinitesimals,

$$(\sin\varphi+\triangle\rho\cos\varphi)=\left(1+\frac{\triangle\mu}{\mu}\right)\sin\varphi.$$

$$\therefore \quad \triangle\rho=\frac{\triangle\mu}{\mu}\tan\varphi.$$

To find $\tan\varphi$ in terms of the observed zenith distance of a body, proceed thus:—

From triangle OPQ, $\dfrac{\sin\psi}{\sin\varphi'}=\dfrac{r'}{r}$.

$$\therefore \quad \sin\psi=\frac{r'}{r}\sin\varphi'.$$

$$\therefore \quad \frac{r'}{r}\sin\varphi' = \frac{\mu}{\mu'}\sin\varphi$$

$$\therefore \quad r\,\mu\,\sin\varphi = r'\,\mu'\,\sin\varphi'.$$

This formula holds good at every point of the path of light and at the earth's surface A. If a is the radius of the earth's surface and μ_0 the refractive index, then

$r\,\mu\,\sin\varphi = a\,\mu_0\,\sin z$, where z is the angle that the ray makes with OA (i. e. z is the zenith distance of the star as seen from A)

$$\therefore \sin\varphi = \frac{a\,\mu_0\sin z}{r\,\mu}.$$

$$\therefore \tan\varphi = \frac{a\,\mu_0\sin z}{\sqrt{r^2\mu^2 - a^2\mu_0^2\sin^2 z}}$$

$$\therefore \quad \triangle\rho = \frac{\triangle\mu}{\mu}\cdot\frac{a\,\mu_0\sin z}{[r^2\mu^2 - a^2\mu_0^2\sin^2 z]^{\frac{1}{2}}}$$

This formula gives the amount by which a ray of light is refracted in passing through two consecutive layers of the earth's atmosphere. The total effect of refraction is therefore the integral of $\triangle\rho$, the limits of integration being μ_0 at the earths' surface and 1 at the outermost layer. Thus we have

$$\rho = a\,\mu_0\sin z\int_1^{\mu_0}\frac{1}{\mu}\left(r^2\mu^2 - a^2\mu_0^2\sin^2 z\right)^{-\frac{1}{2}}\triangle\mu.$$

112. Integration of the differential equation for atmospheric refraction.

The difficulty of performing this integration arises as no definite relation between r and μ is known. An approximate integration may however be performed by the substitution $r = a\,(1+x)$. For practical purposes since the height of the atmosphere is small compared to the earth's radius, x may be considered to be small.

Now $\left(r^2 \mu^2 - a^2 \mu_0^2 \sin^2 z\right)^{-\frac{1}{2}}$

$$=\left(a^2 (1+x)^2 \mu^2 - a^2 \mu_0^2 \sin^2 z\right)^{-\frac{1}{2}}$$

$$=\frac{1}{a}\left(\mu^2 (1+2x) - \mu_0^2 \sin^2 z\right)^{-\frac{1}{2}} \text{ (neglecting } x^2\text{)}$$

$$=\frac{1}{a}\left((\mu^2 - \mu_0^2 \sin^2 z) + 2x\mu^2\right)^{-\frac{1}{2}}$$

$$=\frac{1}{a}(\mu^2 - \mu_0^2 \sin^2 z)^{-\frac{1}{2}}\left(1+\frac{2x\mu^2}{\mu^2 - \mu_0^2 \sin^2 z}\right)^{-\frac{1}{2}}$$

$$=\frac{1}{a}(\mu^2 - \mu_0^2 \sin^2 z)^{-\frac{1}{2}}\left(1-\frac{x\mu^2}{\mu^2 - \mu_0^2 \sin^2 z}\right)$$

This expansion is applicable only if $2x\mu^2$ is less than $\mu^2 - \mu_0^2 \sin^2 z$; and since μ and μ_0 are each very nearly equal to unity, the value of z should not be nearly equal to 90° i. e. the body should nowhere be near the horizon.

$$\therefore \rho = \frac{a\mu_0 \sin z}{a} \int^{\mu_0} \frac{d\mu}{\mu\sqrt{\mu^2 - \mu_0^2 \sin^2 z}}$$

$$-\int_1^{\mu_0} \frac{x\mu^2\, d\mu}{\mu(\mu^2 - \mu_0^2 \sin^2 z)^{\frac{3}{2}}}$$

$$= \mu_0 \sin z \int_1^{\mu_0} \frac{d\mu}{\mu\sqrt{\mu^2 - \mu_0^2 \sin^2 z}}$$

$$-\mu_0 \sin z \int_1^{\mu_0} \frac{x\mu\, d\mu}{(\mu^2 - \mu_0^2 \sin^2 z)^{\frac{3}{2}}}$$

Let this be $\rho_1 - \rho_2$.

Now, $\rho_1 = \mu_0 \sin z \displaystyle\int_1^{\mu_0} \frac{d\mu}{\mu\sqrt{\mu^2 - \mu_0^2 \sin^2 z}}$

$$= \sin^{-1}\left(\frac{\mu_0 \sin z}{\mu}\right)_{\mu_0}^{1}$$

$$= \sin^{-1}(\mu_0 \sin z) - z.$$

Since μ_0 is nearly equal to unity, putting $\mu_0 = 1+\theta$, $\sin^{-1}(\mu_0 \sin z)$ can be expanded as a power series in θ. Let $f(\theta) = \sin^{-1}[(1+\theta)\sin z]$

$$f'(\theta) \equiv \frac{\sin z}{\sqrt{1-(1+\theta)^2 \sin^2 z}};$$

$$f''(\theta) \equiv -\tfrac{1}{2}\frac{\sin z}{[1-(1+\theta)^2 \sin^2 z]^{\frac{3}{2}}}\left(-2(1+\theta)\sin^2 z\right)$$

$$= \frac{(1+\theta)\sin^3 z}{[1-(1+\theta)^2 \sin^2 z]^{\frac{3}{2}}}.$$

$$\therefore f(0) = z;$$

$$f'(0) = \tan z;$$

$$f''(0) = \tan^3 z; \text{ etc.}$$

Now $f(\theta) = f(0) + \theta f'(0) + \frac{\theta^2}{2} f''(0) + \ldots\ldots\ldots\ldots\ldots\ldots$

$$\therefore \sin^{-1}(\mu_0 \sin z) = z + \theta \tan z + \frac{\theta^2}{2}\tan^3 z.$$

$$\therefore \rho_1 = \theta \tan z + \frac{\theta^2}{2}\tan^3 z.$$

$$= (\mu_0 - 1)\tan z + \frac{(\mu_0-1)^2}{2}\tan^3 z$$

In evaluating the second integral, the following law (called Gladstone and Dale's Law) connecting the refractive index μ to the atmospheric density may be used.

$\mu = 1 + c\sigma$ where c is a constant very nearly equal to 0·23 and σ the density of the atmosphere of refractive index μ. Changing the variable μ to σ, we get

$$\rho_2 = \mu_0 \sin z \int_0^{\sigma_0} \frac{x(1+c\sigma)}{[(1+c\sigma)^2 - (1+c\sigma_0)^2 \sin^2 z]^{\frac{3}{2}}}\, c.\,d\sigma$$

$$= c\mu_0 \sin z \int_0^{\sigma_0} \frac{x}{\cos^3 z}\, d\sigma, \text{ as } c\sigma = \mu - 1 \text{ and}$$

$c\sigma_0 = \mu_0 - 1 = 0{\cdot}00029$ and are therefore negligible.

$$\text{Now} \int_0^{\sigma_0} x\, d\sigma = \Big(x\,\sigma\Big)_0^{\sigma_0} - \int_0^{\sigma_0} \sigma\, dx$$

$$= 0 + \int_0^h \sigma\, dx \text{ where } x = h \text{ for } \sigma = 0$$

and $x = 0$ for $\sigma = \sigma_0$

= mass of the column of atmosphere of height h on unit area.

= M (say)

$\therefore \quad \rho_2 = \text{M}.\ c\mu_0 \tan z \sec^2 z$

$= \text{A} \tan z + \text{B} \tan^3 z$; A and B being constants.

Hence total refraction is given by,

$\rho = \rho_1 - \rho_2 = \text{K} \tan z + \text{L} \tan^3 z$ where K and L may be considered constants.

(*Note*: It is to be observed that K and L are constants that may be determined from physical data. But it is always desirable that K and L are directly determined from observations of several stars.)

113. Simpson's formula.

Various other formulae have been suggested for refraction. Thus Simpson assumes that r and μ may be taken to satisfy an equation of the form $r\,\mu^{n+1} = r'$, n being at present unknown and r' being the value of r when μ is unity.

Now $r\,\mu \sin \varphi = a\,\mu_0 \sin z$.

Taking logarithms and differentiating,

$$\frac{dr}{r} + \frac{d\mu}{\mu} + \cot\varphi\, d\varphi = 0.$$

$$\therefore \frac{dr}{r} = -\frac{d\mu}{\mu} \quad \cot\varphi\, d\varphi.$$

From the relation $r\mu^{n+1} = r'$

$$\frac{dr}{r} \qquad \frac{n+1}{\mu}\, d\mu.$$

$$\therefore \frac{n+1}{\mu}\, d\mu = \frac{d\mu}{\mu} + \cot\varphi\, d\varphi.$$

$$\therefore \frac{n}{\mu}\, d\mu = \cot\varphi\, d\varphi.$$

$$\therefore \tan\varphi\, \frac{d\mu}{\mu} = \frac{d\varphi}{n}.$$

$$\text{Now } d\rho = \tan\varphi\, \frac{d\mu}{\mu} = \frac{d\varphi}{n}.$$

$$\therefore \rho = \int_{z'}^{z} \frac{d\varphi}{n}.$$

At the upper boundary of the atmosphere where the ray from any star enters at a zenith distance z',

we have $r\mu \sin\varphi = r' \sin z' = a\,\mu_0 \sin z$

or $a\,\mu_0^{\,n+1} \sin z' = a\,\mu_0 \sin z$

$$\text{or } z' = \sin^{-1}\left(\frac{\sin z}{\mu_0^{\,n}}\right).$$

And at the earths' surface $\varphi = z$.

$$= \left[\frac{\varphi}{n}\right]_{\sin^{-1}\frac{\sin z}{\mu_0^{\,n}}}^{z}.$$

$$= \frac{1}{n}\left\{ z - \sin^{-1}\frac{\sin z}{\mu_0^{\,n}} \right\}.$$

The above formula due to Simpson can be reduced to the form $A \tan z + B \tan^3 z$. If in this formula μ_0 be put equal to $1+\theta$, $\sin^{-1}\left(\frac{\sin z}{\mu_0^{\,n}}\right)$ can be expanded as a power series in θ.

$$\text{Let } \sin^{-1}\left(\frac{\sin z}{(1+\theta)^n}\right) = u.$$

$$\therefore \sin z = (1+\theta)^n \sin u.$$

Taking logarithms and differentiating,

$$\frac{n}{(1+\theta)} + \cot u \frac{du}{d\theta} = 0$$

$$\therefore \frac{du}{d\theta} = -n \frac{\tan u}{1+\theta}.$$

$$\therefore \frac{d^2u}{d\theta^2} \quad -n \left\{ \frac{\sec^2 u}{1+\theta} \frac{du}{d\theta} - \frac{\tan u}{(1+\theta)^2} \right\}$$

$$= -n \left\{ -\frac{n \sec^2 u \tan u}{(1+\theta)^2} - \frac{\tan u}{(1+\theta)^2} \right\}$$

$\therefore$ Putting $\theta = 0$,

$$u_{\theta=0} = z,$$

$$\left(\frac{du}{d\theta}\right)_{\theta=} = -n \tan z.$$

$$\left(\frac{d^2u}{d\theta^2}\right)_{\theta=0} = n \tan z \,(n \sec^2 z + 1).$$

$$\therefore \quad u = z - n\theta \tan z + \tfrac{1}{2}n\,\theta^2 \tan z\,(n \sec^2 z + 1).$$

$$\therefore n\rho = z - u = n\theta \tan z - \tfrac{1}{2}n\,\theta^2 \tan z\,(n \sec^2 z + 1).$$

$$= n \tan z\,[\,\theta - \tfrac{1}{2}(n+1)\,\theta^2\,] - \tfrac{1}{2}n^2\,\theta^2 \tan^3 z.$$

or $\rho = A \tan z + B \tan^3 z.$

114. Bradley's formula.

Bradley deduced his formula from Simpson's result as follows :—

$$\rho = \frac{1}{n}\left(z - \sin^{-1} \frac{\sin z}{\mu_0{}^n}\right)$$

$$\therefore z - n\rho = \sin^{-1} \frac{\sin z}{\mu_0{}^n}$$

$$\therefore \sin(z - n\rho) = \frac{\sin z}{\mu_0{}^n}$$

$$\therefore \quad \frac{\mu_0^n - 1}{\mu_0^n + 1} = \frac{\sin z - \sin(z - n\rho)}{\sin z + \sin(z - n\rho)}$$

$$= \tan \frac{n\rho}{2} \cot \left\{ z - \frac{n\rho}{2} \right\}$$

$$\therefore \quad \tan \frac{n\rho}{2} = \frac{\mu_0^n - 1}{\mu_0^n + 1} . \tan \left\{ z - \frac{n\rho}{2} \right\}$$

Since $\frac{n\rho}{2}$ is in general small,

$$\tan \frac{n\rho}{2} = \frac{n\rho}{2}$$

$$\therefore \quad \rho = \frac{2}{n} \frac{\mu_0^n - 1}{\mu_0^n + 1} . \tan \left\{ z - \frac{n\rho}{2} \right\}$$

115. Determination of the constants in the formula for refraction.

It was observed that the different formulae for refraction could be reduced to the form,

$\rho = A \tan z + B \tan^3 z$. Though the theoretical values for A and B can be found from physical data, they are best determined by actual observations as follows:—

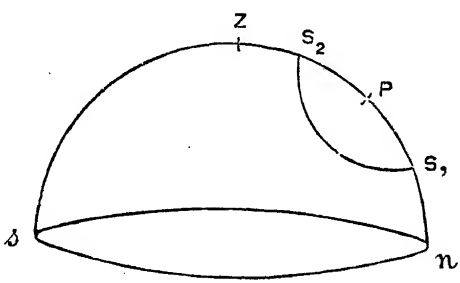


Fig. 79.

Let z_1 and z_2 be the apparent zenith distances of a circumpolar star at its lower and upper culminations and let their values when corrected for refraction be ξ_1 and ξ_2. If δ is the declination of the star and φ, the latitude of the place of observation,

$$Z\,P = 90^0 - \varphi = \frac{\xi_1 + \xi_2}{2}.$$

But $\xi_1 = z_1 + A \tan z_1 + B \tan^3 z_1$ and
$\xi_2 = z_2 + A \tan z_2 + B \tan^3 z_2$.

$$\therefore \quad 180^0 - 2\,\varphi = (z_1 + z_2) + A\,(\tan z_1 + \tan z_2) + B\,(\tan^3 z_1 + \tan^3 z_2).$$

Another circumpolar star gives a similar equation involving A and B. By solving these two, A and B are known. By observing different circumpolar stars, a large number of equations involving A, B and $180^0 - 2\,\varphi$ can be formed. Forming normal equations from these by the method of least squares, the best values of A and B can be found. The values so obtained are $A = 58\cdot''294$ $B = -0\cdot''0668$.

116. Thickness of the homogeneous atmosphere and its refractive index.

The values obtained above, for A and B may be used with advantage in the determination of several useful constants. Thus taking Cassini's formula,

$$\rho'' = \frac{(\mu - 1)}{\sin 1''}\left\{(1 - \frac{h}{a}) \tan z - \frac{h}{a} \tan^3 z\right\}$$

A $\tan z$ + B $\tan^3 z$, where ρ is in seconds.

$$\frac{(\mu - 1)}{\sin 1''}\left\{1 - \frac{h}{a}\right\} = A \text{ and}$$

$$\frac{(\mu - 1)}{\sin 1''}\ \frac{h}{a} \qquad = -B$$

$$\therefore \quad \frac{1 - \dfrac{h}{a}}{\dfrac{h}{a}} \qquad \frac{A}{B}$$

$$\therefore \quad \frac{a}{h} = 1 - \frac{A}{B} \qquad \frac{B - A}{B}$$

$$\frac{h}{a} \quad \frac{B}{B-A}$$

$$= \frac{\cdot 0668}{58\cdot 294+\cdot 0668}$$

$$= \frac{\cdot 0668}{58\cdot 3608}$$

Taking the value for a to be 3957 miles.

$$h = \frac{\cdot 0668 \times 3957}{58\cdot 3608} \text{ miles.}$$

$$4\cdot 5 \text{ miles.}$$

Again $\frac{\mu-1}{\sin 1''} = A-B=58\cdot 3608.$

$$\therefore \quad \mu = 1\cdot 00028.$$

117. Determination of n in Simpson's formula of refraction.

$$n\,\theta - \tfrac{1}{2}n\,(n+1)\,\theta^2 = n\,A.$$

$$\tfrac{1}{2}n\,\theta^2 = -B.$$

$$\text{or } \tfrac{1}{2}n\,(n+1)\,\theta^2 = -(n+1)\,B$$

$$\therefore \quad n\,\theta = n\,A-(n+1)\,B$$

$$\therefore \quad \tfrac{1}{2}n^2\,\theta^2 = \tfrac{1}{2}[\,n\,A-(n+1)\,B\,]^2 = -n\,B,$$

which gives an equation to determine the value for n.

118. General effects of refraction.

The most obvious result of refraction is to increase the true altitude of a heavenly body. This displacement takes place in the vertical plane through the star and hence the azimuth remains unaltered. Due to the effect of refraction the observed declination and hour angle of a star are different from the corresponding true values, and it can be proved that (see Ball's Astronomy page 141) the apparent position of a star will lie on a conic section referred to its mean position as origin. Besides these, the angular distance between two celestial objects undergoes change due to refraction. Again, when the sun or the

moon is observed very near the horizon, the disc appears oval and not circular. This is because, the amount of refraction changes very rapidly for zenith distances close to 90°. The points on the lower limb are raised much more than those on the upper limb, causing thereby a shortening of the vertical diameter.

119. Effect of refraction on the time of rising or that of setting of a body.

Another effect of refraction on a heavenly body is to accelerate the time of rising and to retard the time of setting, by the same amount. Let φ be the latitude of the place of observation, δ be the declination of the body observed, and ρ'' the amount of refraction at the horizon.

$$\cos z = \sin\varphi \sin\delta + \cos\varphi \cos\delta \cos h \quad\ldots\ldots\ldots\ldots(1)$$

If there be a small change in the zenith distance due to refraction, the corresponding change in the hour angle is given by

$$-\sin z\, dz = -\cos\varphi \cos\delta \sin h\, dh.$$

At the horizon this becomes

$$dz = \cos\varphi \cos\delta \sin h\, dh \quad\ldots\ldots\ldots\ldots(2)$$

Also from (1) since $z = 90°$

$$\cos h = -\tan\varphi \tan\delta \quad\ldots\ldots\ldots\ldots(3)$$

Eliminating 'h' between (2) and (3)

$$dz = (\cos^2\delta - \sin^2\varphi)^{\frac{1}{2}}\, dh.$$

Let $dz = \rho''$ and $dh = 15\,t''$.

Then the number of seconds of time of acceleration or retardation is given by

$$t = \frac{1}{15} \cdot \frac{\rho''}{\sqrt{\cos^2\delta - \sin^2\varphi}} \text{ seconds.}$$

t is a minimum when the place of observation is on the equator, and the body is on the celestial equator.

120. Changes in the hour angle, declination and parallactic angle of a body due to refraction.

Let h, δ, η be the true values of the hour angle, declination and parallactic angle of a body. Let P′ and P be the observed and true positions respectively of the body.

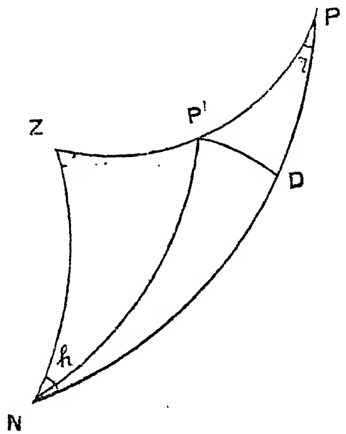


Fig. 80.

Let PP′ be A $\tan z$. Draw P′D $\perp^r$ to NP.

The change in declination due to refraction is given by P D.

i. e., $\Delta\,\delta = K \tan z \cos \eta$ (treating Δ PP′D as a plane triangle)..(1)

This quantity has to be subtracted from the apparent declination to get the true declination. If dh be the decrease in the hour angle due to refraction

$dh \cos \delta = A \tan z \sin \eta$..................................(2)

$\therefore\ dh = A \tan z \sin \eta \sec \delta.$

This should be added to the apparent hour angle to get its true value.

The parallactic angle increases by refraction. From the Δ N Z P, if a is the azimuth of the body, we have $\sin NZ \sin (360^c - a) = \sin \eta \cos \delta$ which does not change due to refraction.

$\therefore \cos\delta \cos\eta \, d\eta - \sin\eta \sin\delta \, d\delta = 0$

Using the result of (1)

$d\eta = A \tan z \sin\eta \tan\delta$.........(3)

This quantity should be subtracted from the apparent parallactic angle to get its true value.

121. Effect of refraction on the apparent distance between two near celestial points.

Let Z be the zenith, A and B the two points, $ZA = x$, $ZB = y$, $AB = d$ and let $Z\hat{A}B$ and $A\hat{Z}B$ be θ and α respectively.

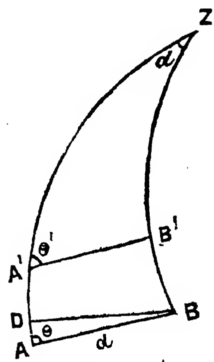


Fig. 81.

Let A′ B′ be the apparent position of AB.

Then $AA' = A \tan x$, $BB' = A \tan y$.

Draw BD $\perp^r$ to ZA.

$\cos d = \cos x \cos y + \sin x \sin y \cos\alpha$.

Differentiating and putting $\triangle x = - A \tan x$, $\triangle y = - A \tan y$, and treating α as a constant, we have

$$\begin{aligned} -\sin d \triangle d &= -\sin x \cos y \triangle x - \sin y \cos x \triangle y \\ &\quad + \cos x \sin y \cos\alpha \triangle x + \sin x \cos y \cos\alpha \triangle y \\ &= A \sin^2 x \cos y \sec x + A \sin^2 y \cos x \sec y \\ &\quad - A \sin x \sin y \cos\alpha - A \sin x \sin y \cos\alpha \\ &= A \sec x \sec y (\sin^2 x \cos^2 y + \cos^2 x \sin^2 y) \\ &\quad - 2 A \sin x \sin y \cos\alpha. \end{aligned}$$

$$= A \sec x \sec y \sin^2 (x-y) + 2 A \sin x \sin y - 2 A \sin x \sin y \cos \alpha.$$

$$= A \sec x \sec y \sin^2 (x-y) + 4 A \sin x \sin y \sin^2 \frac{\alpha}{2}$$

Since both these terms are small, put $x = y = z$ and since d is small $\sin d = d$.

$\sin^2 (x-y) = (x-y)^2 = d^2 \cos^2 \theta.$

and since α is small

$$4 \sin^2 \frac{\alpha}{2} = \alpha^2 = d^2 \sin^2 \theta \operatorname{cosec}^2 z.$$

$$\therefore \ - d \,\Delta d = A \sec^2 z\,(x-y)^2 + \alpha^2 A \sin x \sin y.$$

$$= A\,[\sec^2 z.\, d^2 \cos^2 \theta + d^2 \sin^2 \theta \operatorname{cosec}^2 z. \sin^2 z]$$

$$- \Delta d = A\,d\,[\sec^2 z \cos^2 \theta + \sin^2 \theta]$$

$$= A\,d\,[\cos^2 \theta + \tan^2 z \cos^2 \theta + \sin^2 \theta]$$

$$= A\,d\,[1 + \cos^2 \theta \tan^2 z]$$

This result is in circular measure.

$\therefore$ If d be in seconds, then its circular measure is $d \sin 1''$.

$\therefore$ The change in it due to refraction

$$= A\,d\,[1 + \cos^2 \theta \tan^2 z] \sin 1'' \ldots\ldots\ldots (1)$$

This is the quantity by which the apparent distance has to be increased to get the true distance.

From the Δ A Z B, $\sin d \sin \theta = \sin \alpha \sin y$

This does not change due to refraction and since d is small, it reduces to $d \sin \theta = \sin \alpha \sin y$

$$\therefore \log d + \log \sin \theta = \log \sin \alpha + \log \sin y$$

$$i.e. \quad \frac{\Delta d}{d} + \cot \theta \Delta \theta = \cot y \,\Delta y$$

$$\therefore \ - A\,(1 + \cos^2 \theta \tan^2 z) + \cot \theta \,\Delta \theta$$

$$= - A \tan y \cot y = - A$$

$$\Delta \theta = A \tan \theta \cos^2 \theta \tan^2 z$$
$$= A \sin \theta \cos \theta \tan^2 z$$

This is the quantity that has to be subtracted from the apparent value of θ to get its true value.

122. Effect of refraction on the shape of the sun or moon.

The effect of refraction, in shortening the vertical diameter of the sun or the moon when near the horizon has been referred to, in a previous section. The effect can be investigated mathematically as follows :—

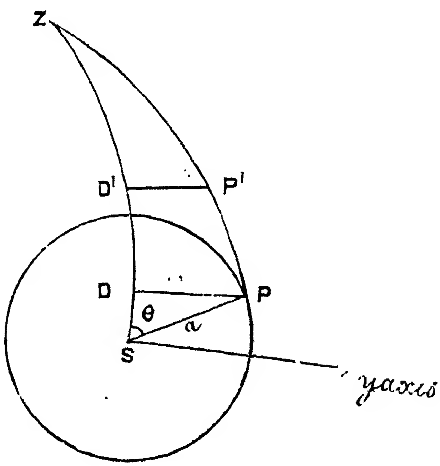


Fig. 82.

Let S be the centre of the sun's disc of angular radius a, Z the zenith, and P a point on the circumference.

Let P D be $\perp^r$ on S Z, and let angle Z S P be θ.

Let P be displaced to P′. Draw P′ D′ $\perp^r$ to Z S.

Take the direction Z S as the x axis and the direction $\perp^r$ to it as the y axis, S being the origin.

Then $y = \mathrm{P'}\,\mathrm{D'} = (1-\mathrm{A})\,\mathrm{P}\;\mathrm{D}\left\{\text{since } \hat{\mathrm{D}} = \frac{\pi}{2} \text{ and therefore } \cos\theta = 0 \text{ in equation (1) of article 121}\right\}$.

$$= a\,(1-\mathrm{A})\sin\theta$$

And $\quad x = \mathrm{S\,D'} = a\cos\theta + \mathrm{D\,D'}$

$$= a\cos\theta + \mathrm{A}\tan\,(z - a\cos\theta)$$

$$= a\cos\theta + \mathrm{A}\,(\tan z - a\cos\theta\sec^2 z)$$

i.e. $x - \mathrm{A}\tan z = a\cos\theta\,(1-\mathrm{A}\sec^2 z)$,

and $\quad \dfrac{y}{a\,(1-\mathrm{A})} = \sin\theta.$

$$\frac{(x-\mathrm{A}\tan z)^2}{a^2\,(1-\mathrm{A}\sec^2 z)^2} + \frac{y^2}{a^2\,(1-\mathrm{A})^2} = 1$$

This is the equation for an ellipse whose centre is $(\mathrm{A}\tan z, 0)$ referred to axes of co-ordinates taken on the tangent plane at S.

Its semi-major axis is $a\,(1-\mathrm{A})$ and

semi-minor axis is $a\,(1-\mathrm{A}\sec^2 z)$

or $\;a\,(1-\mathrm{A}) - a\,\mathrm{A}\tan^2 z$

$$\therefore \text{ Their ratio } = 1 - \frac{\mathrm{A}\tan^2 z}{1-\mathrm{A}}$$

$$= 1 - \mathrm{A}\tan^2 z \text{ nearly.}$$

A is here in circular measure.

Any short horizontal arc is diminished by refraction in the ratio $\dfrac{1-\mathrm{A}}{1}$ as seen in (1) of article 121; and any vertical arc is diminished in the ratio $\dfrac{1-\mathrm{A}\sec^2 z}{1}$, as in this case, θ of article 121 is 0, and therefore the change is $\mathrm{A}\,d\sec^2 z$. Thus in the latter case the reduced length of d becomes $d\,(1-\mathrm{A}\sec^2 z)$.

123. Other effects of refraction.

Among other effects of refraction may be mentioned the following :—

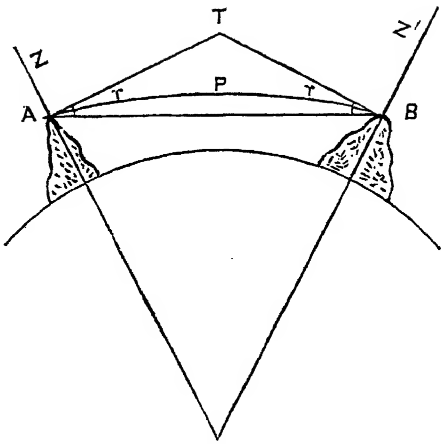


Fig. 83.

(1) The apparent path of a ray of light from one point A to another point B above the earth is not straight, but curved.

Let A and B be two places above the surface of the earth visible from one-another. Let A P B be the path of a ray which is curved owing to the effect of refraction, and let z and z' be the apparent zenith distances of one station as seen from the other.

Then, $T\hat{A}B$ and $T\hat{B}A$ are the refractions due to the atmosphere. We may take both these to be equal to r.

Now, $2r + z + z' + B + A = 360°$.

and $A + B + C = 180°$.

$\therefore 2r + z + z' - C = 180°$.

The above equation gives the value of r, if z and z' are found by observation, and C is calculated from the distance A B and the radius of the earth.

(2) Refraction diminishes the dip, and increases the distance of the visible horizon.

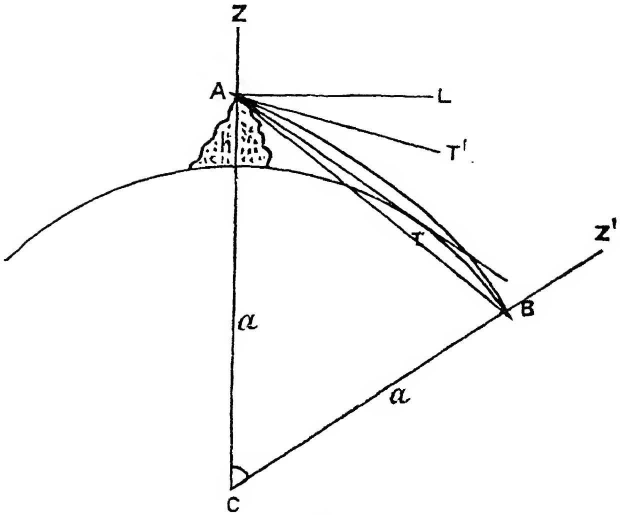


Fig. 84.

Owing to refraction, the path of the ray is curved to the visible horizon from A and one's vision from A goes up to B which is farther away from A than T. Also, the dip is diminished by refraction, as is evident from the figure.

The apparent dip is $L\hat{A}T'$, whereas the true dip is $L\hat{A}T$. Join BA.

$Z'\hat{B}A = 90^\circ + r$, where r is the refraction,

and since $Z\hat{A}B + Z'\hat{B}A = 180^\circ + C$,

$Z\hat{A}B = 90^\circ - r + C$ .... (1)

If a is the radius of the earth, and h is the height of the observer at A,

$$\frac{a+h}{a} = \frac{\sin(90^\circ + r)}{\sin(90^\circ - r + C)} = \frac{\cos r}{\cos(C - r)}$$

$$\frac{h}{a} = \frac{\cos r - \cos(C-r)}{\cos(C-r)} = \frac{2\sin\left(\frac{C}{2} - r\right)\sin\frac{C}{2}}{\cos(C-r)}$$

Since C and r are small

$$\frac{h}{a}=2\left(\frac{C}{2}-r\right)\frac{C}{2}$$

i. e. $\dfrac{2h}{a}=(C-2r)\,C$

Now true dip $=\sqrt{\dfrac{2h}{a}}=C\left(1-\dfrac{2r}{C}\right)^{\frac{1}{2}}=C\left(1-\dfrac{r}{C}\right)$

It has been found that $r=\alpha C$ where α is a constant the value of which depends on the condition of the atmosphere.

If C_1 is the value of C supposing there is no refraction,

$$C_1 = \sqrt{\frac{2h}{a}}$$

$\therefore\ C=C_1(1+\alpha) = C_1+\dfrac{1}{13}\,C_1$ (approximately)

The increase in the value of C is due to the increase in the distance of the visible horizon.

If D' is the apparent dip,

$$\begin{aligned} D' &= Z\hat{A}T'-90^\circ \\ &= Z\hat{A}B-r-90^\circ \\ &= 90^\circ-r+C-r-90^\circ \\ &= C-2r \text{.................from (1)} \\ &= C(1-2\alpha) \end{aligned}$$

But D_1, (true dip if there is no refraction)

$$= C_1 = C(1-\alpha)$$

$$\therefore\quad D' = D_1\,\frac{1-2\alpha}{1-\alpha} = D_1(1-\alpha)$$

$$= D_1-\frac{1}{13}\,D_1$$

i. e. D' is less than D_1.

Hence the apparent dip is less than the true dip.

CHAPTER XII.

PARALLAX.

124. Geocentric Parallax.

The *parallax of* a body is the angle subtended at the body by any two places of observation This is therefore the difference between the directions in which a body is seen by an observer in any two positions. Any two points on the surface of the earth subtend practically no angle at even

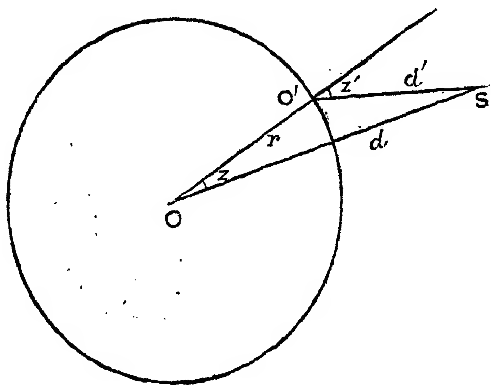


Fig. 85.

the nearest star and so none of the stars has any parallax of the kind mentioned here. At the nearest star α Centauri, this angle is less than 0·″ 00006. On the other hand the moon, the sun and the planets are not so distant as the stars and their parallaxes as viewed from different stations can be determined. As the direction of any of these comparatively near bodies depends upon the point of observation on the earth's surface, this direction is different for different observers, the standard direction being that from the centre of the earth. If O be the centre of the earth and O′ any point on its surface,

and S a body observed from O′, O S O′ is called the *geocentric parallax* of the body.

In the case of even the nearest star the earth's radius does not subtend any appreciable angle at it and so the centre of the sun is taken as the standard direction and the parallax is then called the *annual parallax.* For a star, there is no geocentric parallax.

125. The effect of geocentric parallax.

The effect of geocentric parallax on a body S is to shift its apparent position away from the direction O S by the angle O′ S O. Regarding the earth as spherical, the apparent zenith distance of an object is greater by the amount of parallax. If the true shape of the earth is considered, the effect is to give a shift to S away from the direction of the radius of the earth to the observer's position, which is called the *geocentric zenith.*

If P be the parallax in circular measure, we have from the $\triangle$ OSO′, $\sin P = P = \frac{r}{d} \sin z'$, where z' is the apparent zenith distance of the object S.

When z' is 90° i. e. when the body is at the horizon the parallax is called the *horizontal parallax* (P_0) and its value is $\sin^{-1}\left(\frac{r}{d}\right)$ which is the greatest value for the parallax, considering r and d to be constant. If r is variable and a is the maximum value of r, as in the case of the equatorial radius of the earth, the equatorial horizontal parallax of a body is given by $P_0' = \sin^{-1}\left(\frac{a}{d}\right)$.

The geocentric parallax P of a body at any altitude can be expressed in terms of the horizontal parallax as follows :—

$$\sin P = \sin P_0 \sin z' \qquad \text{.....(1)}$$

or $\quad P = P_0 \sin z'$ since P and P_0 are both small.

Hence we get the approximate law of parallax of a celestial body *viz.*, the parallax of a body varies as the sine of its apparent zenith distance.

The equatorial horizontal parallax of the sun is found to be 8.″8 nearly and that of the moon is about 3422″.

Taking the earth to be a sphere of radius a we find $\sin P_0 = \dfrac{a}{d}$. Hence if the horizontal parallax P_0 of the body and the radius of the earth be known, the distance of the body can be calculated. This is in fact a method of arriving at the distances of neighbouring celestial objects.

To find the sun's distance taking its horizontal parallax to be 8·″8.

If D be the distance of the sun from the earth and a the radius of the earth,

$$\frac{a}{D} = \text{the circular measure of } 8{\cdot}''8$$

$$= \frac{\pi .\ 8{\cdot}8}{180 \times 60 \times 60.}$$

If a is taken to be 3960 miles.

$$D = \frac{3960 \times 180 \times 60 \times 60.}{\pi .\ \ 8{\cdot}8}$$

$$= 92{,}800{,}000 \text{ miles nearly.}$$

126. Horizontal parallax and angular diameter.

The horizontal parallax of a body like the moon is the angular radius of the earth as seen from the moon, so that if a be the earth's radius, and r the distance of the moon, $\sin P = a/r$ and if c be the moon's linear radius, $c/r = \sin m$, where m is the angular radius of the moon.

$$\therefore \quad \frac{c}{a} = \frac{\text{radius of the moon}}{\text{radius of the earth}} = \frac{\sin m}{\sin P} = \frac{m}{P}.$$

This equation enables us to determine the moon's linear diameter, from its angular diameter and its horizontal parallax.

127. Parallax in hour angle and declination,

The effect of parallax in any co-ordinate of the position of a body is called the *parallax in that direction.*

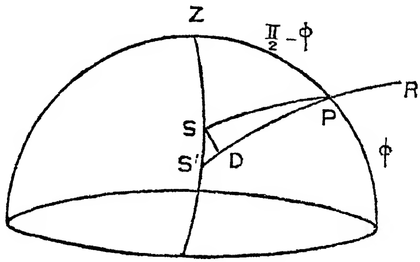


Fig. 86.

Let S′ be the apparent position of an object, and let its geocentric hour angle and declination be h and δ respectively, P and Z being the observer's pole and zenith. Let S represent the standard direction of the body as seen from the centre of the earth, assumed spherical.

Then, $SS' = P_0 \sin z'$ where $z' = ZS'$.

Draw SD $\perp^r$ to S′P.

Then $S\hat{P}S'$ = the parallax in hour angle = $\triangle h$.

and $S'D$ = parallax in declination = $\triangle \delta$

$$\begin{aligned}\text{Now } \triangle h \cos \delta &= SD = SS' \sin SS'D \\ &= P_0 \sin z' \sin SS'D. \\ &= P_0 \sin ZP \sin ZPS' \\ &= P_0 \cos \varphi \sin (h+\triangle h) \text{ where } \varphi \text{ is the observer's latitude.} \\ &= P_0 \cos \varphi \sin h \quad (\text{since } \triangle h \text{ is small}) \\ \therefore \quad \triangle h &= P_0 \cos \varphi \sin h \sec \delta.\end{aligned}$$

To get the parallax in declination, produce S′P to R so that $S'R = 90°$,

$$\therefore \text{PR} = 90^\circ - \text{PS}' = \delta - \triangle\,\delta.$$

$$\begin{aligned}\triangle\,\delta = \text{S}'\text{D} &= \text{SS}' \cos \text{SS}'\text{D} \\ &= \text{P}_0 \sin z' \cos \text{SS}'\text{D}. \\ &= \text{P}_0 \cos \text{ZR}\ [\text{from the } \triangle\ \text{ZS}'\text{R}] \\ &= \text{P}_0 [\sin\varphi \cos(\delta - \triangle\,\delta) \\ &\qquad - \cos\varphi \sin(\delta - \triangle\,\delta)\cos(h + dh)] \\ &= \text{P}_0 [\sin\varphi \cos\delta - \cos\varphi \sin\delta \cos h] \\ &\qquad\qquad \text{nearly.}\end{aligned}$$

128. Spheroidal earth.

When the spheroidal shape of the earth is taken into consideration, the displacement S S′ does not take place in the direction Z S, where Z is the geographical zenith, but in the direction Z′ S where Z′ is the geocentric zenith for the observer.

Now Z Z′ $= (\varphi - \varphi')$ where φ and φ' are the geographical and geocentric latitudes respectively of the place. The corresponding quantities are now obtained by putting φ' for φ in the above formulae and modifying the value of P_0.

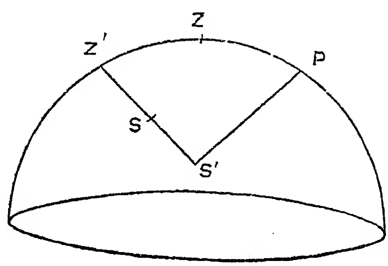


Fig. 87.

Now sin S S′ or S S′ $= \dfrac{\text{O O}'}{d}$ sin Z′ S′ where O O′ is the earth's radius measured from the observer's position.

$\therefore\quad \text{S S}' = \dfrac{a}{d}(1 - c \sin^2 \varphi) \sin \text{Z}'\text{S}'$, where a is the equatorial

radius of the earth, and $c = \frac{a-b}{,}$, b being the polar radius of the earth. The quantity corresponding to P_0 now becomes $\frac{a}{d}(1-c \sin^2 \varphi)$, since $c=e^2\,2$ nearly (See page 47)

129. Parallax in azimuth and altitude.

Just as we had parallax in hour angle and declination, we also have parallax in azimuth and altitude for any body except when it is on the meridian, in which case, there is no parallax in azimuth. When we assume the earth to be spherical in shape, the shift due to parallax takes place only along the vertical circle through the body wherever the body may be situated on the celestial sphere. This is not the case for an observer on the surface of the actual earth, whose shape is spheroidal.

Let Z and Z′ be the geographical and geocentric zeniths respectively of the place of observation and S and S′ the standard and apparent positions of the body.

$ZZ' = \varphi - \varphi'$ where φ and φ' are the geographical and geocentric latitudes of the place. Draw S D and Z′ D′ $\perp^r$ on Z S′ from S and Z′.

d A = Parallax in azimuth

$$= SZD = \sin(dA) = \frac{\sin SD}{\sin SZ}$$

$$\frac{\sin SS' \sin SS'D}{\sin SZ}$$

$$= \frac{a}{d}(1-c\sin^2\varphi)\sin Z'S'.$$

sin SS′D cosec ZS

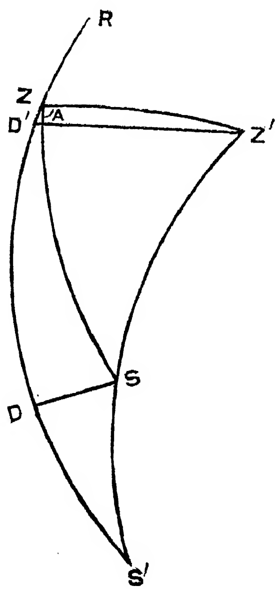


Fig. 88.

$$= \frac{a}{d}(1 - c\sin^2\varphi)\sin(A + dA)\sin(\varphi - \varphi').$$

$$\operatorname{cosec} ZS.$$

$$= \frac{a}{d}(1 - c\sin^2\varphi)\sin A \operatorname{cosec} ZS\,(\varphi - \varphi')$$

(since $\varphi - \varphi'$ is small)

$$dz = S'D = SS' \cos SS'D$$

$$= \frac{a}{d}(1 - c\sin^2\varphi)\sin Z'S' \cos SS'D$$

$$= \frac{a}{d}(1 - c\sin^2\varphi)\,[\cos(\varphi - \varphi')\sin z$$

$$- \sin(\varphi - \varphi')\cos z \cos A]$$

(by producing $S'Z$ to R so that $S'R = 90°$)

130. The fundamental equations of geocentric parallax.

In the present investigation, the earth is assumed spheroidal in shape and the distance of the observer at any station O′ from the centre of the earth is taken to be r, d and d' being the distances of the body from O and O′ respectively. α and δ are the R. A. and declination respectively of the body S as seen from O and α', δ' are the corresponding quantities as seen from O′.

Let φ' be the geocentric latitude of O′. Then the R. A. of the direction O O′ will be given by the sidereal time of observation (T say) and its declination by φ'. Taking O as origin and the axes of x, y and z along directions parallel to (0,0; 90,0; 0,90) in equatorial co-ordinates and

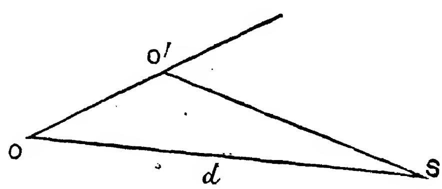


Fig. 89.

projecting O O′, O S and O′S along these axes we get the following fundamental equations of parallax, from which the parallax in R. A. and declination can be determined.

$$d' \cos \alpha' \cos \delta' = d \cos \alpha \cos \delta - r \cos \varphi' \cos T \ldots (1)$$
$$d' \sin \alpha' \cos \delta' = d \sin \alpha \cos \delta - r \cos \varphi' \sin T \ldots (2)$$
$$d' \sin \delta' = d \sin \delta - r \sin \varphi' \ldots\ldots (3)$$

131. Parallax in R. A.

Multiplying (1) by $\sin \alpha$ and (2) by $\cos \alpha$ and subtracting the latter from tbe former we get,

$$d' \cos \delta' \sin (\alpha - \alpha') = r \cos \varphi' \sin (T - \alpha) \ldots\ldots (4)$$

In the above step, $(T - \alpha)$ is the hour angle of the body.

Also multiplying (1) by $\cos \alpha$ and (2) by $\sin \alpha$ and adding we get

$$d' \cos \delta' \cos (\alpha - \alpha') = d \cos \delta - r \cos \varphi' \cos (T - \alpha) \ldots (5)$$

$$\tan (\alpha - \alpha') = \frac{\frac{r}{d} \cos \varphi' \sin (T - \alpha)}{\cos \delta - \frac{r}{d} \cos \varphi' \cos (T - \alpha)}. \qquad (6)$$

Here $\frac{r}{d}$ = Horizontal parallax of the body at the place of observation.

Equation (6) gives the parallax in R. A. of a body. Since $\frac{r}{d}$ is usually a very small quantity, we can write the expression for $(\alpha - \alpha')$ in a series of sines of multiples of $(T - \alpha)$ using a well-known theorem proved on page 76, Chapter V.

Putting $\frac{r}{d} \cos \varphi' = c$,

$$(\alpha - \alpha') = \frac{c \sin (T - \alpha)}{\cos \delta} + \frac{c^2 \sin 2 (T - \alpha)}{2 \cos^2 \delta} + \frac{c^3 \sin 3 (T - \alpha)}{3 \cos^3 \delta} + \ldots\ldots (7)$$

It is to be noted that $(\alpha-\alpha')$ in the above formula is in circular measure and for this series to be rapidly convergent, $\cos\delta$ should not be very small, and therefore the body should be as far away from the poles as possible.

132. Parallax in declination.

Multiply (1) by $\cos\frac{\alpha+\alpha'}{2}$ and (2) by $\sin\frac{\alpha+\alpha'}{2}$, add and then divide by $\cos\frac{\alpha-\alpha'}{2}$. Thus we get,

$$d'\cos\delta' = d\cos\delta - r\cos\varphi'\sec\frac{\alpha-\alpha'}{2}\cos\left(\mathrm{T}-\frac{\alpha+\alpha'}{2}\right)\ldots(8)$$

Let the quantities β and Υ be chosen to satisfy the following equations.

$$\beta\sin\Upsilon = \sin\varphi' \qquad \text{and}$$

$$\beta\cos\Upsilon = \cos\varphi'\sec\frac{\alpha-\alpha'}{2}\cos\left(\mathrm{T}-\frac{\alpha+\alpha'}{2}\right)$$

Now equations (3) and (8) reduce to

$$d'\sin\delta' = d\sin\delta - r\,\beta\sin\Upsilon\ldots\ldots\ldots\ldots(9)$$

and $$d'\cos\delta' = d\cos\delta - r\,\beta\cos\Upsilon\ldots\ldots\ldots\ldots(10)$$

From (9) and (10) it is possible to obtain the following equations as before.

$$d'\cos(\delta'-\delta) = d - r\,\beta\cos(\delta-\Upsilon)\ldots\ldots\ldots\ldots(11)$$

and $$d'\sin(\delta'-\delta) = r\,\beta\sin(\delta-\Upsilon)\ldots\ldots\ldots\ldots(12)$$

$$\therefore\quad \tan(\delta'-\delta) = \frac{\frac{r}{d}\,\beta\sin(\delta-\Upsilon)}{1-\frac{r}{d}\,\beta\cos(\delta-\Upsilon)}\ldots\ldots\ldots\ldots(13)$$

Equation (13) gives an expression for $(\delta'-\delta)$ similar to (6) for $(\alpha-\alpha')$.

Let $\frac{r}{d}\,\beta = \mathrm{K}$.

$$\text{Then, } (\delta'-\delta)=K\sin(\delta-\Upsilon)+\frac{K^2}{2}\sin 2(\delta-\Upsilon)$$
$$+\frac{K^3}{3}\sin 3(\delta-\Upsilon)+\ldots\ldots(14)$$

133. The determination of the parallax of a body by meridian observations.

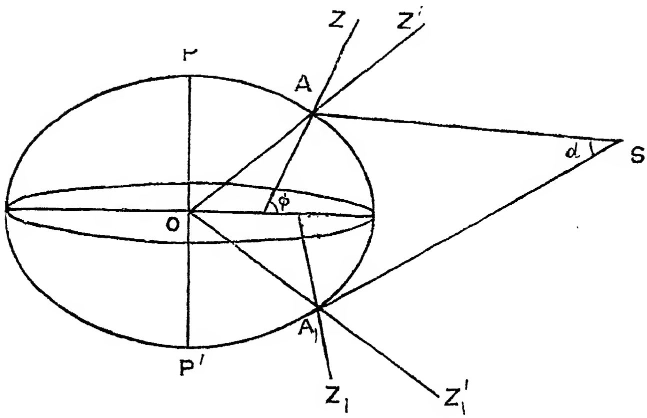


Fig. 90.

Let A and A_1 be two stations on the surface of the earth and on the same meridian, one in the northern hemisphere of latitude φ, and the other in the southern hemisphere of latitude φ_1, and S the body whose parallax is to be determined.

The angle $A S A_1$ is the change in the declination of S as seen from A and A_1 and this is determined by observing the difference of the differences in the meridian zenith distances of S and a star X near S as observed from A and A_1. It is better if the star and the moon have approximately the same R. A. (so that X and S may be on or nearly on the same declination circle) and do not differ much in altitude. As the star's declination does not get affected by the observer's change of position from A to A_1 since the star has no geocentric parallax, the difference in

the two values is due to the parallax of S as seen from the two stations and is given by the angle $A\,S\,A_1$. The direct determination of the declination of S by observation of the zenith distance of S is not reliable as that will introduce corrections for refraction etc., which are not known sufficiently accurately for the purpose. Therefore the difference in the declinations of S and the star is taken for each station; let them be x and x_1. The value of $A\hat{S}A_1$ obtained from the difference of the two readings x and x_1 as observed at A and A_1 is given by $\alpha = x - x_1$. Now α is also equal to $A\hat{S}O + A_1\hat{S}O$. Let φ and φ_1 be the geographical latitudes of A and A_1, φ' and φ_1' their geocentric latitudes, r and r_1 the radii OA and OA_1, a the equatorial radius of the earth, d the distance OS, z and z_1 the geographic zenith distances ZAS and Z_1A_1S.

$$\frac{\sin ASO}{\sin Z'AS} \quad \frac{r}{d} \quad \frac{a\,(1-c\sin^2\varphi)}{d}\text{; and } Z'\hat{A}S = (z-\varphi+\varphi')$$

$$\therefore \quad A\hat{S}O = \frac{a}{d}(1-c\sin^2\varphi)\sin(z-\varphi+\varphi')$$

[since $A\hat{S}O$ is small.]

$$\text{and } A_1\hat{S}O = \frac{a}{d}(1-c\sin^2\varphi_1)\sin(z_1-\varphi_1+\varphi_1')$$

$$\therefore \quad \alpha = \frac{a}{d}\Big\{(1-c\sin^2\varphi)\sin(z-\varphi+\varphi') + (1-c\sin^2\varphi_1)\sin(z_1-\varphi_1+\varphi_1')\Big\}$$

Now P_0' = the equatorial horizontal parallax of S

$$= \frac{a}{d} = \frac{\alpha}{(1-c\sin^2\varphi)\sin(z-\varphi+\varphi') + (1-c\sin^2\varphi_1).\sin(z_1-\varphi_1+\varphi_1')}$$

If it is not possible to select the two stations A and A_1 on the same meridian, it is possible to get the value of $A\,S\,A_1$ from observations at a station A_2 not on the

meridian $A A_1$, provided we allow for the change of declination of S between its transits over the meridians of A_1 and A_2. The best results are obtained when the value of $A S A_1$ is as large as possible and therefore the stations should be as near as possible to the poles of the earth.

If the form of the earth be assumed spherical it is easy to see that the formula for the horizontal parallax is reduced to $P_0 = \dfrac{\alpha}{\sin z + \sin z_1}$

In the above method of determining parallax, errors due to refraction do not affect the result seriously, as the body and the star observed are chosen to have nearly the same altitude and as the effect on the difference of their declinations is very little.

From observations of the type mentioned, it has been found that the mean horizontal parallax of the moon is $57'$-$2 \cdot'' 71$. This corresponds to a mean distance of 238,900 miles, which is the value obtained by expressing the horizontal parallax in circular measure and dividing the mean radius of the earth by the parallax so expressed.

134. A more accurate method of determining the distance of the moon.

As the distance of the moon from the earth is not very great, the horizontal parallax for the moon is large and so to get the lunar distance very accurately the expression for lunar parallax in series is generally used. When the moon is on the meridian, expression (14) on page (197) for the geocentric parallax of the moon in declination becomes,

$$(\delta' - \delta) = \frac{r}{d}\,\frac{\sin(\delta - \varphi')}{\sin 1''} + \frac{r^2}{d^2}\,\frac{\sin 2(\delta - \varphi')}{\sin 2''} + \dots\dots\dots (1)$$

Now, the true and apparent R. A. of the moon being the same, $\alpha' = \alpha = T$ (the sidereal time) and

therefore $\beta = 1$ and $\gamma = \varphi' =$ the geocentric latitude of the place.

Equation (1) provides us with a method of determining d by observations made from two stations. When the moon is at the meridian of a station A, its apparent declination δ' is obtained from the observed zenith distance by a transit circle. Substituting the value in (1) we get an equation to determine δ and d, since φ' and r are known. Let a second observation be made from station B and let δ_1', δ_1, r_1 etc., be the corresponding quantities for station B. Now we have a second equation between δ_1 and d. From these two equations δ and d are determined, provided $(\delta_1 - \delta)$ is either known or negligible. Now d is the geocentric distance of the moon.

To determine $(\delta_1 - \delta)$ accurately, it is desirable that A and B should be nearly on the same meridian so as to make the interval between the two observations as short as possible. This quantity vanishes if the two stations are on the same meridian; otherwise, this is known from the difference of time between the two observations and the moon's hourly motion in declination.

The equations are first solved by neglecting the second and higher powers of $\left(\frac{r}{d}\right)$. The value of d so obtained is substituted later in those terms and the equations are again solved as simultaneous equations between d and δ. [For further details regarding this method, see Ball's Astronomy.]

135. The parallax of an exterior planet and that of the sun.

The principle of the above method in determining the parallax of an exterior planet consists in observing it early in the morning and also in the evening after sunset, at a time when the planet's longitude differs from that of the sun by 180°. The planet is then said to be in opposition

and is therefore much nearer the earth than at other times. During the interval between the two observations, the change in the observer's position with reference to the centre of the earth and the planet gives the necessary base line for the measurement of the parallax of the planet.

A star whose position in the sky is near the planet will not undergo any appreciable shift due to parallax as its distance from us is very great; therefore any change in the

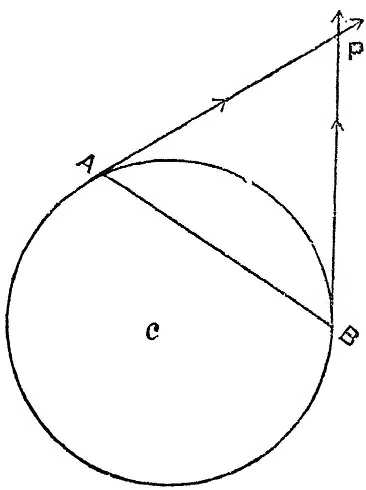


Fig. 91.

angular distance between the star and the planet should be attributed to the parallax of the planet. It is easy to see that the apparent R. A. of the planet is greater in the morning and is less in the evening, than the corresponding geocentric values, whereas these quantities will be the same for the star. Therefore the difference between the observed R. A. of the star and that of the planet on these two occasions will have variation in opposite directions from the corresponding geocentric values. This difference can be measured by means of the heliometer or filar micrometer or by measurement of the co-ordinates of the

planet and the star from a photographic plate exposed to that region.

Let m be the difference between the observed R. A. of the planet and that of the star so that $m=(\alpha'-\alpha_0)$, α_0 being the R. A. of the star, and α' the R. A. of the planet observed. Let α be the geocentric R. A. of the planet.

Now $(\alpha'-\alpha)=(\alpha'-\alpha_0)+(\alpha_0-\alpha)$

$$=m+(\alpha_0-\alpha)=-\frac{r}{d}\cos\varphi'\sec\delta\sin h,$$

where φ' is the geocentric latitude of the place of observation and r and d are respectively the distances of the observer and the planet from the earth's centre.

If the observed quantities for the evening observation be m_1, α_1, δ_1, etc., we get

$$m_1+(\alpha_0-\alpha_1)=-\frac{r}{d_1}\cos\varphi'\sec\delta_1\sin h_1 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Again for the next morning observation we get a similar equation. $m_2+(\alpha_0-\alpha_2)=-\frac{r}{d_2}\cos\varphi'\sec\delta_2\sin h_2 \ldots\ldots\ldots\ldots (2)$

Subtracting (1) from (2) the star's geocentric R. A. is eliminated and we get,

$$m_2-m_1+(\alpha_1-\alpha_2)=-r\cos\varphi'\left\{\frac{\sec\delta_2\sin h_2}{d_2}-\frac{\sec\delta_1\sin h_1}{d_1}\right\}$$

Now $(\alpha_2-\alpha_1)$ is the increase in the geocentric R. A. of the planet, during the interval between the two observations, due to the relative motion of the planet, and this quantity can be deduced by calculation, or by a series of observations. Also, the accuracy of the equation is not much affected by putting for δ_2, δ_1, d_2 and d_1 the mean values of these quantities. Hence we have the equation,

$$(m_2-m_1)=(\alpha_2-\alpha_1)-\frac{r}{d}\cos\varphi'\sec\delta\,(\sin h_2-\sin h_1) \ldots . (3)$$

Here (m_2-m_1) is given by the measures obtained from the photographic plate or the observed differences in R. A. and r, $\cos\varphi'$, $\sec\delta$ etc. are all known quantities. Thus d is determined from equation (3).

If a and b are respectively the distances of the earth and the planet from the sun when the planet is in opposition, we have $(b-a)=d$ as the planet is beyond the earth and $\frac{b}{a}$ is known from Kepler's law. Hence 'a' the sun's distance is known. It is assumed that the earth and the planet are moving in concentric orbits with the sun at the centre.

136. Best circumstances for observation.

When the planet is east of the meridian, the sign of its hour angle is negative and if the observations are made when the planet is 6 hrs. west and 6 hrs. east of the meridian we have $\sin h_2 - \sin h_1 = 2$ which is the maximum value of the expression, and r will be best determined if $(m_2 - m_1)$ is as large as possible. For this, omitting the variations in the R. A of the planet, $\cos \varphi'$ and $\sin h_2 - \sin h_1$ should both be maximum. Hence a station near the equator is selected for the observation and the evening and morning observations are made as far as possible to satisfy the equation $h_2 = 24 - h_1 = 6$ hours.

In the above method, only observations made on a single star and a planet are used. In practice, several stars in the neighbourhood are used and from each of them the value of d is deduced. In 1877 when Mars was in opposition, Sir David Gill made a series of observations on it and the stars near it, from the island of Ascension, and the result he deduced for solar horizontal parallax was $8.''778$ with a probable error of $0''\cdot012$.

In the case of Mars, there are two causes which lead to some error in the observation. One is due to the dispersion of light in the atmosphere at considerable zenith distances which gives a coloured fringe to the disc of Mars, blue above and red below, and this systematically gives an increased parallactic displacement; and the second is due to the difficulty to get at the difference of the R. A. of a star and that of a planet which has an appreciable

disc. Both these are reduced very much, if instead of Mars, a minor planet like Eros is chosen for observation when it comes to opposition; in the case of this planet, the image is only a point like a star and both the above-mentioned difficulties are considerably reduced.

The value of the horizontal parallax of the sun derived from observations made at several stations at the opposition of Eros in 1900 was 8.″806 corresponding to 92,900,000 miles for the mean distance of the sun from the earth. This is taken as the *Astronomical unit of distance.*

137. Solar parallax from the constant of aberration.

If K is the constant of aberration, we have already shown on page 158 that

$K=\dfrac{2\pi\, a \operatorname{cosec} 1''}{CT\,(1-e^2)^{\frac{1}{2}}}$ where C is the velocity of light and T the length of year.

If P is the equatorial horizontal parallax of the sun,

$P=\dfrac{\rho}{a} \operatorname{cosec} 1''$ where ρ is the equatorial radius of the earth and a the radius of the earth's orbit.

$$\text{Hence K. P}=\frac{2\pi\,\rho \operatorname{cosec}^2 1''}{CT\,(1-e^2)^{1/2}}$$

Now $\rho=6378\cdot4$ *k. ms*; $C=299,860$ *k. ms.* per second and $T=31{,}558{,}149$ seconds; $e=0{\cdot}01674$.

Using the above values, we get

K. P = 180·21, K and P being expressed in seconds of arc.

K is known to be 20″·45 and hence P is obtained to be 8·″812

138. Annual Parallax.

The distances of the stars are so great that the diameter of the earth does not subtend any appreciable angle at even the nearest of them. Hence from the known length of the diameter of the earth, the stellar distances

could not be calculated. The radius of the earth's orbit round the sun does however subtend a small angle at some of the nearer stars and this forms a means of knowing the distances of such stars. Assuming the standard direction to any star as that from the centre of the sun, the apparent direction of the star as seen from any position of the earth in its orbit will vary according to the distance and direction of the earth from the sun. The maximum angle which the radius vector to the earth from the sun subtends at any star is called its *annual parallax*.

139. Effect of parallax on the position of a star.

Let O, X, and S represent respectively the position of the earth, a star and the sun at any time. Draw the celestial sphere with O as centre and on it let s, and x respectively represent the apparent position of the sun and the star.

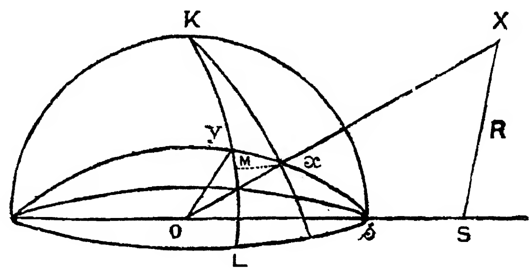


Fig. 92.

Let K be the pole of the ecliptic and let Y represent the standard position of the star on the celestial sphere, as seen by an observer on the sun. The effect of parallax is to give the star a shift from Y to x in the direction Y s.

If XS=R and OS=r we get $\dfrac{\sin S\hat{O}X}{\sin X} = \dfrac{R}{r}$

Or the circular measure of $X = \dfrac{r}{R} \sin sx$, X being small

Parallax of star in seconds of arc $= Yx = X$

$$= \frac{r}{R \sin 1''} \sin sx.$$

Now the maximum value for $Yx = \frac{r}{R \sin 1''}$ which is the annual parallax of the star, and this is the parallax of the star when $S\hat{O}X$ is a right angle. Denoting this by P, we obtain the apparent shift of the star on the celestial sphere due to parallax to be $P \sin sx$ or $P \sin sY$ towards the direction occupied by the sun at any moment.

140. Effect of parallax on the latitude and longitude of a star.

Draw through Y and x the secondaries KY, Kx to the ecliptic; and also draw $xM \perp^r$ to YL. Let the sun's longitude be S and let β, λ be the longitude and latitude respectively of the star at the standard position Y.

If $d\beta$ be the change in longitude due to parallax,

$d\beta \cos \lambda = xM$

$= Yx \sin Y$, (by treating triangle YxM as a plane triangle)

$= P \sin Ys. \sin LYs$ (since Ys is nearly equal to xs)

$= P \sin Ls = P \sin (S-\beta)$

$\therefore \ d\beta = P \sin (S-\beta) \sec \lambda$ ....(1)

Also $d\lambda = -Yx \cos Y$

$= -P \sin Ys \cos LYs$

$= -P \sin YL \cos Ls$, by producing YL to 90°

$= -P \sin \lambda \cos (S-\beta)$ ....(2)

From equations (1) and (2), it could be seen that parallax displaces any star from its mean position Y to any place x on an ellipse whose semi-major axis is P parallel to the ecliptic, and semi-minor axis is $P \sin \lambda$. The equation to this apparent elliptic orbit on the tangent plane of the celestial sphere at the mean position of the star is given by $\frac{x^2}{P^2} + \frac{y^2}{P^2 \sin^2 \lambda} = 1$. The star occupies any point on this ellipse during the course of the sidereal year.

141. Effect of parallax on the R. A. and declination of a star.

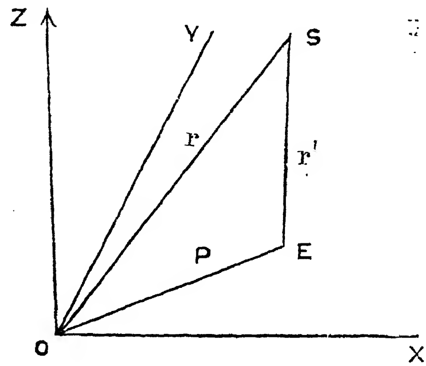


Fig. 93.

Let α, δ be the R. A. and declination of a star S referred to O the centre of the sun as origin and let E be the centre of the earth; let OS be r, $ES = r'$ and $OE = \rho$. If ω be the obliquity of the ecliptic and S the heliocentric longitude of the earth, we have the resolved parts of r' along any three rectangular axes equal to the sum of the resolved parts of ρ and r, along the same axes.

Therefore, $r' \cos \alpha' \cos \delta' = r \cos \alpha \cos \delta + \rho \cos S$ ...(1)

$r' \sin \alpha' \cos \delta' = r \sin \alpha \cos \delta + \rho \cos \omega \sin S$ ....(2)

$r' \sin \delta' = r \sin \delta + \rho \sin \omega \sin S$ ....(3)

From (1) and (2) $\tan \alpha' = \dfrac{r \sin \alpha \cos \delta + \rho \cos \omega \sin S}{r \cos \alpha \cos \delta + \rho \cos S}$ ......(4)

Since $(\alpha' - \alpha)$ is a very small quantity we have by Taylor's Theorem, $\tan \alpha' = \tan (\alpha + \alpha' - \alpha)$

$= \tan \alpha + (\alpha' - \alpha) \sin 1'' \sec^2 \alpha +$ terms containing higher powers of $(\alpha' - \alpha)$, which can be neglected.

$$\therefore \ (\alpha' - \alpha) \sin 1'' \sec^2 \alpha = \frac{r \sin \alpha \cos \delta + \rho \sin S \cos \omega}{r \cos \alpha \cos \delta + \rho \cos S} - \frac{\sin \alpha}{\cos \alpha}$$

$$= \frac{\rho (\sin S \cos \omega \cos \alpha - \sin \alpha \cos S)}{\cos \alpha (r \cos \alpha \cos \delta + \rho \cos S)}$$

$\therefore$ Parallax of a star in R. A. $=(\alpha'-\alpha)''$

$$=\frac{\rho}{r\sin 1''}\,\frac{(\cos\alpha\cos\omega\sin S-\sin\alpha\cos S)}{\cos\delta\,(1+\frac{\rho}{r}\cos S\sec\alpha\sec\delta)}$$

$$=\frac{\rho}{r\sin 1''}\sec\delta\,[\cos\alpha\sin S\cos\omega-\sin\alpha\cos S]\ldots\ldots\ldots(5),$$

omitting terms containing higher powers of $\frac{\rho}{r}$ than the first. Here $\frac{\rho}{r\sin 1''}=P$, the annual parallax of the star.

Squaring and adding (1) and (2) and using only terms up to the first power of ρ, as ρ is small compared with r

$$r'^2\cos^2\delta'=r^2\cos^2\delta+2\,r\,\rho\,(\cos\delta\cos\alpha\cos S+\cos\delta\sin\alpha\sin S\cos\omega)\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

$\therefore$ $r'\cos\delta'=r\cos\delta+\rho\,(\cos\alpha\cos S+\sin\alpha\sin S\cos\omega)$, omitting higher powers of ρ than the first.

$$\therefore\quad \tan\delta'=\frac{r\sin\delta+\rho\sin S\sin\omega}{r\cos\delta+\rho\,(\cos\alpha\cos S+\sin\alpha\sin S\cos\omega)}$$

But $\tan\delta'=\tan(\delta+\delta'-\delta)=\tan\delta+(\delta'-\delta)\sin 1''\sec^2\delta$

$\therefore$ $(\delta'-\delta)\sin 1''\sec^2\delta=\tan\delta'-\tan\delta$

$$=\frac{r\sin\delta+\rho\sin S\sin\omega}{r\cos\delta+\rho\,(\cos\alpha\cos S+\sin\alpha\cos\omega\sin S)}-\frac{\sin\delta}{\cos\delta}$$

$$=\frac{\rho\,(\sin S\sin\omega\cos\delta-\sin\delta\cos\alpha\cos S-\sin\delta\sin\alpha\cos\omega\sin S)}{\cos\delta\,[r\cos\delta+\rho\,(\cos\alpha\cos S+\sin\alpha\cos\omega\sin S)]}$$

i. e. $$(\delta'-\delta)''=\frac{\rho}{r\sin 1''}\Big\{\sin S\sin\omega\cos\delta-\sin\delta\cos\alpha\cos S-\sin\delta\sin\alpha\cos\omega\sin S\Big\}\ldots\ldots(7),$$

omitting higher powers of $\frac{\rho}{r}$ than the first.

142. Measurement of stellar parallax.

In the determination of the annnal parallax of a star either of the formulae for the parallax of a star in R. A.

or declination can be used. If $\triangle \alpha$ be the change in R. A. of a star of heliocentric R. A. α and declination δ due to parallax we have from (5) of article 141,

$\triangle \alpha \cos \delta = (\alpha_1 - \alpha) \cos \delta = P X$, where α_1 is the geocentric R. A. of the star and

$$X = [\cos \alpha \cos \omega \sin S - \sin \alpha \cos S] \quad \text{.....................(1)}$$

X is called the parallax factor for the star in R. A. and it varies with the sun's longitude throughout the year.

Let A be the star whose distance has to be determined by observation and let B be a faint star assumed to be practically at infinite distance and therefore having no appreciable parallax, but very near A in angular distance. Let α_0 be the R. A. of B. On any day the difference between the geocentric R. A. α_1 of A and α_0 of B could be measured. Let $\alpha_1 - \alpha_0 = m_1 = (\alpha_1 - \alpha) + (\alpha - \alpha_0.)$ Since B is regarded practically at an infinite distance, its heliocentric R. A. is α_0 only.

$\therefore$ $(\alpha - \alpha_0)$ in m_1 is a constant quantity.

$$\therefore \quad m_1 = X_1 P \sec \delta + (\alpha - \alpha_0) \quad \text{..................................(2),}$$

where X_1 is the parallatic factor on the particular date of observation. The observation is repeated some months later and let a similar result be also recorded.

i. e. $m_2 = X_2 P \sec \delta + (\alpha - \alpha_0)$ ....(3)

Eliminating $(\alpha - \alpha_0)$ from (2) and (3) we have

$m_2 - m_1 = P \sec \delta \, (X_2 - X_1)$ giving the annual parallax of the star A as $P = \dfrac{(m_2 - m_1) \cos \delta}{X_2 - X_1} = \dfrac{\rho}{r \sin 1''}$ ....(4),

where ρ is the earth's distance from the sun and r the distance of the star from the sun. Therefore r is known.

143. Conditions leading to the most accurate determination of parallax.

The result given by (4) of article 142 may have some observational error, and let e be the error in the measured quantity (m_2-m_1). The annual parallax P will have an error of $\frac{e \cos \delta}{X_2-X_1}$ and this error is least when X_2-X_1 is greatest.

In equation (1) of article 142, let $a \cos b=\cos \alpha \cos \omega$ and $a \sin b=\sin \alpha$

so that $X=a \sin (S-b)$ ....(1)

$\therefore$ The greatest numerical value of X is a and this occurs when $(S-b)=90^\circ$ or 270°.

Also $a^2=\cos^2 \alpha \cos^2 \omega+\sin^2 \alpha$.

or $a=(1-\sin^2 \omega \cos^2 \alpha)^{\frac{1}{2}}=$ the maximum numerical value of X, and b is given by $\tan b=\tan \alpha \sec \omega$. Hence when X is maximum, the sun's longitude S is given by $S=\tan^{-1}(\tan \alpha \sec \omega)+90^\circ$ or $\tan^{-1}(\tan \alpha \sec \omega)+270^\circ$. Therefore the dates when the parallax factor attains its greatest numerical value can be calculated. If the interval between the two dates be six months,

$$X_1=-X_2$$

and P is given by $\frac{(m_2-m_1) \cos \delta}{2 X_2}$.

The modern method in determining stellar parallax consists in taking photographs on or near the most favourable dates determined by consideration of the parallax factor, of the region of the sky in which the star (called parallax star) whose distance is to be measured is situated. Several faint stars called comparison stars and lying near the parallax star are also selected for observation. From photographic plates, taken at six months intervals, many values for P of the type (4) of article 142 are obtained.

The best fitting value is then chosen as the horizontal parallax of the star. By this method, the nearest star proxima centauri is found to have an annual parallax of $0''{\cdot}76$.

144. Parsec and light-year.

A star having an annual parallax of $1''$ is said to be at a distance of one *parsec*. Taking the earth's mean distance of $92{\cdot}9\times10^{6}$ miles as unity and $\sin 1''$ as $\frac{1}{206265}$, the parsec is found to be $206265\times92{\cdot}9\times10^{6}$ or $19{\cdot}16\times10^{12}$ miles. Therefore it is easy to see that even the nearest stars are at distances of the order of million times the distances of the planets from the sun. A star of parallax $0''{\cdot}001$ is at a distance of 1000 parsecs or $19{\cdot}16\times10^{15}$ miles.

These stellar distances are so great that it is only convenient to express them by parallax, by parsec or by what is called *light year*. This last term signifies the time in years that a ray of light from the star takes to reach us. From the velocity of light, it is known that it travels a distance of $6{\cdot}88\times10^{12}$ miles in a year and therefore a star of parallax $1''$ or 1 parsec distance is said to be at a distance of $\frac{19{\cdot}16\times10^{12}}{6{\cdot}88\times10^{12}}$ or $3{\cdot}26$ light years. It is found then that even the nearest star is situated at a greater distance than the above value. The table on page 212 gives the parallaxes of some of the prominent stars.

TABLE OF PARALLAXES OF A FEW OF THE BRIGHT STARS

Name of star	Magnitude	Approximate annual parallax (in seconds)	Distance in millions of times the distance of the sun from the earth.	Time in years that light takes to reach us (light-years)
α centauri	0·2	0·75	0·28	4·4
Sirius	−1·4	0·37	0·56	8·8
Procyon	0·5	0·31	0·69	11
Altair	0·9	0·28	0·74	12
Aldebaran	1·1	0·17	1·4	22
Capella	0·2	0·12	1·7	27
Vega	0·1	0·12	1.7	27
Polaris	2·1	0·07	3·0	47
Arcturus	0·2	0·024	8·7	140

CHAPTER XIII.

THE MOON.

145. Introduction.

Of all the celestial bodies, the moon is the nearest to us and is perhaps the next in importance to the sun. Together with the sun, it causes the tides and its light though borrowed from the sun is welcomed with pleasure by us at night, when there is nothing else except the twinkling light of the stars to illumine the earth. Its rapid motion among the stars, its enormous size and the succession of its beautiful phases have attracted the attention of all, even from very early times. The moon revolves round the earth in an elliptical orbit. Its mean distance from the earth is 238,840 miles, its greatest distance being 252,972 miles and least distance 221,614 miles. Its linear diameter is 2163 miles. When it is nearest the earth, it is said to be in *perigee*, and when farthest, it is said to be in *apogee*.

146. The apparent motion of the moon

The apparent motion of the moon is more rapid than that of the sun and this could be studied by observations of its motion relative to the stars. It is also inferred from the daily retardation in the time of moonrise. The average daily motion of the moon relative to the stars is about 13°.

147. Synodic and sidereal months

The sidereal month is the period occupied by the moon in making a revolution of 360° with respect to any star. Its average length is 27 days 7 hours 43 minutes and 11.6 seconds. The variation in its length is caused by the perturbing forces on the moon. Another period usually associated with the moon is *the synodic month*

which is its period of revolution relative to the sun. This is termed *a lunation* and is longer than a sidereal month, since the sun too has a motion of about 1° relative to the stars. The average length of a lunation is 29 days 12 hours 44 minutes 2.87 secs. The variation in this period is also due to the variation in the eccentricity of the lunar orbit caused by the perturbing forces due to the sun and the planets. *The tropical period* is the name given to the period of revolution of the moon relative to the first point of Aries. This is about 6.85 secs. shorter than the sidereal period, as the first point of Aries has a slow retrograde motion along the ecliptic. Its length is 27.32158 days.

148. Relation between sidereal and synodic months

The sidereal and synodic periods of revolution of the moon are connected with the sidereal year in the manner indicated by the following equation.

$$\frac{360}{\text{Sidereal period}} - \frac{360}{\text{Synodic period}} = \frac{360}{\text{Sidereal year}}$$

or if S be the length of the sidereal month in days

S′ be the length of the synodic month ,,

and y be the length of the sidereal year ,,

$$\frac{1}{S} - \frac{1}{S'} = \frac{1}{y}$$

149. Phase of the moon or that of a planet.

The variation in the visible portion of the moon's disc is a very striking phenomenon. This is known as the moon's phase. This is due to the fact that only part of its illuminated surface is turned towards our view. At any time, half the surface of the moon or the planets is illuminated by the sun and only part of this light is reflected back upon the earth. So observers on the earth can see only that portion of the illuminated surface which is turned towards their line of sight. By the term *phase*, we mean the ratio between the area of the apparent

illuminated disc of a celestial body and the area of the whole disc.

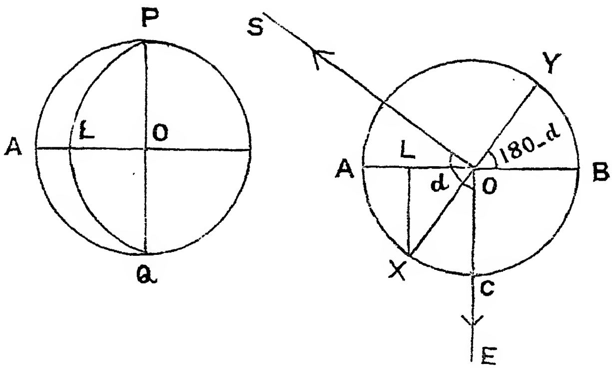


Fig. 94.

In the figure let AX CBY represent the celestial body (moon or any planet) and OS and OE the directions towards the sun and the earth respectively. It is only the projection of the surface corresponding to AX perpendicular to AB, that is visible to observers on the earth, and the apparent area which is visible can be easily calculated. The elliptic curve PLQ is the projection of the visible boundary on which X lies and L is the projection of the point X.

The area of the semi-ellipse $PLQ = \frac{\pi}{2} PO.\ OL$

$= \frac{\pi}{2} a^2 \cos(180^\circ - d)$, where d is the is the angle SOE and a, the radius of the moon's disc.

$= -\frac{\pi}{2} a^2 \cos d$

$\therefore$ Area of $PAQL = \frac{\pi}{2} a^2 (1+\cos d)$

$$\therefore \text{Phase} = \frac{\frac{\pi}{2} a^2 (1+\cos d)}{\pi a^2} = \tfrac{1}{2}(1+\cos d).$$

Here d is called *the elongation of the earth* and it is the difference between the celestial longitudes of the sun and the earth as observed from the moon. The moon is said to be *in conjunction or new* when it has the same geocentric longitude as the sun and it is said to be *in opposition or full* when the difference of the longitudes is 180°.

150. The successive phases of the moon.

The different phases of the moon and its corresponding positions relative to the sun and the earth are shown

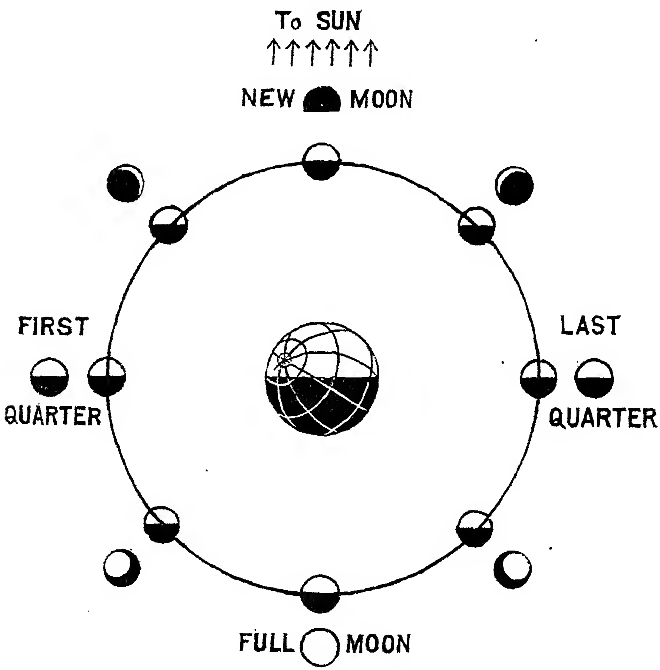


Fig. 95.

in figure (95). When the moon is between the sun and the earth, its illuminated half is wholly turned away from the earth, and therefore it is not seen at all. This is the reason, why the moon is invisible on new moon day, when it is said to be in geocentric conjunction with the sun. Three or four days later, when part of the illuminated

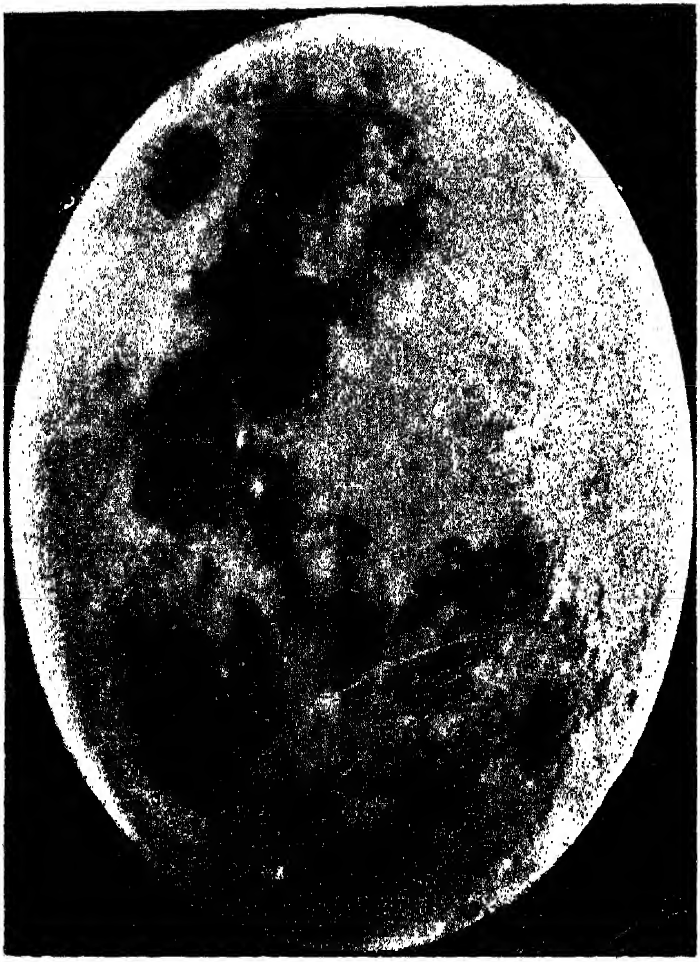


Plate No. 1.

A photograph of the Full Moon taken at the Trivandrum Observatory with the 5" telescope at 9 P. M. on 8th January 1936 a little before the commencement of the total eclipse of the moon.

surface of the moon is turned towards the earth as in the next position (anticlockwise) one could see the moon in crescent form, with the circular boundary nearer the sun. Again about $7\frac{1}{2}$ days later, when the moon's elongation is nearly 90°, it is seen with the western half of its disc bright and the moon is then said to be in *the first quarter*. When the moon's elongation from the sun is 90° exactly, the moon is said to be *in quadrature*. This takes place about 15 minutes later. The half moon is also called *dichotomized moon*. About 11 days after the new moon, more than half the illuminated surface of the moon is turned towards the earth, and the corresponding appearance of the disc as indicated in the diagram is called *the gibbous moon*. When the elongation of the moon is 180°, the whole of the illuminated surface is directed towards the earth and the full disc is seen. This is called *full moon*. During the remaining part of the lunation, the same appearances are repeated in the reverse order, first gibbous, then half, then crescent and finally the next new moon.

151. The surface structure of the moon.

The lunar surface is seen to be of a rugged nature and is full of mountains and valleys. There are many lofty peaks, numerous regions resembling craters of volcanoes, and other large areas looking darker than the surrounding portions. These places are comparatively lower than the surrounding places and are known as "maria" (seas) though there is no water on the surface of the moon. Many of the craters possess, besides the round mountain fencing, lofty peaks at the centres. The origin of these craters on the lunar surface is not definitely known. One supposition is that the moon showed great volcanic activity long time ago and that these craters are the tops of extinct volcanoes. Another theory is the likelihood of these being formed by the

bombardment of numerous meteors, though it is difficult to imagine the existence of a cluster of meteors of the required sizes to create the huge craters seen on the moon.

152. Determination of the heights of lunar mountains.

The lunar surface contains many mountains and ranges of hills, some of them being as high as 15000 feet. These mountains are very sharp and their surfaces have undergone no changes since their formation, as there is no rain or any other influence to act on them and make them wear out. The landscape on the moon will be perfectly clear, as there is no atmosphere round the moon. Hence everything would appear white in the sun or black in the absence of sunlight. The heights of these lunar mountains can be determined by any one of the following methods :—

First method.

Let P be the top of a hill just illuminated by the rays of the sun on the dark side of the moon. By a micrometer its distance from the edge A can be measured. What we really observe is the angle subtended by PL at the eye the observer.

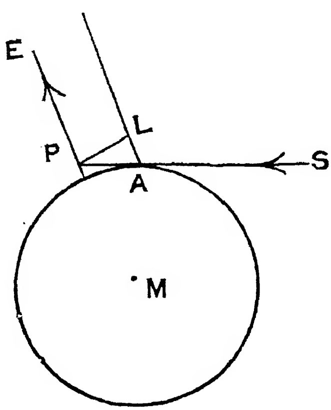


Fig. 96.

Let $P A L = \theta =$ elongation of the moon, the angle subtended by $P L = d$ (in circular measure), and the moon's dis tance from the earth $= a$.

Then $P L = d\, a$ and $A P = \dfrac{a\, d}{\sin \theta}$.

If h is the height of the lunar mountain,

$A P = \sqrt{2 r h}$, where r is the radius of the moon.

Hence $\sqrt{2 r h} = \dfrac{d\, a}{\sin \theta}$ or $h = \dfrac{d^2 a^2}{\sin^2 \theta .\, 2 r}$.

Second method.

The position of an observer at a certain height above sea-level enables him to see the sunrise earlier and the acceleration in the time of rising of the sun is known from the dip of the horizon. Conversely, from the acceleration of the time of rising, the dip and the height of the observer can be known. Now, this acceleration in the time of sunrise at the top of a lunar mountain, can be observed. This is the difference in the time of just seeing a bright speck on the dark side and the time it takes for the elliptic curve to pass through this spot. From this, the height can be calculated.

153. The moon's orbit.

The moon's R. A. and declination can be observed daily by the transit circle. From a series of such observations, the orbit of the moon on the celestial sphere can be known. The relative distances of the moon can be calculated from the measures of the angular diameters on different days. But it should be noted that the measures of the angular diameters could be taken only from the surface of the earth and not from its centre. So this distance has to be corrected to get the corresponding distance from the earth's centre. These distances and the corresponding directions of the moon enable us to draw its orbit relative to the earth's centre.

The moon's orbit, as has been mentioned already, is an ellipse, with the earth at one of its foci and is in a plane through the centre of the earth inclined at an angle of about 5°-$8'$ to the ecliptic. The points of intersection of this plane with the ecliptic, are known as the *ascending and descending nodes.* The period of sidereal revolution of the moon in its orbit is about $27\frac{1}{3}$ days. The eccentricity of the orbit is found to be nearly 1/18. This can be easily determined by comparing the greatest and least distances of the moon from the earth, corresponding to the least and the greatest angular diameters.

Let d_1 and d_2 be the greatest and least angular diameters respectively, and a, the semi-major axis of the lunar orbit.

Then $\dfrac{d_2}{d_1}=\dfrac{a\,(1-e)}{a\,(1+e)}=\dfrac{1-e}{1+e}$, where e = eccentricity.

$\therefore \quad e=\dfrac{d_1-d_2}{d_1+d_2}=\dfrac{19-17}{19+17}=\dfrac{1}{18}$, $\dfrac{19}{17}$ being the ratio between the greatest and the le

154. Perturbations of the lunar orbit.

The moon does not always describe the same elliptic orbit relative to the earth. The plane of the ellipse and its dimensions are subjected to a slow variation. The elliptic path itself moves, and gets deformed.

(1) *Retrograde motion of the moon's nodes.*

The moon's nodes have a retrograde motion of about 19° per year along the ecliptic. This phenomenon is similar to precession and by this, the pole of the lunar orbit moves about the pole of the ecliptic, performing a complete revolution in 18·6 years. As a result of this, the inclination of the lunar orbit to the equator is also subject to periodic variations from $18^{\circ}-18'$ to $28^{\circ}-36'$.

(2) *Progressive motion of the apse line.*

The apse line of the lunar orbit has a direct motion, similar to the apse line of the earth's orbit round the sun It takes 3232·6 days or about 9 years for it to make a complete sidereal revolution.

(3) The inclination of the lunar orbit to the ecliptic is itself not constant, being subject to a variation from $4^\circ-57'$ to $5^\circ-20'$. This corresponds to the change in the obliquity due to nutation (See Page. 140).

(4) The eccentricity of the lunar orbit and the length of the sidereal period are also subject to small changes.

155. The luni-solar day.

The moon always shows the same portion of its surface to the earth. Hence, the period of its rotation about its axis is the same as its period of revolution about the sun. Hence the length of a solar day at the moon is about $29\frac{1}{2}$ days. The length of the sidereal day for a lunar observer would however be $27\frac{1}{3}$ days nearly.

156. Librations of the moon in latitude and longitude.

Owing to the elliptial character of the moon's orbit, its velocity becomes a maximum at perigee and minimum at apogee, but its rotation about its axis is uniform. Therefore at apogee, the velocity of rotation is more than the velocity of translation and at perigee the velocity of translation exceeds that of rotation. Thus when it is near perigee, the moon will present to us a little more of its western side, and when it is near apogee, it will show us more of its eastern side, than could be expected normally. This phenomenon is called *libration in longitude* and by this we are able to see about $7\frac{3}{4}^\circ$ more of the moon's surface on either side of the region normally in front of us.

The moon's axis is inclined at $83\frac{1}{2}^\circ$ to its orbit. Therefore during the course of the moon's revolution, the

northern part and at other times the southern part are turned to our view. This is known as *libration in latitude.* And its effect is to make the moon's poles oscillate to and fro.

157 Diurnal libration.

Owing to the earth's rotation, the observer is carried with a certain velocity from west to east and when the moon is rising, this velocity helps the observer to see a little more of the western side of the moon, than he would have seen, if he were underneath the moon, as at the moment of its transit. Similarly, when the moon is setting, he sees a little more of its eastern side. This phenomenon is called *diurnal libration* and its greatest value is $57'$. This is simply an effect of parallax, for, when the moon is rising, an observer at the centre of the earth or at a point of the earth where the line joining the moon and the earth's centre meets, is vertically below the moon, whereas, one on the meridian is at a distance 'a' from the centre. Similarly, anyone in the northern hemisphere would see more of the northern portion of the moon, and one in the southern hemisphere would see more of the southern portion, than a man at the equator.

158. Total lunar surface visible.

At any time, an observer can see only the portion of the lunar surface bounded by a cone whose vertex is the observer and which touches the moon. This is slightly less than half the lunar surface. As a consequence of the different librations mentioned above, we are able to get a knowledge of about 60 % of the lunar surface, in the course of one revolution of the moon about the earth.

159. Lunar orbit relative to the sun.

The moon moves round the earth in an ellipse with the earth as one of its foci, but the earth moves round the sun in a similar orbit, and therefore it is interesting

to see what kind of orbit, the moon is describing with respect to the sun. In one year, the moon goes round the earth about $12\frac{1}{2}$ times and hence one should expect the moon's orbit to cut the ecliptic in 25 places, if the plane

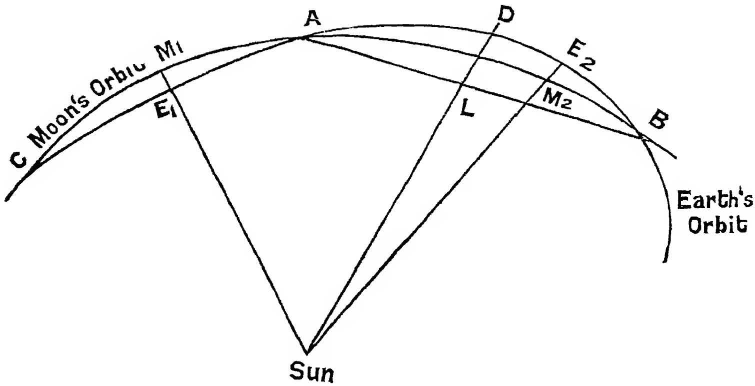


Fig. 97.

of the motion of the earth, and that of the motion of the moon were the same. It might be inferred from this that the moon's path is in some places concave and in other places, convex to the sun. But it is not so. The path is everywhere concave to the sun as could be seen from the above figure drawn to illustrate the motion of the moon and the earth from one full moon to the next new moon.

In the above figure, where the moon's orbit is shown to intersect the earth's orbit in C, A and B, the lunar orbit can be convex if at all, only between A and B, where the moon's orbit lies inside the earth's orbit. Join AB. Now, AB being nearly $\frac{1}{25}$ of the lunar orbit, it should subtend an angle $\frac{2\pi}{25}$ at the sun S.

Assuming ADB to be part of a circle of radius a with the sun as centre, and if L is the foot of the perpendicular from the sun on the chord AB,

$$L\,D = a\left(1-\cos\frac{\pi}{25}\right)=\frac{a}{2}\,\frac{\pi^2}{25^2}=\frac{3\,a}{400} \text{ nearly}$$

= thrice the moon's distance from the earth. Therefore the moon lies between A B and A D B, when it is inside the earth's orbit in this part of its course. Hence its orbit is concave to the sun here, and so also in other places.

The velocity of the moon tangential to its orbit is 2300 miles per second, and the gravitational pull exerted on it by the earth and the sun makes it deviate from its straight path by $\frac{1}{20}$ of an inch in a second and thus gives it a concave shape with reference to the sun.

160. Physical conditions on the lunar surface.

From continued observations, it has been established that there is no life on the moon. It has no atmosphere, no water and hence no vegetation. The non-existence of atmosphere is established by the sudden disappearance of stars, when they are occulted by the moon. If the moon had an atmosphere, the rays coming to us from the star, grazing the moon would get bent by refraction, and this would make the disappearance gradual and not sudden. The fact that the boundary of the lunar surface seen on the solar disc during a solar eclipse is perfectly sharp, leads us to the same conclusion regarding the non-existence of atmosphere. Again, the mass of the moon and the consequent gravitational pull will not be sufficient to keep the molecules of air, if any, on to the lunar surface, against their tendency to fly away due to their molecular movements. These molecules have at lunar temperatures a velocity larger than $1\frac{1}{2}$ miles per second which is the limiting velocity for a particle to fly away from the moon.

161. Temperature of the moon's surface.

Lunar heat is partly heat obtained from reflected sunlight and partly heat radiated out by the moon, after absorbing heat energy from the sun. Every portion of the lunar surface is exposed to the sun in a cloudless sky for about 14 days and so it attains a temperature of about 120° C, but as soon as sunset takes place, the temperature drops to about $-10°$ C and reaches a minimum of $-80°$ C during a night extending over two weeks. In fact, the temperature would even go lower but for the stored up heat in the moon, which it possesses on account of some other portion of the moon receiving solar rays. Ordinarily, on the moon, the temperature before sunrise is $-250°$ F and after sunrise it is $+200°$ F.

162. The albedo of the moon.

The ratio of the amount of sunlight reflected to the amount received by a spherical body is called its *albedo*. The moon has an albedo of 0·073. Therefore about 93 % of the rays of sun's light go to heat the moon's surface. This low albedo is due partly to the irregularity of the moon's surface, and partly to the poor conductivity of the volcanic dust on the lunar surface not emitting even stored up heat. The brightness of the full moon is $\frac{1}{465,000}$ of that of the sun. Even if the whole of the sky be packed with full moons, the light obtained would not be more than 20 % of the sun's light. When the moon is waning (getting reduced in its phase), the light falls down rapidly with the elongation, until it is reduced to $\frac{1}{1000}$ of the light of the full moon, when the elongation of the moon is 20°.

163. Lunar influence on the earth.

The main influence of the moon on the earth is in producing the phenomenon of the tides. The sun also

exerts a similar influence on the earth. Besides this, there are very minute disturbances of terrestial magnetism and atmospheric pressure depending upon the position of the moon in its orbit. A brief explanation of how tides are caused may be given as follows.

In the following explanations it is assumed that the earth is spherical and that it is covered all over by an ocean of uniform depth.

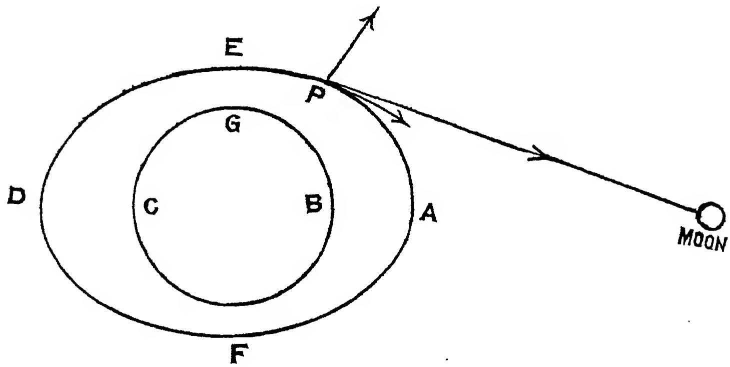


Fig. 98.

The attraction exerted by the moon on a particle of water at A is more than that at a point B on the earth. In the same manner, a point C on the earth is subject to a greater force of attraction from the moon than a particle of water at D. As a result of the lunar attraction, the surface of an ocean of uniform depth should at any time assume the form of a spheroid, having the moon on its major axis produced. As the moon is revolving round the earth the crests A and D would also be rotating round the earth in the course of a lunar month. Any place like A or D would get high tide twice in a lunar day and the corresponding place of low tide would then be E or F. The earth's rotation and the irregularity of the ocean boundaries bring about such a modification to the above result, that the actual crests are formed at places 90° distant

from the point below the moon. If the lunar orbit coincided with the equatorial plane of the earth, the two poles would have always low tide only, but the moon is moving in such a manner that its declination changes from 28° north to 28° south and so the crest of the tide travels over a region of the earth between latitudes 28° north and 28° south.

164. The tide-generating force.

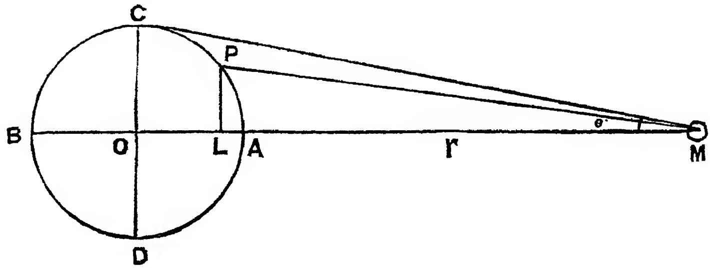


Fig. 99.

Let m be the mass of the earth, a its radius, and m' the mass of the moon (M) situated at a distance r.

The resultant attraction between the earth and the moon is $\frac{\Upsilon\, m\, m'}{r^2}$, where Υ is a constant.

Now the acceleration of unit mass at A due to the attraction of the moon is $\frac{\Upsilon\, m'}{AM^2}$, and those at O and B are $\frac{\Upsilon\, m'}{r^2}$ and $\frac{\Upsilon\, m'}{BM^2}$ respectively.

Hence the acceleration towards M of a particle at A relative to the centre of the earth

$$=\Upsilon\, m'\left(\frac{1}{AM^2}-\frac{1}{r^2}\right)=\frac{\Upsilon\, m'}{AM^2.\, r^2}\left(r^2-AM^2\right)$$

$$=\frac{\Upsilon\, m'.\, a\,(r+AM)}{AM^2.\, r^2}=\frac{\Upsilon\, m'\, a}{r^2}\left(\frac{2\,r-a}{(r-a)^2}\right)$$

$$=\frac{2\,\Upsilon\,m'\,a}{r^3}\,\frac{\left(1-\dfrac{a}{2\,r}\right)}{\left(1-\dfrac{a}{r}\right)^2}$$

$$=\frac{2\,\Upsilon\,m'\,a}{r^3}\left(1-\frac{a}{2r}\right)\left(1+\frac{2\,a}{r}\right)$$

$$=\frac{2\,\Upsilon\,m'\,a}{r^3},\ \text{omitting all the powers of}\ \left(\frac{a}{r}\right)$$

from the expansion.

Similarly the acceleration of a particle at B is $\dfrac{2\,\Upsilon\,m'\,a}{r^3}$ relative to O and away from it.

Now the acceleration of a particle at C is given by $\dfrac{\Upsilon\,m'}{CM^2}$ along C M and this can be resolved into $\dfrac{\Upsilon\,m'}{CM^2}\cos\theta$ along O M and $\dfrac{\Upsilon\,m'}{CM^2}\sin\theta$ along C O.

Since θ is small,

$$\cos\theta=1,\ CM=OM\ \text{and}\ \sin\theta=\frac{a}{CM}=\frac{a}{OM}\ \text{nearly.}$$

$\therefore$ The acceleration of C parallel to $OM=\dfrac{\Upsilon\,m'}{OM^2}$

= acceleration of O along O M.

And the acceleration of C towards O is $\Upsilon\,m'\,a$

Similarly, a particle at D has also an acceleration of $\dfrac{\Upsilon\,m'\,a}{r^3}$ towards O.

Now consider any other point P on the earth's surface, and let P L be the perpendicular from P on O M.

The difference between the attractive forces exerted by the moon at P and L, tends to give P an acceleration towards L, equal to $\dfrac{\Upsilon\,m'.PL}{r^3}$.

Again, the difference between the forces exerted by the moon at O and L tends to give L an acceleration of $\frac{2\Upsilon m'.OL}{r^3}$ relative to O and away from it.

Hence the accelerations at a point P relative to O are $\frac{2\Upsilon m'.OL}{r^3}$ and $\frac{\Upsilon m' PL}{r^3}$ respectively along OM and perpendicular to it.

Thus we see that the tide-generating force due to any body like the moon at a point of the earth's surface is directly proportional to its mass and inversely proportional to the cube of its distance.

165. Diurnal, semidiurnal and fortnightly tides.

At any place, the high tide occurs when the moon is nearest the zenith or nadir in the course of its diurnal rotation and the height of the tide will be greater or smaller according as this distance is smaller or greater. Therefore when the moon's declination is $d°$ north, and when it transits at Q the high tide for a place of latitude φ is higher

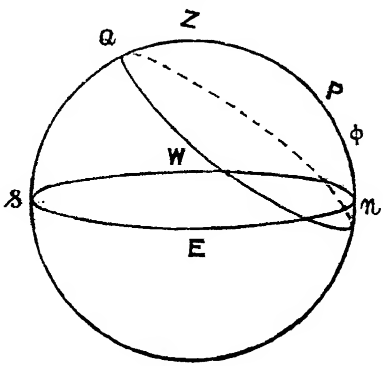


Fig. 100.

than what it would be when the moon is having the lower transit. Thus the real high tide for the place is occurring only once in a lunar day, though there are two high tides, each occurring at intervals of half a lunar day. The length of the lunar day is found to be 24 hrs. 50 mins. 32 secs., as

it is the interval between two transits of the moon at a place, and therefore, the period of one lunation ($29\frac{1}{2}$ mean solar days) is equal to $28\frac{1}{2}$ lunar days.

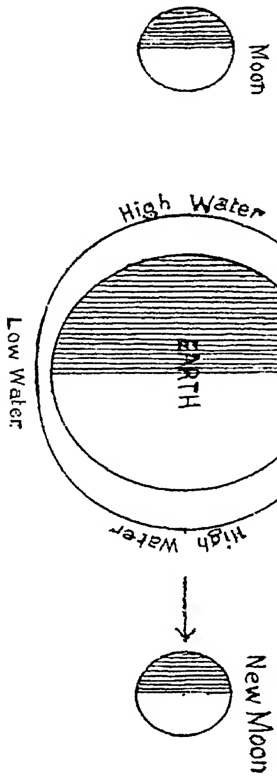
The moon's meridian zenith distance and declination go through a complete cycle of changes in the course of a lunar month; but in half a month, their values differ from the previous values only in sign. Hence the tides reach the same height at a place once in a fortnight.

166. Spring and neap tides.

As has been mentioned already, the tides are caused both by the sun and the moon, though the latter is responsible for $\frac{7}{10}$ of the total effect. The height of the tide in a place may be taken to be the algebraic sum of the heights of the tides that would be caused by the sun and the moon separately.

At the time when the moon is full or new, the total effect of the tides due to the sun and the moon is to add to each other and therefore both high tide and low tide are most marked at that time. These are called *spring tides.* The height of the spring tide is equal to $\left(1+\frac{3}{10}\right)$ of the lunar tide alone. (See Fig. 101, where the earth is supposed to be covered with a shell of water and not to be rotating)

When the moon is at the first or last quarter, the high tide caused by the moon at the sub-solar point of the earth is destroyed somewhat by the action of the sun, whereas the high tide caused by the sun at the sub-lunar point is also partially destroyed by the moon. The action of the moon being the more important, predominates. This is called the *neap tide* and its height is equal to $\left(1-\frac{3}{7}\right)$ of the lunar tide. (See Fig. 102).
Spring tides are most marked when the sun and the moon are in perigee while neap tides are best seen when the moon is in apogee.


Moon
High Water
Low Water
EARTH
Low Water
High Water
New Moon

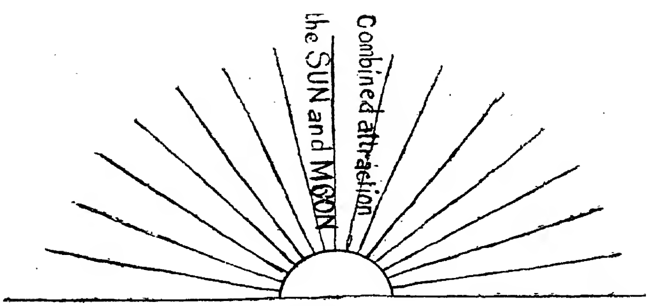

Combined attraction
the SUN and MOON

Fig. 101.

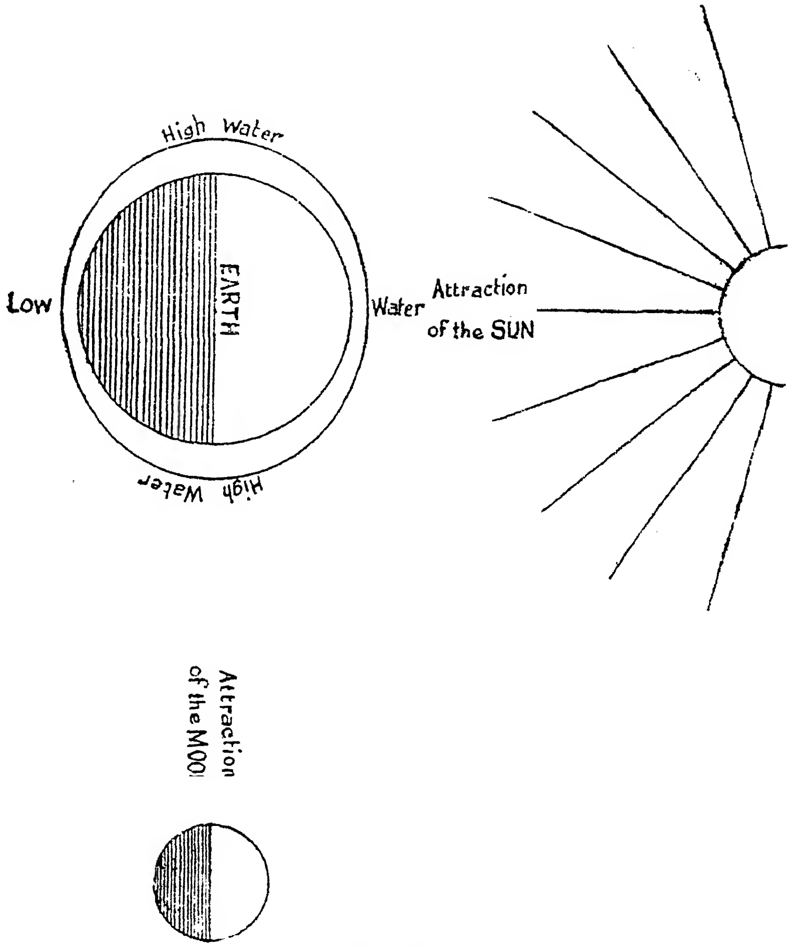


Fig. 102.

167. The priming and lagging of the tides.

When the moon is at any other position relative to the sun, than at quadrature or syzygy, their combined action in creating tides is discussed as follows:—

Assuming the moon to be at rest, let the sun and therefore the sub-solar point move round the earth in the clockwise direction, as the moon gets older and older. When the moon is three or four days old, let S_1 be the

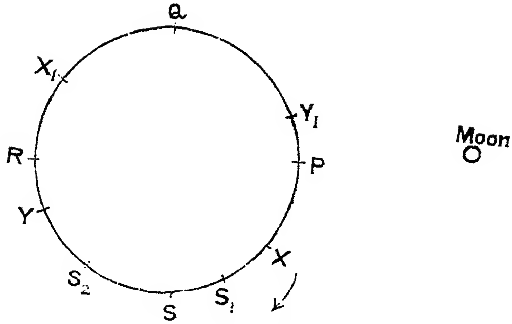


Fig. 103.

sub-solar point. The combined effect of the sun and the moon is to keep the crest between some point X between P and S and its antipode X_1 between Q and R, X being west of P. The high water therefore occurs earlier than it would have occurred under the influence of the moon alone. The tides are then said to *prime*.

After the first quarter, when the sub-solar point is at S_2, the high tides are somewhere near Y and Y_1, which are stations east of R and P. These high tides then occur later everyday. This is said to be *the lagging of the tide.*

168. Tidal friction and its effects.

1. Owing to the friction got into play between the earth and the oceans when the water moves, the crest of the tidal wave is always kept at a point east of the point underneath the moon or the point diametrically opposite. The attraction of the moon tends to draw this water back towards the point beneath the moon. Thus a couple is generated, which tends to diminish the angular velocity of rotation of the earth and thereby to increase the length of the sidereal day. This effect is quite imperceptible, and it cannot change the length of the sidereal day by more than ·006 of a second in another 1000 years. Against this tendency for the increase in the length of the

sidereal day, there is however another tendency for its shortening also, caused by the shrinkage of the earth due to gradual cooling and the consequent increase of the velocity of rotation. Probably both these small changes working in opposite directions, keep the length of the sidereal day constant.

2. Another effect of the tidal friction is to increase the distance of the moon from the earth and thereby the length of the sidereal period of revolution. This is due to the couple exerted by the earth on the moon equal and opposite to that which the latter exerts on the former. This couple increases the areal velocity of the moon with respect to the earth and therefore the length of the radius of the moon's orbit. When the radius of the orbit increases, the angular velocity in the orbit decreases (this is also the angular velocity of rotation of the moon), and then the sidereal period increases. The ultimate effect of tidal friction will be to make the sidereal day and the sidereal month equal in length, by decreasing the angular velocity of rotation of both the earth and the moon. After the lapse of millions of years when the day and the month become equal in length (about $58\frac{1}{3}$ sidereal days) we will have the wonderful phenomenon of the earth and the moon rotating, showing the same face to each other.

3. The fact that the moon is always showing the same side of its surface to us can be explained by considerations of tidal friction which might have existed in the moon years ago, when its surface contained probably oceans and other liquid matter in the form of molten liquid ejected by the active valcanoes existing in the moon. The earth's mass being 81 times that of the moon, should produce considerably greater tides on the moon, and therefore the frictional effects caused by such tides should have brought about the equalisation of the moon's periods of rotation and revolution, which would have been different, years ago.

4. It is observed that Mercury is having the same duration of time both for its rotation about its own axis, and for its revolution about the sun. This seems to be the case with Venus also. These two planets being very near the sun, the sun's tide-raising force, which is proportional to $\frac{1}{r^3}$, should be considerably larger on these planets than on the earth. It is possible that the tidal friction should have made the two periods equal in the case of these planets. To bring about such a state of affairs on the earth or the more distant planets, it may take a considerably longer time or it may not be possible at all, as the tidal friction decreases very rapidly with the distance of a planet.

169. The Metonic cycle, the golden number, and the epact.

Many religious festivals in various countries have been regulated by the moon's phases even from pre-historic times and to fix the date of these ceremonies before hand, it was necessary to predict the moon's phases from a knowledge of the length of the tropical year and the synodic period of the moon. Even so early as 433 B. C. an Athenian astronomer named Meton and another astromer called Euctemon, discovered that after a cycle of 19 years the new moons and full moons as well as the other phases of the moon occurred on the corresponding dates as in the previous cycle. If therefore a regular record of the different phases of the moon for a cycle of 19 years be made, one could make out from it the dates of any of the phases of the moon for the next cycle or any other cycle after that. In some parts of ancient Greece, some of these dates were inscribed in letters of gold upon public monuments and even now the number which marks the rank of any one of the years in a cycle is called the *golden number* of that year. How the Metonic cycle of 19 years brings back the same phase of the moon for a few cycles at least,

could be seen from the fact that 235 lunations occupy $235 \times 29 \cdot 5306$ or 6939·69 days, and 19 tropical years cover $19 \times 365 \cdot 2422$ or 6939·6 days, giving a difference of about 2 hours only, between these two periods. It is found that the cycle now in use is such that it might be said to have been started from the year 1. B. C. Hence to determine the golden number of any year, add 1 to the number representing the year, divide the sum by 19, and then the remainder gives the golden number. When the remainder is zero, the golden number is 19. Thus the golden number for 1936 is 18.

The moon's age on the 1st of January of any year is called the *Epact* of that year. The 1st of January 1. B.C. happened to be a new moon day. The golden number of that year is 1 and the epact is zero and will be the same for the first tropical year of a Metonic cycle. As the length of a tropical year is 365·242 days and as 12 lunations occur in 12×29.5306 or 354·367 days, the moon will be 10·875 days, 21·75 days, 32·625 days, (i. e. 3·094 days) older at the beginning of the 2nd, 3rd and 4th year than at the beginning of the 1st year. If the civil year had not differed from the tropical, we would have had the following rule to find the epact for any year :—i. e., subtract 1 from the golden number, multiply by 10·875 and divide by 29·53; the remainder will give the moon's age on 1st January which is the epact of that year. This however is not the rule adopted in finding the epact for the movable ecclesiastical feasts of the calendar, owing to the fact that the length of the civil year is different from that of the tropical year, and that the length of the lunation is not constant. The rule now adopted in finding the epact is to diminish the golden number by 1, multiply that by 11, and then divide by 30; the remainder is taken

as the epact. The actual age of the moon on the 1st January of a year would differ as much as even a day from the epact calculated by the fixed rule now adopted, but the rule is usually adhered to owing to its being simple.

CHAPTER XIV

THE PLANETS AND THEIR MOTIONS

170. Introduction.

The ancients used the name 'planets' to denote those celestial bodies which have a motion relative to the fixed stars. They included the sun and the moon in their list of planets, but not the comets and the meteors, about whose motions little was known to them. By planets, we now denote only those bodies which like the earth revolve round the sun in definite orbits. Including the earth, there are nine bodies and a group of more than a thousand comparatively small objects, all moving in elliptical orbits round the sun. Of these Uranus, Neptune, Pluto and the group of small bodies known as Asteroids or minor planets which move in orbits lying between the orbits of Mars and Jupiter, were not known to the ancients.

171. Bode's law of planetary distances.

It is now known that all the planets move round the sun in elliptic orbits having the sun for a common focus. The famous law known as the Bode's law gives in a remarkable manner the mean distances of some of the nearer planets, though there is no theoretical explanation for the derivation of the law. The law however does not hold good for the more distant planets. The law states that the relative distances of the planets from the sun are roughly given by adding 4 to the numbers, 0, 3, 6, 12, 24, 48, 96, 192, 384. The law holds to a high degree of approximation for planets up to Jupiter, as can be easily verified by comparison with the table at the end of this chapter.

A few words in general about the physical and other features of the planets may not be out of place. It should

be noted at the outset that all the planets move round the sun in the same direction (direct) and that they also rotate about their own axes in the same direction.

172. Mercury.

This is the nearest planet to the sun. It is small in size and has no satellite. Of the orbits of all the planets, the orbit of Mercury has the maximum eccentricity. The inclination of its orbit to the ecliptic is also the greatest excepting that of the minor planet Ceres. Its period of rotation about its axis is the same as its period of revolution round the sun (88 days nearly), whereas the planet Pluto takes 250 years to go round the sun once. Mercury can be best seen by the naked eye only for a short time either before sunrise or after sunset.

Since the planet's orbit has a high eccentricity, it has a large libration in longitude which enables us to see in the course of its revolution much more than half of its surface. The planet has not any well defined marks by which we can easily observe its period of rotation, as in the case of a planet like Mars. From the faint markings, however, Schiaparalli has inferred that the planet always turns the same face towards the sun just as the moon is turning the same face towards us.

The surface temperature of Mercury is found to be 675° F at the centre of the side turned towards the sun; the other side should correspondingly be very cold. The very high temperature of the sun-lit side of Mercury strengthens our conclusion that the period of rotation of Mercury is the same as that of its period of revolution. The average temperature on Mercury is 343° F, while that on earth is 57° F and that on Pluto is −380° F.

Mercury has an albedo of 0·14, which is the fraction of the incident light reflected by it. This low value indicates that possibly the planet has no atmosphere of the

kind that we have on the earth. Moreover when the planet is seen to transit across the disc of the sun, its sharp and black disc shows that there is no atmosphere surrounding the planet; the rays of the sun coming to us grazing the planet do not show any sign of refraction.

The mass of Mercury is very small compared to that of the earth. Consequently the gravitational force exerted by it is so small that a particle having a velocity of $2\frac{1}{4}$ miles a second is likely to fly away from the planet's grip. Now, molecules of air on the planet will fly about with a considerably higher velocity, so that any atmosphere given to the planet is not likely to be kept long by it.

Mercury is thus found to be nearly like the moon in its physical characteristics, having a rough surface of volcanic dust.

173. Venus.

This is the brightest of all the planets. Its diameter is about 7,700 miles and so its size is nearly the same as that of the earth. Its mass, density and surface gravity are nearly comparable to that of the earth. The gravitational pull exerted by venus is only 15 % less than that of the earth. Since it is nearer to the sun than the earth it has an average temperature which is 90° higher than that of the earth.

This planet, like Mercury and the moon, shows phases and it is best seen before sunrise or after sunset. There are no well defined markings on it which will help us to determine its period of rotation. But some astronomers claim that they find some markings and that they have found the period of rotation to be about $23\frac{1}{3}$ hrs. Spectroscopic method of determining the rotation period has not yielded any definite result, and values varying from 12 hrs. to 30 days have been obtained. The true period of rotation

is still unknown. Perhaps it might have a period of rotation which is the same as its period of revolution, in which case there cannot be any wind or rain on the planet.

The planet has an albedo of 0·76, showing that it has a high reflecting power caused by the thick clouds covering the surface. These clouds render it impossible for us to see the surface of the planet. The extra clouds and foggy appearance are due to more water remaining in a state of evaporation than in the case of the earth. This is what should be expected from the higher average temperature prevailing there. As water could exist in both liquid and vaporous conditions, there should be seas and rivers on the planet.

The existence of an atmosphere on this planet is inferred from the bright ring that surrounds the disc of the planet during its transit across the sun's disc, the bright ring being formed by the refraction caused by the atmosphere in Venus. Again the crescent Venus is found to extend a little beyond the geometrical shape expected from its elongation. This is also due to the refraction caused by the atmosphere in Venus. From a study of the light sent to us by Venus, it is found that there is no oxygen in Venus. Hence there cannot be any vegetation there. What we are able to study is only the outer regions of the atmosphere of the planet; nothing about the composition of the inner part of the atmosphere is known.

174. Mars.

This is the nearest of the superior planets to the earth. It is most favourably seen during its opposition, (i.e. when its geocentric longitude differs from that of the sun by 180°) when it is nearest the earth. Though its mean distance from the sun is 141·5 million miles, it comes to a distance of about 48 million miles from the earth, at opposition. The diameter of the planet is 4200 miles, so

that its volume is nearly $\frac{1}{7}$ that of the earth. Its mass is about $\frac{1}{10}$ of the earth's mass.

From a study of the alterations in the positions of certain markings on the surface of Mars, the period of rotation of the planet is found to be $24\frac{2}{7}$ hrs. nearly. The two poles of the planet are surrounded by what are known as ice caps which have a periodical variation in their dimensions due to melting. Besides these, there are other markings of different shades of colour on the surface of Mars, which may be due to water, vegetation or bare land surfaces. These markings are also subject to seasonal changes, which are attributed to particles of ice or an atmosphere of carbondioxide or due to rain falling on a dry surface like that of the moon.

Mars is surrounded by an atmosphere containing very little of oxygen or water vapour, and its density is less than half of the density of our atmosphere. As there is alternation of heat and cold, it might be inferred that there are particles of water, ice and snow on the planet.

Martian seasons.

The axis of rotation of Mars instead of being perpendicular to its plane of motion has an inclination of 25°—$10'$. The inclination of the earth's axis, we know, is only 23°—$27'$. Hence the Martian seasons are more pronounced than ours. The eccentricity of the orbit of Mars is greater than that of the orbit of the earth, so that there is a difference of 26 million miles between the maximum and minimum distances of Mars from the sun whereas this difference in the case of the earth is only 3 million miles. Hence Martian climate will be warmer when the planet is nearer the sun and colder when it is farther away. This effect is superimposed on the Martian seasons, in the same manner as a similar effect is noticed for the earth. Now Mars is farthest from the sun (as in the case of the earth also) when it is winter for

the southern latitudes and summer for the northern latitudes. Therefore the seasons of the southern latitudes are accentuated by this cause in a more pronounced degree than for us.

Temperature in Mars.

When hottest, the equatorial temperature is 60° F; but this will fall down quickly with the sunset, as there are no clouds to keep up the warmth. Near the equator the temperature falls 40° below zero, a few hours after sunset. Near the poles the temperature is $-100°$ F.

Mars is not likely to have a growth of vegetation as we have around us; for, vegetation if existed would give plenty of oxygen which is very little seen in the Martian atmosphere. On the whole Mars is likely to be as uninhabitable as the moon in spite of the existence of water and atmosphere; for, it shows extremes of climatic conditions and its agreeable warm temperature lasts only for a few hours. Mars is now in a condition which the earth will reach many millions of years hence.

Satellites of Mars.

Mars possesses two small satellites, whose diameters are about 40 and 10 miles respectively. The bigger one (Deimos) is at a mean distance of 14,600 miles from Mars and it revolves round the planet in 30 hrs. 18 minutes while the smaller one (Phobos) is at a mean distance of 5826 miles and goes round the planet in 7 hrs. 39 minutes. Owing to the more rapid revolution of Phobos, it would appear to rise in the west, and set in the east 4 ½hrs. later to an observer in Mars, which has a period of rotation of 24 hrs. 37 mins. 22·6 secs.

175. Minor planets or Asteroids.

Next to Mars, there are about a thousand small bodies known as the Asteroids moving in space whose mean distance from the sun is very nearly 2·8 astronomical units,

The first of these bodies to be discovered was Ceres, when search was made for a planet moving between the orbits of Mars and Jupiter. Pallas, Juno and Vesta, the brightest of the Asteroids were discovered later and now the list of minor planets has swollen to more than 1250. When a photographic plate, exposed to the region of the ecliptic and following the motion of the stars, shows any trail, it should be due to a planet and in this way the discovery of the minor planets was rendered easy. Stars will appear as points in such a plate.

There are two theories about the origin of these numerous bodies. One is that these are broken pieces of a large planet which, owing to the disturbance exerted by a planet like Jupiter, split into two, and these again into many more fragments, the splitting taking place so long as these bodies are within what is called the danger zone of the bigger planet. A second suggestion is that at one time there was a ring of bodies round the sun all moving uniformly as a whole, just as any one of the rings of saturn, and that the influence of the planets on these different masses brought about the present change in their common orbit. The largest minor planet is 480 miles in diameter and some of the smallest are less than 50 miles in diameter. The total mass of all the minor planets so far discovered is less than that of Mars.

None of the asteroids has an atmosphere, as is shown by their low reflecting power. Their small masses won't be able to keep an atmosphere, even if one existed originally. Some of the asteroids have their actual orbits extending beyond the orbit of Jupiter, while others have their orbital dimensions smaller than those of Mars. The minor planet Eros comes nearer to the earth than Mars in its opposition and so it is now used in the determination of solar parallax. As the planet is very small in size, it is not difficult to find the centre of the disc of the

planet and so observations on it would yield better results than in the case of Mars in determining the solar parallax.

176. Jupiter.

This is the largest and most massive planet in the solar system, though in apparent brightness it is only second to Venus. When it is in opposition, it is more than five times as bright as the brightest star.

The sidereal period of Jupiter is 11·86 years, so that it remains in each of the signs of the zodiac nearly for an year. The period of rotation of the planet about its axis is 10 hrs. and this rapid rotation has made the planet obviously nonspherical, its polar diameter being 82,800 miles and its equatorial diameter 88,700 miles. Its volume is 1300 times that of the earth and its mass 318 times as large. On the surface of Jupiter each one of us will weigh nearly two and a half times our present weight. The velocity of a point on its equator is 28,000 miles a second as against 1040 miles for a corresponding point on the earth.

Surface markings of Jupiter.

The surface of Jupiter is marked by belts of different colours and a few spots which occasionally undergo variations in size. These spots serve as good points for determining the rotation period of the planet. It is found that the different zones of Jupiter have different periods of rotation. At the equator, the period is nearly 9 hrs. 50 mins. and near the pole it is 9 hrs. 56 mins. This variation is probably due to the planet being still in a gaseous or semi-solid condition. The planet has a high albedo of 0·62 and this is due to the thick clouds of carbon dioxide, ammonia, methane and other compounds that are enveloping the planet. The brightness of the planet decreases from the centre to the limb,

The temperature of the surface of the planet is near $-140°$ C, which shows that very little heat is coming from the interior. The surface is warmed only by sun's heat.

Satellites of Jupiter.

Jupiter has nine satellites four of these being known even from the time of Galileo. These four were discovered in 1610 by Galileo and they have periods ranging from thirty hours to $16\frac{3}{4}$ days. These satellites show the same face to the planet, giving proof of their axial rotation period being the same as their period of revolution about the planet. They show some faint markings on their discs which serve to determine their period of axial rotation. They are moving very nearly in the plane of the equator of the planet and the first three of them pass through the shadow of the planet at every revolution. They also transit across the planet's disc and their shadows then may be seen as black dots on the disc of the planet.

Roemer in 1675 determined the velocity of light by observing the eclipses of the major satellites of Jupiter. The periods of revolution of the satellites were known with extreme accuracy, and from these their times of eclipse were calculated by him. When the observed times of eclipse were compared with the calculated values, it was found that when the earth was nearer Jupiter the observed times were earlier than the calculated times and when the earth was more distant, the observed times were later. This difference in the time was explained as being due to the shorter or greater distance that light had to travel on these occasions. Now the greatest difference between the observed and the calculated times for the accelerated and retarded eclipses should be equal to the time taken by light to travel a distance equal to the diameter of the earth's orbit. Roemer got a value of 22 mins. for this difference, as against the recently determined value of

$16\frac{1}{2}$ mins. From the known value for the diameter of the orbit, the velocity of light was determined.

Observations of the times of the eclipses of these satellites enable us also to determine the longitude of the place of observation, as the Greenwich times of these eclipses are calculated and given in the Nautical Almanac. Now, if from any station one of these eclipses be observed, the difference in the observed time and the Greenwich time gives the longitude of the place. The eighth and the ninth satellites revolve in the retrograde direction about the planet. This is in the direction opposite to that in which almost all bodies rotate and revolve in space.

177. Saturn.

This planet is unique in having a system of rings consisting of millions of bodies all moving round it. Before the invention of the telescope, this was the most distant planet known to the ancient people.

The mean distance of this planet from the sun is 886 million miles and the eccentricity of its orbit is ·056 and its orbit is inclined to the ecliptic at an angle of $2\frac{1}{2}°$. Its sidereal period is about $29\frac{1}{2}$ years (10,759 days) and its synodic period 378 days. It is more oblate in size than Jupiter itself. Its mean diameter is about nine times that of the earth and its mass is 95 times that of the earth. It is the least dense of all the planets. Its equator is inclined at 27° to its orbital plane. The planet has a high albedo of 0·72, and it is covered by dense clouds as in the case of Jupiter and these give rise to absorption bands when examined by a spectroscope. These clouds contain less of ammonia and more of methane than in the case of Jupiter.

The planet's disc shows a series of belts running parallel to the equator and some spots which are seen occasionally on the planet, help us to determine the rotation period of the planet. Different zones of the

planet are having different periods of rotation, showing that the planet is still in a gaseous condition.

The concentric system of rings of Saturn form a striking feature of that planet. There are three of these, one within the other, separated by a small interval of distance. The division between the first and the second is sharp and clear; but not so is the division between, the second and the third, which is the ring nearest the planet. The third and the innermost is semi-transparent and the edge of the planet can be seen through it. These rings are all very thin not more than 70 miles in thickness as is proved by their disappearance when the earth comes to the plane of these rings once in about 15 years. During half the period of sidereal revolution of the planet one side of the ring is visible to us and during the other half the other side is visible.

The rings are comprised of small satellites as has been demonstrated spectroscopically by Keeler in 1895. He showed that the linear velocity of the different parts of the rings decreased with the distance from the centre. If each of the rings formed a solid piece, the velocity at the rim will be much greater than at any point inside. Clerk Maxwell had shown that such a solid piece of matter cannot be in stable equilibrium. Saturn has ten satellites, the largest of which is Titan discovered in 1655. The most distant satellite, Phoebe, moves round Saturn in the retrograde direction, unlike the remaining nine satellites.

178. Uranus.

This is the first planet that was discovered by the aid of the telescope. Sir William Herschel discovered it on March 13th, 1781 and its orbit which was computed by Laplace was shown to be beyond that of Saturn. The sidereal period of Uranus is about 84 years and its orbit is nearly in the plane of the ecliptic. The eccentricity of its orbit is 0·047.

The disc of the planet being very small, its diameter could not be measured very accurately and therefore the actual diameter of 31,000 miles deduced from the measured angular diameter is only approximate. Uranus has four satellites all revolving round the planet in the retrograde direction and nearly perpendicular to the equator of the planet and from these satellites its mass could be determined. It is found to be about 15 times that of the earth. Its density is only a quarter of that of the earth. This planet is ordinarily visible to the naked eye as one of the faintest stars in the sky.

The albedo of this planet is about 0·6, and this high reflecting power is due to the thick atmosphere of clouds surrounding the planet. Though occasionally faint dark bands appear on the disc, there are no definite markings on the planet which would help us to determine its rotation period. However a rotation period of about $10\frac{3}{4}$ hrs. was announced by Lowell in 1911.

179. Neptune.

This planet was originally taken to be a star and its discovery as a planet was due to the calculations of the eminent astronomers Adams in England and Leverrier in France who simultaneously solved the problem of fixing the position of an unknown planet, which caused certain discrepancies in the observed motions of Uranus. It was Bessel who suggested the problem, and the discovery of the planet in the predicted position was then considered to be the greatest triumph of the law of gravitation formulated by Newton.

The mean distance of the planet from the sun is nearly 30 times that of the earth and it moves in an orbit very nearly circular. It has a sidereal period of about 165 years correponding to a motion of about 2° in the sky per year. This small motion together with its faintness was responsible for the planet being mistaken for a star.

When once it was suspected to be a planet, its motion was detected, and it amounted to about 20″ per day. Relative to the earth it will come to 133″ per day at conjunction.

The diameter of Neptune is 33,000 miles and itsmass is about seventeen times that of the earth. The mass is determined from its satellite which revolves about the planet in 5 hrs. 21 mins. The density of the planet is 1·3 and so its physical condition is similar to that of Saturn or Uranus, in its possessing a dense atmosphere, probably containing the same substances.

180. Pluto.

For a long time, slight differences between the observed and calculated positions of Uranus and Neptune were noted, and these were attributed to the existence of an unknown planet; but as these differences were very small, the position of the supposed planet could not be easily fixed and so it was not possible to assign a fixed place to this planet from calculated results, and then to verify it by photographic observations. But persitant observation at the Lowell Observatory yielded the expected result and Pluto was discovered on the 13th of March 1930 to be a planet having a sidereal period of 249 years and receiving about $\frac{1}{1600}$th of the radiation received by the earth. No satellite has been known to exist for this planet.

Beyond Pluto, even if there be a planet, its discovery is bound to be very difficult, firstly because its motion relative to the stars would be very slow and secondly because it may be too faint to be seen or photographed by telescopes of moderate size. Moreover, the gravitational pull exerted by the sun at such a distance is so small that the motion of a planet beyond Pluto could not be controlled so definitely by the sun, as in the case of the other nearer planets.

181. Kepler's laws of planetary motion.

Before the time of Kepler it was thought that each planet moved in a circular orbit round the sun with its centre not coinciding with the position of the sun. The true laws of planetary motion were discovered by the Danish astronomer John Kepler (1571–1630) who made a thorough study of the extensive planetary observations made by his predecessor Tycho Brahe (1564–1601). It was between 1607 and 1620 that Kepler arrived at his three famous laws, which well fitted with the observations of Tycho Brahe. The wonderful accuracy of these laws, deduced after many years of observation shows the intuitive genius of the great astronomer Kepler. The first two of these laws have already been mentioned and verified in the case of the earth's orbit round the sun (See Chapter VI). The three laws are as follows:—

(1) The orbit of each planet round the sun is an ellipse with the sun at one of the foci.

(2) Each planet moves round the sun in such a way that the radius vector joining the sun and the planet describes equal areas in equal intervals of time.

(3) The squares of the sidereal periods of any two planets are proportional to the cubes of the semi-major axes of their orbits or of their mean distances from the sun.

The first two laws deal with the motion of any one planet, whereas the third deals with the periods and mean distances of any two planets of the solar system. Therefore, if the period of revolution of any one planet round the sun is known, its mean distance from the sun can be calculated in terms of earth's distance by means of the third law. Thus the determination of the absolute distances of any one planet either from the sun or from the earth enables us to determine the distances of all the

other planets, it being assumed of course that the periodic times of all the planets are known.

It was left to Newton to discover the true physical significance of these laws which were found to be remarkably true by observation. He deduced all these laws mathematically from the hypothesis that each planet moves under an attractive force existing between the sun and the planet, the force of attraction being given by $\gamma . \frac{m_1 m_2}{r^2}$ where m_1 and m_2 are the masses of the sun and the planet, r the distance between them and γ is a constant which is the same for all the planets and which is called *the constant of gravitation*.

182. Kepler's method of plotting the orbit of a planet round the sun.

It was the observations made on the planet Mars and his consequent success in plotting the orbit of this planet that enabled Kepler to deduce his first two laws. From the observed right ascension and declination of the sun, the geocentric longitude of the sun for any date is known, and from this, the heliocentric longitude of the earth is known (since these two differ by 180°). Thus at any time of observation the position of the earth on the ecliptic

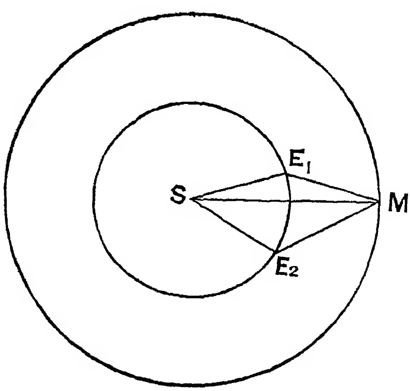


Fig. 104.

plane is known. The sidereal period of Mars is also known from the synodic period, which being the interval between two consecutive oppositions could be easily observed.

Let the two curves shown in fig. (104) represent the orbits of the earth and of Mars. They may or may not be in the same plane. The earth's orbit is supposed to be circular as was assumed in Tycho Brahe's hypothesis. Let the planet Mars be observed when it is at any position M, from the earth E_1. Now, after a period equal to the sidereal period of Mars, the planet again comes back to its original position M. But the earth will now be in a different position E_2. The direction $E_2 M$ is again observed. The orbit of the earth being supposed to be known, $S E_1$, $S E_2$ and $E_2 \hat{S} E_1$ are known quantities. Hence $S \hat{E}_1 E_2$, $S \hat{E}_2 E_1$ and $E_1 E_2$ are also known.

Now $S \hat{E}_1 M$ and $S \hat{E}_2 M$ are known from the two observations and therefore $E_2 \hat{E}_1 M$ and $E_1 \hat{E}_2 M$ can be found. Thus the triangle $E_1 E_2 M$ is completely determined.

$S E_1$, $E_1 M$ and $S \hat{E}_1 M$ now being known, SM and $E_1 \hat{S} M$ can be calculated, and these give respectively the distance of Mars from the sun and the direction of Mars relative to the earth. Thus the point M can be plotted on the paper. In this way any number of positions of Mars in its orbit can be plotted, provided we make on it sets of two observations each, separated by an interval equal to the sidereal period of the planet. Such a method of plotting the orbit of Mars showed its orbit to be very nearly an ellipse with the sun at a focus.

183. Relation between the sidereal and synodic periods of a planet.

The sidereal period of a planet as in the case of the moon is the time that it takes to make one complete revolution in its orbit about the sun relative to the stars.

The synodic period of a planet is the interval between two conjunctions of the same kind or two oppositions with the earth, relative to the sun.

Let S be the synodic period of a planet, T its sidereal period and Y the earth's sidereal period or the length of a year. Then, we have as in the case of the moon, $\frac{360}{S} = \frac{360}{T} - \frac{360}{Y}$

or $\frac{1}{S} = \frac{1}{T} - \frac{1}{Y}$ for a planet whose orbit lies within that of the earth. Such a planet is called an *inferior planet* and its speed in its orbit is greater than that of the earth. In the case of *a superior planet,* i. e. a planet whose orbit lies out side that of the earth, the corresponding relation between the synodic and sidereal periods is given by $\frac{1}{S} = \frac{1}{Y} - \frac{1}{T}$ and here, the orbital velocity of the planet is less than that of the earth.

From the above relations, the sidereal period of a planet can be calculated from the observed synodic period and vice versa.

184. The elongation of a planet.

Before discussing the changes in the elongation of a planet, we shall define here some of the terms that occur in our explanation.

The elongation of a planet is the difference between the geocentric longitude of the planet and that of the sun. When the orbit of the planet is coplanar with that

of the earth, the elongation is equal to the angle subtended at the earth by the line joining the sun and the planet.

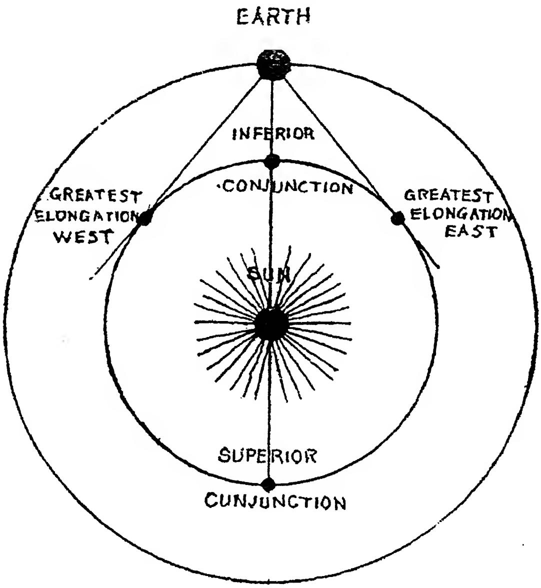


Fig. 105.

Elongation of an inferior planet.

A planet is said to be *in conjunction with* the sun when it has the same geocentric longitude as the sun, and it is said to be *in opposition with* the sun when its geocentric longitude differs from that of the sun by 180°.

It can be easily seen from a figure that in the course of a synodic period, an inferior planet has two positions at which it has the same geocentric longitude as the sun. But in one of these positions the planet has the same heliocentric longitude as the earth and in this position the planet is said to be in *inferior conjunction* with the sun. In the other position, the heliocentric longitude of the planet differs from that of the earth by 180° and in this position the planet is said to be in *superior conjunction*

with the sun. It is also to be noted that an inferior planet can never be in opposition with the sun.

In the case of a superior planet, it can be seen that in the course of a synodic period, the planet is once in conjunction and once in opposition with the sun (Fig. 108).

185. Changes in the elongation of an inferior planet.

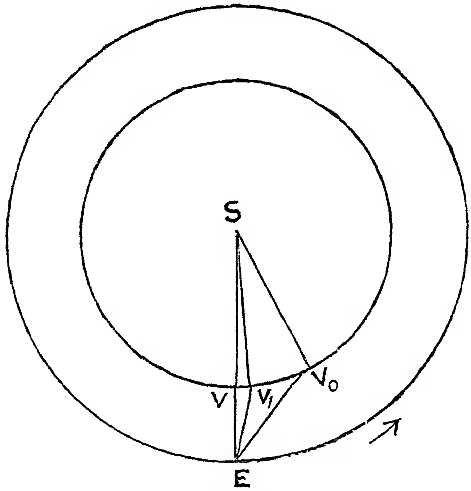


Fig. 106.

Let S be the sun and V, E the positions of an inferior planet V and the earth E at inferior conjunction, the orbits of the planet and the earth being supposed to be circular. At this position the elongation of the planet is zero and the planet rises with the sun and sets with it, so that the planet cannot be seen at any time of the day by a terrestrial observer. Now the radius vector S V moves more rapidly than the radius vector S E, so that if V_1 be the position of V relative to the earth after a small interval of time, the earth may be supposed to be at rest and the elongation of the planet would have increased from 0° to $S\hat{E}V_1$. Now, the apparent motion of V relative to E is retrograde i. e. from east to west. For a terrestrial observer

the planet V will be seen in the east just before sunrise at an altitude depending upon the angle S E V_1.

The elongation of the planet gradually increases and every morning the altitude of the planet just before sunrise is seen to be greater than that in the preceeding morning. This phenomenon goes on till the planet and the earth take such positions that the line joining them is a tangent to the planet's orbit (Fig. 105). The elongation of the planet now, should be clearly a maximum and the planet is seen in the east at its maximum height from the horizon just before sunrise. At this time the planet's motion relative to the earth is direct, and so it would have appeared stationary a little while ago.

From this point onwards the elongation of the planet becomes less and less and hence the altitude of the planet just before sunrise also becomes less and less, until the elongation becomes zero at superior conjunction. At this time the planet rises at the same moment as the sun. (Fig. 105)

After superior conjunction the elongation of the planet increases and an observer on the surface of the earth sees the sun rising when the planet is still below the horizon. Thus the planet is no longer seen in the east in the early morning. The elongation of the planet gradually increases and the altitude of the planet just after sunset becomes greater and greater. This goes on till the elongation of the planet becomes a maximum in the west followed by a stationary position and then a retrograde motion relative to the earth. Thereafter the elongation of the planet decreases and the altitude of the planet just after sunset becomes less and less until the planet is again in inferior conjunction with the sun. (Fig. 105)

186. Changes in the elongation of a superior planet.

Let a superior planet be in opposition at J_1 when the earth is at E. (Fig. 107) After some time when the earth

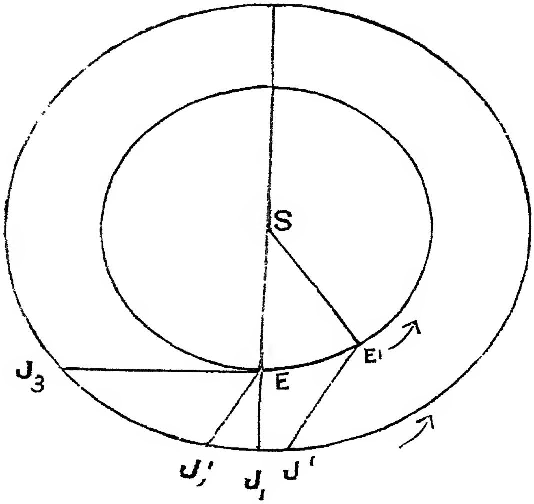


Fig. 107.

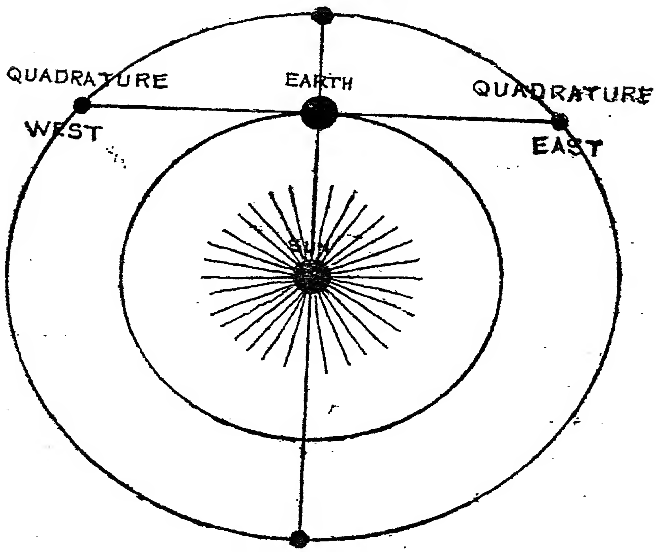


Fig. 108.
Elongation of a superior planet.

moves to E′, let the superior planet move to J′, so that the line joining the earth and the planet rotates in a clockwise direction with respect to $E J_1$. Assuming the earth to be at rest, the apparent direction of motion of the superior planet after opposition is retrograde. Therefore the elongation changes after opposition, from 180° to 90°, when the superior planet is said to be at *quadrature.* (Fig. 108) At this time the combined velocities of the earth and the planet in their orbits make the planet move directly relative to the earth. The planet should therefore have changed its direction of motion from retrograde to direct some time before its elongation reached 90°. After this, direct motion continues till the elongation of the planet is zero, when the planet is said to be in conjunction. After conjunction the elongation of the planet increases till it comes to the stationary position soon after second quadrature. Thenceforward the motion is retrograde. Fig. 108 clearly shows the different positions of a superior planet, during its sidereal period.

187. Relation between the geocentric and heliocentric co-ordinates of a planet.

In the following investigations, rectangular axes are chosen with the centre of the sun as origin and their directions towards the first point of Aries, and the points whose longitudes and latitudes are respectively (90,0) and (0,90).

Let a be the distance of the earth from the sun, L its longitude and let b be the distance of the planet whose rectangular co-ordinates with respect to the sun are x, y, z and whose heliocentric longitude and latitude are λ and β.

Let λ', β' be the geocentric longitude and latitude of the planet and r its distance from the earth.

The co-ordinates of the planet and the earth are then related to one another as follows:—

$$b \cos \lambda \cos \beta = a \cos L + r \cos \lambda' \cos \beta' \quad \ldots\ldots(1)$$
$$b \sin \lambda \cos \beta = a \sin L + r \sin \lambda' \cos \beta' \quad \ldots\ldots(2)$$
$$b \sin \beta = r \sin \beta' \quad \ldots\ldots(3)$$

From (1) and (2), $b \cos\beta \cos(\mathrm{L}-\lambda) = a + r \cos\beta' \cos(\mathrm{L}-\lambda')$

$b \cos\beta \sin(\mathrm{L}-\lambda) = \quad r \cos\beta' \sin(\mathrm{L}-\lambda')$

$$\therefore \tan(\mathrm{L}-\lambda') = \frac{b \cos\beta \sin(\mathrm{L}-\lambda)}{b\cos\beta\cos(\mathrm{L}-\lambda)-a} \qquad \ldots\ldots\ldots\ldots(4)$$

(4) gives λ' the geocentric longitude of the planet, from its heliocentric co-ordinates, and the known values L, a and b.

Again from (1), (2) and (3), by squaring and adding we get $r^2 = a^2 - 2\,a\,b\cos(\mathrm{L}-\lambda)\cos\beta + b^2$ ....(5)

(5) gives the distance of the planet from the earth.

Also from (1) and (2),

$$r^2\cos^2\beta' = b^2\cos^2\beta - 2\,a\,b\cos(\mathrm{L}-\lambda)\cos\beta + a^2 \ldots(6)$$

Now from (3) and (6)

$$\tan\beta' = \frac{b\sin\beta}{[a^2 - 2\,ab\cos\beta\cos(\mathrm{L}-\lambda) + b^2\cos^2\beta]^{\frac{1}{2}}} \ldots\ldots\ldots(7)$$

(7) gives the geocentric latitude of the planet. From the geocentric co-ordinates, the heliocentric co-ordinates are obtained in the same manner.

188. Analytical investigation of the geocentric motion of a planet.

Let the earth and the planet move round the sun in coplanar circular orbits of radii a and b respectively and let their heliocentric longitudes be L and λ, and let the geocentric longitude and distance of the planet be λ' and r.

The co-ordinates of the planet and the earth are related to one another by the equations,

$$r\sin\lambda' = b\sin\lambda - a\sin\mathrm{L} \ldots\ldots\ldots\ldots(1)$$

$$r\cos\lambda' = b\cos\lambda - a\cos\mathrm{L} \ldots\ldots\ldots\ldots(2)$$

$$\therefore \quad \tan\lambda' = \frac{b\sin\lambda - a\sin\mathrm{L}}{b\cos\lambda - a\cos\mathrm{L}} \ldots\ldots\ldots\ldots(3)$$

(3) gives the geocentric longitude of the planet.

Also $r^2 = a^2 - 2\,a\,b\cos(\mathrm{L}-\lambda) + b^2$ ....(4)

Differentiating (1)

$$r \cos \lambda' \frac{d\lambda'}{dt} + \sin \lambda' \frac{dr}{dt} = b \cos \lambda \frac{d\lambda}{dt} - a \cos \mathrm{L} \frac{d\mathrm{L}}{dt}$$

$$\frac{\cos \lambda}{b^{\frac{1}{2}}} \quad \frac{\cos \mathrm{L}}{a^{\frac{1}{2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots(5),$$

since $\frac{d\lambda}{dt}$ and $\frac{d\mathrm{L}}{dt}$ are (by Kepler's law) proportional to $b^{-3/2}$ and $a^{-3/2}$ and the constant of proportion can be taken to be unity by a proper choice of the units of time and distance.

Similarly, differentiating (2),

$$-r \sin \lambda' \frac{d\lambda'}{dt} + \cos \lambda' \frac{dr}{dt} = -\frac{\sin \lambda}{b^{\frac{1}{2}}} + \frac{\sin \mathrm{L}}{a^{\frac{1}{2}}} \ldots(6).$$

Multiplying (5) by (1) and (6) by (2) and adding we get $r^2 \frac{d\lambda'}{dt} = a^{\frac{1}{2}} + b^{\frac{1}{2}} - \left(\frac{a}{b^{\frac{1}{2}}} + \frac{b}{a^{\frac{1}{2}}}\right) \cos(\mathrm{L} - \lambda) \ldots\ldots\ldots\ldots\ldots\ldots(7)$

Eliminating $(\mathrm{L}-\lambda)$ from (4) and (7), we get

$$r^2 \frac{d\lambda'}{dt} = a^{\frac{1}{2}} + b^{\frac{1}{2}} \quad \frac{a b^{-\frac{1}{2}} + b a^{-\frac{1}{2}}}{2ab} \cdot (a^2 + b^2 - r^2). \qquad (8)$$

(8) gives the angular velocity of the planet relative to the earth.

From (7) it is seen that the planet has no angular velocity relative to the earth, if $\cos(\mathrm{L}-\lambda) = \frac{a^{\frac{1}{2}} b^{\frac{1}{2}}}{a - a^{\frac{1}{2}} b^{\frac{1}{2}} + b}$. This gives two real values for $(\mathrm{L}-\lambda)$ corresponding to the positions when the planets are stationary as seen from one another.

189. Stationary Points.

Assuming the orbits of two planets A and B to be circular and in the same plane, we can find the condition that the planets may be stationary as seen from one another. Let the sun occupy the centre O of the two circles (see Fig. 109) and let v_1 and v_2 be the linear velocities of the

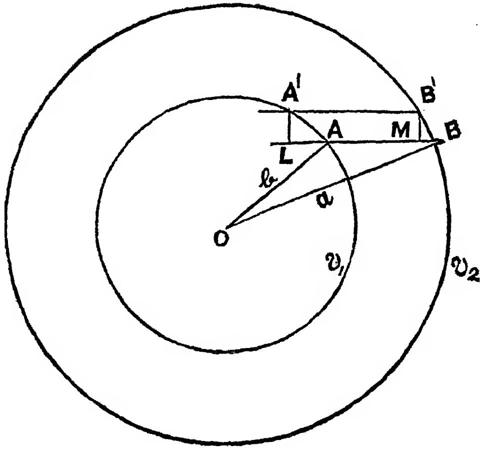


Fig. 109.

planets A and B in their orbits. If the planets are stationary at A and B, and if A′ and B′ are the positions of the planets a little later, A′ B′ should be parallel to A B, and therefore A′ L = B′ M, L and M being the feet of the perpendiculars on A B from A′ and B′. The arcs A A′ and B B′ are proportional to v_1 and v_2. Hence the condition for the two planets to be stationary is that v_1 cos O A L equals v_2 cos O B M.

or $v_1 \cos OAB + v_2 \cos OBA = 0$........................(1)

The angle subtended at the sun by the stationary points in the orbits of two planets.

Projecting O A on O B we get

$a = b \cos AOB + AB \cos ABO$

Projecting O B on O A we get

$b = a \cos AOB + AB \cos OAB.$

$$\frac{\cos ABO}{\cos OAB} = \frac{a - b \cos AOB}{b - a \cos AOB} = -\frac{v_1}{v_2} \text{ from (1)}$$

$$\text{or } \cos AOB = \frac{b\, v_1 + a\, v_2}{a\, v_1 + b\, v_2}$$

Also if T_1 and T_2 are the sidereal periods of the planets, using Kepler's law of planetary motion we get

$$\frac{b^{\frac{3}{2}}}{a^{\frac{3}{2}}} = \frac{T_1}{T_2} = \frac{2\pi b}{v_1} \Big/ \frac{2\pi a}{v_2} = \frac{v_2 b}{v_1 a} \text{ or } \frac{b^{\frac{1}{2}}}{a} = \frac{v_2}{v_1}$$

$$\cos AOB = \frac{a^{\frac{1}{2}} b + b^{\frac{1}{2}} a}{a^{\frac{3}{2}} + b^{\frac{3}{2}}} = \frac{\sqrt{ab}}{a - \sqrt{ab} + b}$$

The inferior planet is stationary once again after superior conjunction at a point A_1 such that $A_1OB = AOB$.

∴ The angle subtended at the sun by the two stationary points in the orbit of the inferior planet is given by

$$2 \cos^{-1} \frac{\sqrt{ab}}{a - \sqrt{ab} + b}$$

The motion of either planet during the interval in which the inferior planet describes $A_1 A$ is retrograde and therefore the period of time during which the motion of either planet is retrograde is given by

$\frac{S}{360} \times 2 \cos^{-1} \frac{\sqrt{ab}}{a - \sqrt{ab} + b}$, where S is the synodic period.

The time of direct motion is obviously

$$S - \frac{S}{180} \times \cos^{-1} \frac{\sqrt{ab}}{a + b - \sqrt{ab}}$$

190. Stationary Points when the planet moves outside the ecliptic plane.

Let the earth and the planet be moving in circular orbits that are not coplanar.

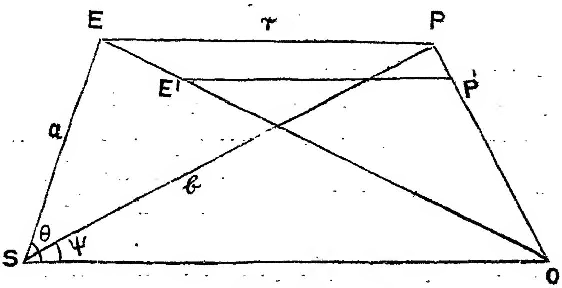


Fig. 110.

If a planet P is stationary when the earth is at E, and if E′ and P′ are the positions of the earth and the planet in their orbits at the next instant E P is parallel to to E′ P′ and therefore E E′ and P P′ are coplanar, and so intersect at some point O. Join S O where S is the position of the sun. Also E O and P O are $\perp^r$ to S E and S P respectively and if S E $= a$ and S P $= b$ we have $\frac{EO}{PO} = \frac{EE'}{PP'} = \frac{v_1}{v_2}$ where v_1 and v_2 are the linear orbital velocities of E and P. But $\frac{v_1}{v_2} = \frac{a^{-1/2}}{\quad^{-1/2}}$. (see Page 263)

$$\therefore \quad \frac{EO}{PO} = \frac{a^{-1/2}}{b^{-1/2}} \qquad \frac{EO}{a^{-1/2}} = \frac{PO}{b^{-1/2}} = K$$

$$\therefore \quad SO^2 = a^2 + \frac{K^2}{a} = b^2 + \frac{K^2}{b}.$$

$$\therefore \quad \frac{K^2\,(a-b)}{a\,b} = (a^2 - b^2)$$

$$\therefore \quad K^2 = a\,b\,(a+b)$$

$$\therefore \quad SO = \sqrt{a^2 + a\,b + b^2}$$

$$EO = \sqrt{b\,(a+b)} \text{ and } PO = \sqrt{a\,(a+b)}$$

Again if $\theta = ESO$ and $\psi = PSO$, we have these angles given by $\theta = \sec^{-1} \frac{\sqrt{a^2+b^2+a\,b}}{a}$ ....(1) and

$$\psi = \sec^{-1} \sqrt{a^2+b^2+a\,b} \qquad .(2)$$

If φ be the angle subtended by E P at S at the stationary instant, we have,

$\cos\varphi = \cos\theta \cos\psi + \sin\theta \sin\psi \cos i$, where i is the inclination of the planetary orbit to the ecliptic.

From (1) and (2), $\cos\varphi = \frac{a\,b + \sqrt{a\,b}\,(a+b)\cos i}{a^2 + a\,b + b^2}$

191. Phase of a planet.

By the term *phase of a planet* is meant the ratio of the visible part of the disc of the planet to the whole disc.

The phase of a planet can be obtained as in the case of the moon. (See article 149).

An inferior planet or the moon would show all phases from 0 to 1 whereas a superior planet has always more than half phase.

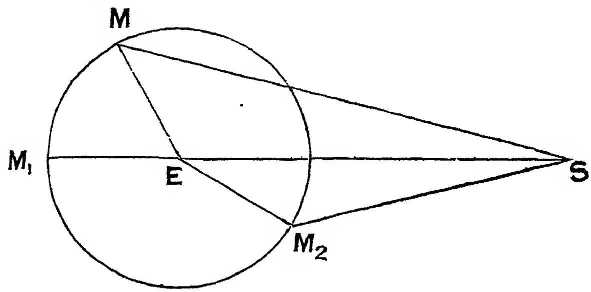


Fig. 111.

If in Fig. 111 M is the moon, the angle subtended by the line joining the earth and the sun at M, is EMS. For this position, the phase is $\frac{1}{2}(1+\cos M)$ which is more than half. When the moon is at M_1, $\cos M_1 = 1$ and so the phase is 1. When the moon is at M_2, the angle M_2 is obtuse and hence $\cos M_2$ is negative. Then $\frac{1+\cos M_2}{2}$ is less than $\frac{1}{2}$ and so the

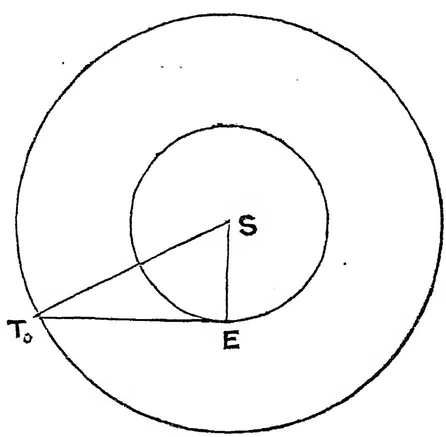


Fig. 112.

Dimensions of the sun and the planets and the orbital elements of the latter.

Name	Semi-diameter in miles.	No. to give mass of the sun	Density (water=1)	Semi-major axis of orbit in astronomical units.	Eccentricity.	Sidereal period in tropical years	Synodic period in days	Inclination of the orbit to the ecliptic.
								° ′ ″
Sun	432,000	1	1·42					
Mercury	1504	9,000,000	3·73	0·387099	0·2056	0·24085	115·88	7–0–12·4
Venus	3,788	403,490	5·21	0·723331	0·0068	0·61521	583·92	3–23–38·2
Earth	3963 (Eql. Dia) 3950 (Pl.... ...)	329,390	5·53	1·000000	0·0167	1·00004	... ...	0–0–0
Mars	2,108	3,093,500	3·95	1·523688	0·0933	1·88089	779·94	1–51–0·4
Jupiter	44,350 (Eql. Dia) 41,390 (Pl. ...)	1047	1·34	5·202803	0·0484	11·86223	398·88	1–18–25·2
Saturn	37,530 (Eql. Dia) 33,580 (Pl. ...)	3502	0·69	9·538843	0·0558	29·45772	378·09	2–29–27·9
Uranus	15,440	22,870	1·36	19·190978	0·0471	84·01529	369·66	0–46–22·2
Neptune	16,470	19,314	1·33	30·070672	0·0086	164·78829	367·49	1–46–34·6
Pluto	—	—	—	39·597000	0·2537	249·17	366·70	17–9–0·0

Dimensions of the satellites of the planets and their orbits.

Planet	Satellite	Mean distance in Astronomical units	Sidereal period in days	Eccentricity of orbit	Diameter in miles
Earth	1. Moon	0·002570	27·321661	0·0549	2163
Mars	1. Phobos	0·000063	0·318910	0·0170	—
	2. Deimos	0·000157	1·262440	0·0031	—
Jupiter	1. Io	0·002820	1·769138	Small and variable.	2109
	2. Europa	0·004486	3·551181	... ...	1865
	3. Ganymede	0·007156	7·154553	... ...	3273
	4. Callisto	0·012587	16·689018	... ...	3142
	5. —	0·001207	0·498179	... ...	—
	6. —	0·076605	250·620000	0·155	—
	7. —	0·078516	260·070000	0·207	—
	* 8. —	0·157200	738·900000	0·380	—
	* 9. —	0·158100	745·000000	0·248	14
Saturn	1. Mimas	0·001240	0·942422	0·019	370
	2. Enceladus	0·001591	1·370218	0·005	460
	3. Tethys	0·001969	1·887803	0·000	750
	4. Dione	0·002522	2·736916	0·002	900
	5. Rhea	0·003523	4·517503	0·001	1150
	6. Titan	0·008166	15·945452	0·029	3550
	7. Themis	0·009600	20·850000	—	—
	8. Hyperion	0·009893	21·276665	0·119	—
	9. Iapetus	0·023798	79·330820	0·029	—
	*10. Phoebe	0·086593	550·450000	0·166	—
Uranus	* 1. Ariel	0·001282	2·520383	—	—
	* 2. Umbriel	0·001786	4·144183	—	—
	* 3. Titania	0·002930	8·705876	—	—
	* 4. Oberon	0·003919	13·463262	—	—
Neptune	* 1. Triton	0·002364	5·876833	—	—

* Denotes retrograde motion.

CHAPTER XV.

TRANSIT OF AN INFERIOR PLANET.

193. Introduction.

Transit of an inferior planet like Venus or Mercury takes place at inferior conjunction when the planet goes across the sun's disc. The planet then appears as a black dot, and could be seen either by projection or in a photograph.

In olden days, before other methods such as the observations on Mars or on a minor planet were devised, transit observations of this kind were useful in determining the sun's distance. (See Chapter 12). The circumstances of a transit are different for observers situated in different places, as their relative directions are different. The ratio between the parallaxes of the sun and the planet is got either by knowing their periodic times $\left(\frac{T_1^2}{T_2^2}=\frac{a_1^3}{a_2^3}\right)$ or by observing the greatest elongation of the inferior planet. Also, by comparing the circumstances of the transit as seen from different places, the actual parallaxes of the sun and the planet could be determined as is described in Section 194.

Of the two transits possible, that of Venus is more important as it gives better results for the following reasons:—

(1) Mercury is so near the sun that its parallax is nearly the same as that of the sun and therefore the relative displacement of the planet as seen from two stations is small.

(2) It moves so rapidly across the sun's disc that there is very little time for accurate observation.

(3) The eccentricity of Mercury's orbit is so large that, from the observed displacement, the ratio of the

distances could not be accurately got, as the method is only applicable in the case of nearly circular orbits.

194. Solar parallax.

There are three methods by which the sun's distance could be derived from transit observations.

(1) Observe simultaneously from two stations the relative positions of the planet. This could be done either from a study of the photographs taken from the two stations or by measures of the position of the planet relative to the sun's disc.

Let P and p be the horizontal parallaxes of the sun and the planet. Now if the geocentric zenith distance of the planet be z, its displacement due to parallax would be $p \sin z$ away from the zenith.

For the same reason, the sun is displaced by $P \sin z$. Therefore the planet is brought nearer the sun's lower limb by $(p-P) \sin z$. Hence observe the position of the planet on the sun's disc from two stations at one of which the sun is vertical and at the other the sun only just rises, the corresponding zenith distances being $0°$ and $90°$.

$\therefore$ The relative displacement is 0 in the 1st place and $p-P$ in the 2nd place.

$\therefore$ $p-P$ is the angle between the two directions to the planet. If the two stations are at the opposite extremities of the earth's diameter we get $2(p-P)$ as the value. From this, $p-P$ could be found.

If r and r' be the distances of the earth and Venus from the sun $\frac{r'}{r}=T^{\frac{2}{3}}$ years, where the sidereal year is the unit. Also at the middle of the transit, the planet is in conjunction and hence its distance from the earth is $r-r'$.

$$\therefore \quad \frac{p}{P}=\frac{r}{r-r'} \text{ or } \frac{P}{p-P}=\frac{r-r'}{r'}=\frac{r}{r'}-1$$

$$=\frac{1}{T^{\frac{2}{3}}}-1$$

From this, we can find P the sun's horizontal parallax.

In the case of Venus $\frac{r'}{r}=\frac{7}{10}$. The displacement of Venus on the sun's disc at a place where the zenith distance is z, is $(p-\mathrm{P}) \sin z$ which is equal to $\frac{7}{3}$ P $\sin z$. The apparent position of Venus on the sun's disc may be observed either by measurement with a micrometer or by photography.

(2) *Delisle's Method.*

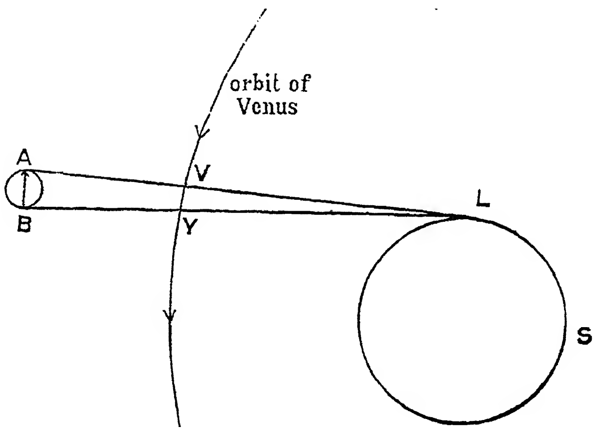


Fig. 114.

Here the difference between the times at which the transit begins or ends at two different places is observed. Let A and B be two diametrically opposite positions on the earth's equator and in the plane of Venus's orbit. When Venus reaches V, transit begins at A, when it reaches Y transit begins at B, and during the interval Venus moves through V Y or the angle V L Y $=2$ P. The rate of motion of Venus about the sun$=\frac{360}{\mathrm{S}}$, where S is the synodic period. 2 P or V L Y is got from the known interval. Similarly, observe the end of transit also from A and B and get another value for P. Even if A and B

are not diametrically opposite, the method can be applied after suitable modifications.

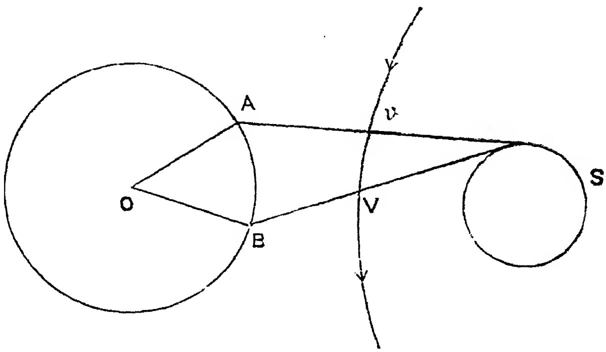


Fig. 115.

In Fig. 115, A S B is known from the synodic period and the interval between the times of beginning of transit as seen from A and B. O A and O B the radii are also known. Hence A S or B S can be found. The times of observation are Greenwich times so that the difference is taken from times reckoned from a common epoch. The longitudes of A and B should be known very accurately to compute the corresponding Greenwich times.

Example. The synodic period of Venus = 584 days. The difference of time observed from two stations A and B at the extremities of the earth's diameter = $11\frac{1}{2}$ minutes. Find solar parallax.

The relative angular motion of V in 1 min. $= \dfrac{360 \times 60 \times 60}{584 \times 24 \times 60}$
$= 1''.541.$

Therefore in $11\frac{1}{2}$ minutes Venus moves through $11\frac{1}{2} \times 1.541 = 17''.72$

$\therefore$ Parallax of the sun $= 8''.86$

(3) *Halley's method.*

The following method (due to Halley) was discovered in 1766 and was used for the transits of Venus in 1761 and 1769.

In this, the times of duration of transits at two stations A and B diametrically opposite are taken, A and B being such that one is in the north latitude, the other in the south latitude and A B being $\perp^r$ to the path of Venus (i. e. nearly $\perp^r$ to the ecliptic)

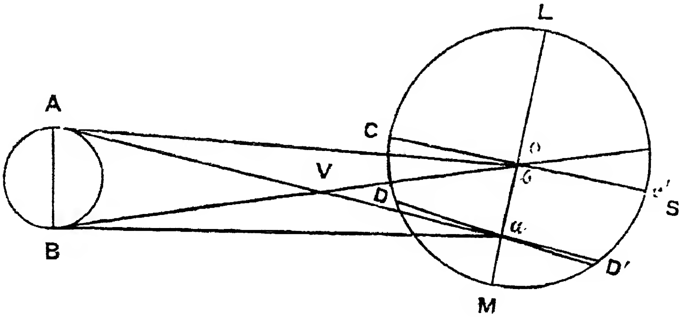


Fig. 116.

Let the plane of the paper be $\perp_r$ to the plane of the planet's motion and A B be on the plane of the paper and L M be the diameter of the sun's disc $\perp_r$ to the direction between the Earth and Venus. Now a and b are the relative positios of V seen at conjunction from the two stations. Now angular value of ab = V A b = A V B − A bB $= 2\,(p - \mathrm{P})$. The observed times are times taken by the planet to travel 2 a D and 2 b C.

If o be the centre of the sun's disc, $o\,b = \sqrt{\mathrm{R}^2 - b\mathrm{C}^2}$ and $o\ a = \sqrt{\mathrm{R}^2 - a\,\mathrm{D}^2}$ where R is the radius of the sun's disc.

The rate of motion of Venus across the sun's disc is found as follows:—

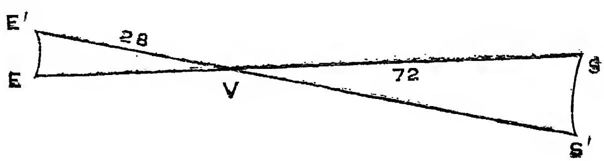


Fig. 117.

S V separates from S E by $1''{\cdot}54$ (from synodic period) per minute or $1'-32''{\cdot}4$ per hour.

The ratio of the distances of Venus from the earth and the sun is given by $\frac{28}{72}$

$$\therefore \quad \frac{\text{angular velocity of E V}}{\text{angular velocity of S V}}=\frac{1/\text{E V}}{1/\text{S V}}=\frac{\text{S V}}{\text{E V}}=\frac{72}{28}=\frac{18}{7}$$

$\therefore$ E V separates from E S with an angular velocity

$$\frac{18}{7}\times 1'-32''{\cdot}4 \text{ per hour.}$$

$=4'$ per hour or $4''$ per min. nearly.

Now from the known times of the transit, the lengths of the chords are known. Also the sun's angular diameter is known. Hence $o\,b$ and $o\,a$ are known and hence $b\,a$ which is $2\,(p-\text{P})$ is known.

$$\text{P}=\frac{7}{18}(p-\text{P}) \text{ giving the value of P.}$$

195. Difficulties of observing the duration of transit.

If the duration of transit could be observed correct to a second we can get the parallax correct to $0{\cdot}01''$. In practice it is difficult to observe the exact time of beginning and end of a transit due to the following reasons:—

(1) The planet Venus is not a point but a disc, and the first contact or the last contact could not be exactly observed since the planet is visible only after it has fully entered the area of the sun's disc. So the internal contacts alone are observed and correction has to be applied for the semidiameter of Venus to get the duration of transit.

(2) The second difficulty is the occurrence of 'black drop' i. e. the planet appears elongated or connected to the edge of the sun for some time after the second contact.

(3) The atmosphere of Venus is also to some extent the cause of uncertainty in fixing the exact time of contact.

196. Advantages and disadvantages of Halley's and Delisle's methods.

In Halleys' method only intervals of time are required and not exact times of contact. So the Greenwich times and the exact longitudes of the places need not be known.

In Delisle's method these have to be known accurately. In applying this method, stations could be chosen on the equator at opposite extremities of the diameter and thus one could get the greatest effect of parallax. But in Halley's method stations have to be chosen in north and south latitudes and two ideal stations for the purpose are the two poles. These however could not be chosen and hence the maximum effect of parallax cannot be used in this method.

197. Frequency of transits.

The orbit of Mercury is inclined to the ecliptic at about 7° and for the occurrence of a transit the planet should be very near either of its nodes at its inferior conjunction. The earth passes through these nodes some times near November 9 or May 7. Therefore transit of Mercury can occur in any year near November 9 or May 7. The corresponding dates for Venus are in December and in June. For the occurrence of a transit of mercury the conjunction has to take place when Mercury is within $2^\circ-10'$ of the node, taking its mean distance into consideration. Now the orbit of Mercury being elliptic, the distance of the planet from the earth is shorter in May than its mean distance and longer in November. This would make the limit for November getting larger and so the transits more frequently occur during November. From a study of the synodic period of Mercury, we find that 22 synodic periods are equal to 7 years nearly or more closely 41 periods correspond to 13 years or to a better approximation 145 periods are very nearly equal to

46 years. Hence transits of Mercury can be expected to recur at intervals of 7, 13 or 46 years. The May transit will recur only after 13 or 46 years, as its limit is very small.

The following dates give some of the future transits of mercury :—

1940	November	12
1953	...	13
1960	...	6
1973	...	9
1986	...	12
1999	...	14
1957	May	5
1970	...	9

The inclination of the orbit of Venus to the ecliptic is about $3\frac{1}{2}°$ and the transit limit for Venus is $1° - 42'$ on either side of the node. Now 5 synodic periods of Venus cover nearly 8 years or to a better approximation 152 synodic periods cover 243 years. The transits of Venus can be expected then to occur at the same node once in 8 years or 243 years; but the first period of recurrence does not happen generally more than once, as the relative positions of the earth and planet during the second period change very much from their initial values. The position of the planet relative to a node will be also nearly the same at intervals of $105\frac{1}{2}$ and $121\frac{1}{2}$ years. The following are the dates of the transits of Venus showing the various periods :—

	1761	June	5th	
				243 years
	2004	June	7th	
				8 years
	2012	June	5th	
$121\frac{1}{2}$ years (2004–2125)				$105\frac{1}{2}$ years
	2117	December	10th	
				8 years
	2125	December	8th	

CHAPTER XVI

ECLIPSES

198. Introduction.

The word eclipse means a swoon, and astronomically it is applied to the darkening of a celestial body like the sun by the interposition of a body between the earth and the body as in the case of the solar eclipse, or to the passage of a body into the shadow cast by another body as in the case of a lunar eclipse when the moon passes through the shadow cast by the earth. If at a full moon, the centres of the sun, earth, and moon are nearly in a straight line, the moon's disc is either wholly or partially inside the earth's shadow and therefore we get a lunar eclipse at such oppositions when there is very little difference between the sun's and moon's latitudes. Similarly if at a conjunction or a new moon, the sun and moon have very little difference in latitude, the moon cuts the rays from the sun either wholly or partially and we thus get a total or partial solar eclipse.

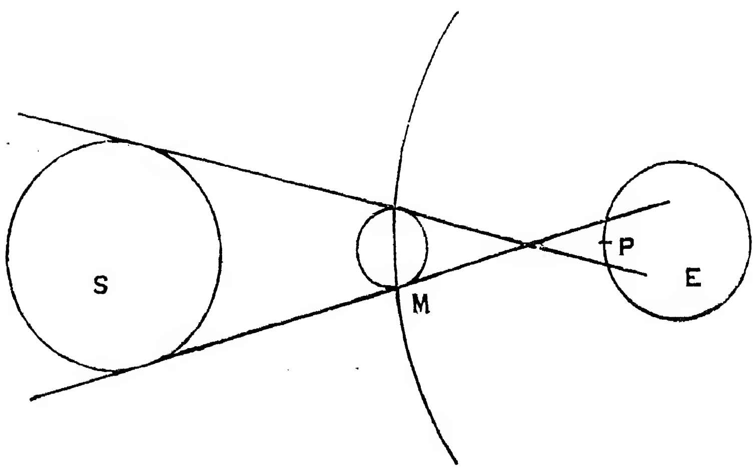
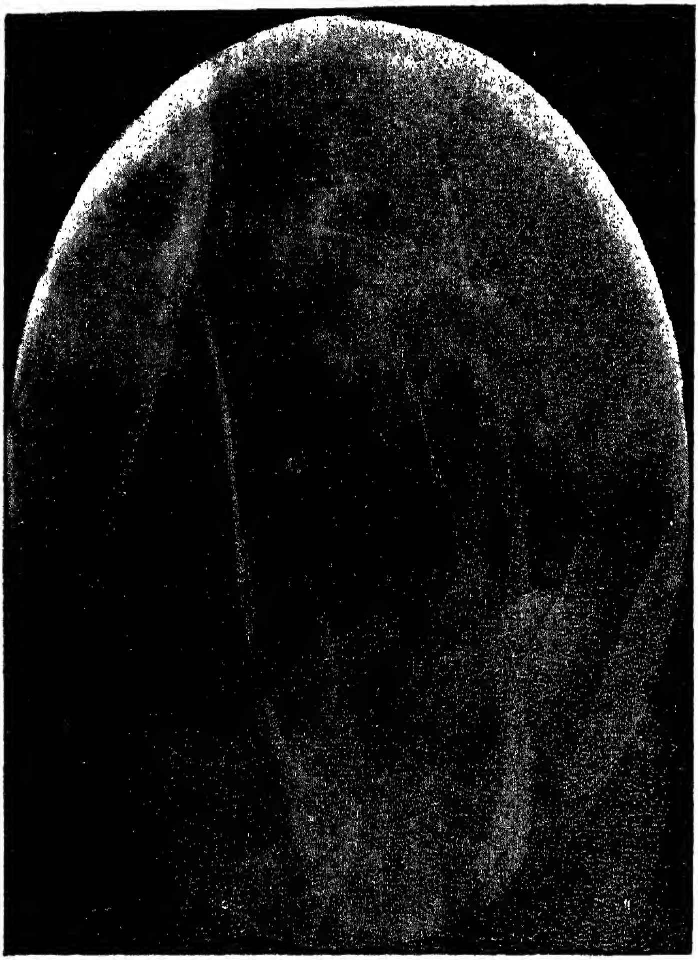


Fig. 118.



CURRENT SCIENCE.

Plate No. 2.

A photograph of the eclipsed moon taken at the Trivandrum Observatory at 10 P. M. on 8th January 1936. The eclipse commenced at 9-36 P. M. In the photograph, only less than half the surface of the moon is visible.

The condition for the occurrence of a total solar eclipse visible from some place on the earth is that the common tangent cone between the sun and the moon, should contain that place within it. If however the common tangents meet before they reach the earth as in the fig. 118, then an observer at a point P will be able to see only the outer edge of the sun as a bright ring but not the central part which is obstructed from his view by the moon. Such a phenomenon is called an *annular eclipse*. Now during an annular eclipse the moon is able to eclipse only the central portion of the sun's disc; for the angular diameter of the moon is less than that of the sun as seen from P. During a total eclise of the sun, the moon subtends at the observer's eye an angular diameter greater than that of the sun. Owing to the moon's path round the earth being elliptic, its angular diameter varies from $28'$-$46''$ to $33'$-$32''$ and owing to the apparent orbit of the sun round the earth also being elliptic, the angular diameter of the sun is also subject to a variation from $31'$-$32''$ to $32'$-$36''$. Thus the nature of the solar eclipse at some place, (i. e. whether it is total or annular) is determined by the relative distances of the earth, the moon and the sun at the time of conjunction. If the moon's orbit had been in the plane of the ecliptic, we should have a lunar eclipse at every opposition and a solar eclipse at every conjunction; for on either of these occasions, the centres of the sun, the moon and the earth would be in the same straight line. From a knowledge of the sizes of the earth and the sun and their distances apart, we can get the length of the earth's shadow, and its mean length is found to be 857200 miles. The mean radius of the moon's orbit is about 240,000 miles, so that the moon must pass through the shadow (and not beyond it) at every opposition if it moved in the ecliptic plane.

The fact that the lunar orbit is inclined to the ecliptic at an angle of about $5^\circ - 8'$ makes it impossible for the

occurrence of an eclipse at every conjunction or opposition unless the difference of their latitudes is very little on such an occasion. Therefore an eclipse can occur when the moon is at syzygy, only if it is very near the ecliptic and therefore very near the line of nodes. If near conjunction the shortest distance between the centre of moon's disc and the centre of sun's disc be always greater than the sum of their apparent angular radii, then there would be no solar eclipse at that conjunction (Fig. 121). Similarly near an opposition there would not be a lunar eclipse if the moon's shortest distance from the axis of the earth's shadow be greater than the sum of the moon's angular radius and the radius of the section of the earth's shadow at the lunar distance. (Fig. 120)

199. Expression for the angle between the common tangent and the line of centres for two circles.

The condition for the occurrence of an eclipse can be obtained from the following expression for the angle between the line of centres and the common tangent to two circles.

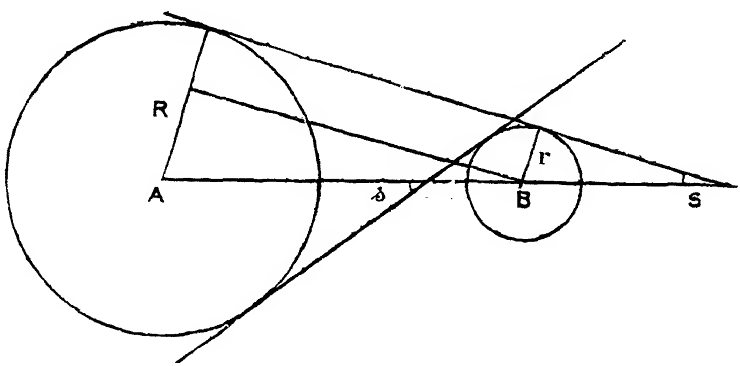


Fig. 119.

Let S be the angle between the direct common tangent to two circles of radii R and r and their line of centres and let s be the angle between the line of centres

and the inverse common tangent. If D be the distance between the two circles, then

$$\sin S = \frac{R - r}{D} = \frac{R}{D} - \frac{r}{D}$$

and $$\sin s = \frac{R + r}{D} = \frac{R}{D} + \frac{r}{D}$$

200. Condition for the occurrence of a lunar eclipse.

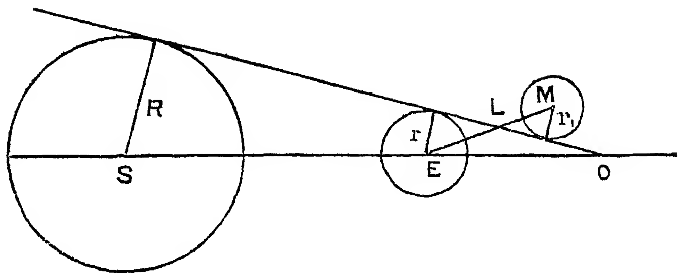


Fig. 120.

A lunar eclipse can occur at an opposition only if the angular distance between the moon's centre and the centre of the earth's shadow as seen from the earth's centre be less than angle M E O at any time during the opposition.

Now, $M\hat{E}O = M\hat{L}O - L\hat{O}E$

$= \dfrac{r_1 + r}{d} - \dfrac{R - r}{D}$ where R, r, r_1 are the radii of the sun, the earth, and the moon and D, d, the distances between the earth and the sun and the earth and the moon respectively.

But $\dfrac{r}{d}$ is the moon's horizontal parallax (p)

$\dfrac{r}{D}$ is the sun's horizontal parallax (P)

$\dfrac{r_1}{d}$ is the angular radius of the moon as seen

from the centre of the earth (m).

and $\frac{R}{D}$ is the angular radius of the sun as seen from the centre of the earth (s)

Hence at the time of first contact of a lunar eclipse, the angular distance of the moon from the axis of the shadow cone is given by $p+P+m-s$. The total eclipse commences when the moon is just completely immersed in the shadow cone and that occurs when the angular distance is $(p+P+m-s)-2m$ i. e. $p+P-m-s$.

Owing to the absorption caused by the earth's atmosphere, the angular radius of the section of the shadow cone at the moon's distance is increased by 2%. Therefore the angular radius of the umbral section is taken as $(p+P-s)\frac{51}{50}$ and that of the penumbral section is taken as $(p+P+s)\frac{51}{50}$

201. Condition for a solar eclipse.

In the same manner, the angular distance of the moon's centre from the line SE, when a solar eclipse commences, is got as follows:—

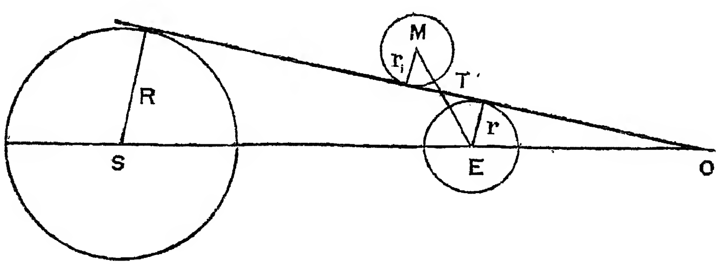


Fig. 121.

$$S\hat{E}M = \hat{O} + E\hat{T}O = \frac{R-r}{D} + \frac{r+r_1}{d} = s-P+p+m.$$

The totality of a solar eclipse commences when the angular distance becomes $(s-P+p+m)-2m$
$=s-P-m+p$.

Taking the mean values of s, m, p and P to be 16′, 15′, 57′ and 8″ respectively, it is possible to get the geocentric angular distance of the moon's centre at the commencement of any kind of eclipse.

202. Approximate method of determining ecliptic limits.

It is found by substitution in the previous results that a lunar eclipse cannot occur at an opposition unless the moon's latitude be then less than about 56′ and that no solar eclipse could occur at a conjunction unless the moon's latitude be then less than 88′. Since the moon's latitude increases with its distance from the line of nodes, the limiting distances from the nodes could be known for the occurrence of an eclipse. These limits are known as the *ecliptic limits.* They are usually measured along the ecliptic.

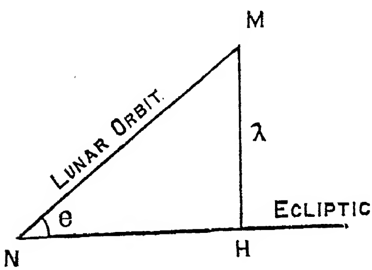


Fig. 122.

Let N be one of the nodes and let M denote the position of the moon in its orbit, inclined at angle θ ($5°-8'{\cdot}32$) to the ecliptic at the time when an eclipse is just possible. If λ be the moon's latitude, we have

$\theta . \sin NH = \lambda$. (from spherical geometry) giving the value of N H approximately when θ and λ are known.

An alternate method is to treat the triangle N M H as a plane triangle, and then $NH = \lambda \cot \theta$.

The lunar and solar ecliptic limits are found to be nearly 11° and 17° respectively taking the geocentric latitudes of the centre of the moon's disc to be 56′ and 88′ respectively at the beginning of these eclipses.

203. Major and minor ecliptic limits.

The distances of the sun and the moon from the earth are varying, and so also are their parallaxes; and therefore the geocentric latitude of the moon at the time of an eclipse is a varying quantity. Moreover the inclination of the lunar orbit to the ecliptic is also subject to a change from 4°-57′ to 5°-20′. These render the ecliptic limits variable, and the least and the greatest values of these limits are called the *Minor* (inferior) and the *Major* (superior) ecliptic limits.

For a lunar eclipse at present, the major and minor ecliptic limits are about 12°-5′ and 9°-30′ respectively. This would mean that a lunar eclipse must take place if the moon's longitude does not differ from that of the node by 9°-30′. These and the corresponding limits for a solar eclipse are derived subsequently,

204. Lunar ecliptic limits determined more accurately.

The lunar ecliptic limits could be determined with a greater degree of accuracy as follows :—

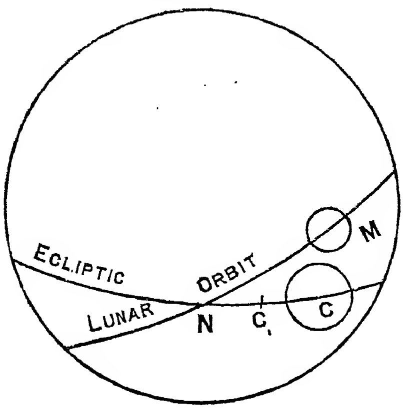


Fig. 123.

Let M be the position of the moon, and C the centre of the earth's shadow at time t after the moon's passage of the node.

Let C_1 be the position of C when the moon was at N.

Let $N C_1 = x$ and $M C = y$ and let t be the time taken by the moon to go from N to M. The longitude of C is that of the sun increased by 180°.

Let θ be the sun's hourly rate of change of longitude and φ the moon's angular velocity in its orbit, both being in radians. Also, let i be the inclination of the lunar orbit to the ecliptic.

Then $$CM^2 = y^2 = (x + t\,\theta)^2 + \varphi^2 t^2 - 2\,\varphi\, t\,(x + \theta\, t) \cos i$$

$$= x^2 + 2\,x\,t\,(\theta - \varphi \cos i) + t^2\,(\theta^2 + \varphi^2 - 2\,\theta\varphi \cos i)$$

Putting coefft. of $t^2 = K^2$.

$$CM^2 = \left(\frac{x^2}{K^2} + \frac{2\,x\,t}{K^2}\,(\theta - \varphi \cos i) + t^2\right) K^2$$

$$= K^2\left(t + x\,\frac{(\theta - \varphi \cos i)}{K^2}\right)^2 + x^2 - \frac{x^2}{K^2}\,(\varphi \cos i - \theta)^2$$

$$= K^2\left(t + x\frac{(\theta - \varphi \cos i)}{K^2}\right)^2 + x^2\left(1 - \frac{\varphi^2 \cos^2 i + \theta^2 - 2\,\theta\,\varphi \cos i}{K^2}\right)$$

$$= K^2\left(t + x\,\frac{(\theta - \varphi \cos i)}{K^2}\right)^2 + \frac{x^2}{K^2}\Big(\theta^2 + \varphi^2 - 2\,\theta\,\varphi \cos i$$

$$- (\theta^2 + \varphi^2 \cos^2 i - 2\,\theta\,\varphi \cos i)\Big)$$

$$= K^2\left(t + x\,\frac{(\theta - \varphi \cos i)}{K^2}\right)^2 + \frac{x^2}{K^2}\,\varphi^2 \sin^2 i,$$

It is easily seen that the minimum value for y is given by

$$y_0 = \frac{x\,\varphi \sin i}{(\theta^2 + \varphi^2 - 2\,\theta\,\varphi \cos i)^{\frac{1}{2}}}$$ when the first term vanishes.

If $\frac{\theta}{\varphi}$ be l we get from the above

$$x = y_0\,(1 - 2\,l \cos i + l^2)^{\frac{1}{2}} \operatorname{cosec} i.$$

Now l is the ratio of the moon's sidereal period to the year and its mean value is about $\frac{3}{40}$ and $i = 5°\text{·}12'$. Hence

$x = 10\text{·}3\, y_0,$

The angular distance of the centre of the moon from the earth at the commencement of a lunar eclipse is given by $\frac{51}{50}(p+P-s)+m.$ (See article 200)

Now for a lunar eclipse to occur near a node we should have $y_0 < \frac{51}{50}(p+P-s)+m.$

or $x < 10{\cdot}3\left(\frac{51}{50}(p+P-s)+m.\right)$

Taking $P=9''$, $p=3422''$, $s=960''$ and $m=935''$.
we get $x < 9^\circ{\cdot}9$.

For a total lunar eclipse the corresponding value obtained is $x < 10{\cdot}3\left(\frac{51}{50}(p+P-s)-m.\right)$

i. e. $x < 4^\circ{\cdot}6$.

These are the lunar ecliptic limits.

205. Accurate determination of solar ecliptic limits.

The solar ecliptic limits are obtained analytically by the following method :—

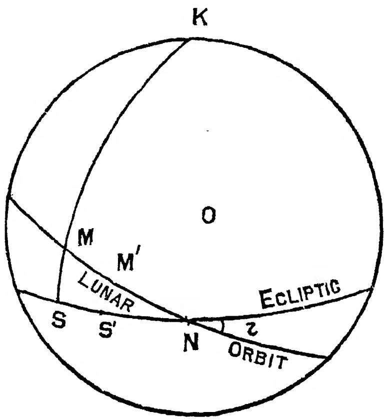


Fig. 124.

Let the conjunction take place when the moon is at M with β as latitude, and the sun (S) is near the node N

and let i be the inclination of the lunar orbit to the ecliptic.

Let M′, S′ be the positions of the moon and the sun at any other time. Taking M S N as a plane triangle the longitude of the moon has increased now by $MM' \cos i$ and that of sun by SS'. If m be the ratio of these quantities and if $MM' = x$, $SS' = m\,x \cos i$.

Now $SN = \beta \cot i$ and $MN = \beta \operatorname{cosec} i$.

$\therefore \quad S'N = \beta \cot i - m\,x \cos i$.

and $M'N = \beta \operatorname{cosec} i - x$.

If $M'S' = y$, we have from triangle M′ S′ N

$$y^2 = (\beta \cot i - m\,x \cos i)^2 + (\beta \operatorname{cosec} i - x)^2 - 2(\beta \cot i - m\,x \cos i)(\beta \operatorname{cosec} i - x) \cos i.$$

This is a quadratic in x, and making the terms containing x a perfect square we get

$$y^2 = (1 - 2m \cos^2 i + m^2 \cos^2 i)\left(x - \frac{\beta \sin i}{1 - 2m \cos^2 i + m^2 \cos^2 i}\right)^2 + \frac{\beta^2 (1-m)^2 \cos^2 i}{1 - 2m \cos^2 i + m^2 \cos^2 i}$$

From the above it is clear that the minimum value of y is given by

$$y_0 = \frac{\beta (1-m) \cos i}{(1 - 2m \cos^2 i + m^2 \cos^2 i)^{\frac{1}{2}}}$$

$$= \frac{\beta (1-m) \cos i}{(\sin^2 i + (1-m)^2 \cos^2 i)^{\frac{1}{2}}}$$

$$= \frac{\beta}{\left(\dfrac{\sin^2 i}{(1-m)^2 \cos^2 i} + 1\right)^{\frac{1}{2}}} = \beta \cos g \text{ where } \tan g = \frac{\tan i}{1-m}$$

Now since i is nearly $5° - 12'$, $\tan i$ is $\frac{1}{11}$ and $m = \frac{3}{40}$

$$\text{i. e. } \cos g = \frac{(1-m) \cos i}{(1 - 2m \cos^2 i + m^2 \cos^2 i)^{\frac{1}{2}}}$$

$$= \cos i - 0{\cdot}0006 \cos i$$

$\therefore$ g is very nearly equal to i.

$\therefore$ $y_0=\beta\cos g=\beta\cos i=\beta\left(1-2\sin^2\frac{i}{2}\right)$

Now at the beginning of a partial eclipse of the sun, the angular distance of the moon from the radius vector to the sun is $s-\mathrm{P}+p+m$. (See article 201). Therefore for the occurrence of a solar eclipse y_0 or $\beta\left(1-2\sin^2\frac{i}{2}\right)$ should be less than $s-\mathrm{P}+p+m$.

i. e. $\beta<\dfrac{(s-\mathrm{P}+p+m)}{1-2\sin^2\frac{i}{2}}<(s-\mathrm{P}+p+m)$

$$+2\sin^2\frac{i}{2}(s-\mathrm{P}+p+m)<(s+p+m-\mathrm{P})+0'\cdot4.$$

Now the mean value of $(s-\mathrm{P}+p+m)$ is $1^\circ-28'.6$.

Also taking for s, p and m their maximum values $16'\cdot3$, $61'\cdot5$ and $16'\cdot8$ and for P the least possible value $0'\cdot1$ generally we find that a solar eclipse is impossible if the moon's latitude is greater than $1^\circ-34'\cdot9$ at the time of a conjunction. Taking the least values of s, p, and m as $15'\cdot8$, $53'\cdot9$, and $14'\cdot7$, and assuming for P the greatest value $9''$, a solar eclipse must generally happen if the geocentric latitude of the moon at the time of conjunction be less than $1^\circ-24'\cdot7$. The distance of the sun from the node at the time of new moon is given by $\tan i=\dfrac{\tan\beta}{\tan(\mathrm{S\,N})}$

Now this distance SN is greatest when β is maximum and i is minimum (4°-$58'\cdot8$) and then SN is found to be $18^\circ\cdot5$. This is called the superior ecliptic limit. Thus an eclipse is impossible if new moon occurs when the sun is beyond this distance from a node.

The least value of SN is called the inferior ecliptic limit, and it corresponds to the least value of β 1°-$24'.7$ already calculated and the greatest value of i which is $5^\circ-18'\cdot6$. From these the inferior ecliptic limit is found

to be $15^\circ.4$. If the sun at the time of the new moon be within $15^\circ.4$ of a node, a solar eclipse must take place at that conjunction.

If conjunction takes place when the sun is between the inferior and superior ecliptic limits, an eclipse may or may not happen. On such an occasion the actual calculation of the angular distance of the moon from the sun alone can settle the question.

The two ecliptic limits for the occurrence of a total solar eclipse can also be calculated in a similar way from a knowledge of the value of the moon's angular distance from the sun at the beginning of totality.

206. Synodic period of the moon's nodes.

The moon's node has a daily retrograde motion of about $3'-11''$ on the ecliptic for reasons similar to the precessional movement of the first point of Aries, and the sun's mean daily motion is about $59'-8''$. Hence the sun's daily motion relative to the node is $62'-19''$, giving the period of sun's revolution relative to the node as $\frac{360 \times 60}{62\frac{19}{60}} = 346{\cdot}6$ days. Therefore the sun's position on the ecliptic relative to the node is always the same as it was $346{\cdot}6$ days before.

207. Number of eclipses when the sun is near a node.

We shall next see how many eclipses are likely to occur when the sun passes through a node of the lunar orbit. The interval between a full moon and a new moon is about $14\frac{3}{4}$ days; and during this time the sun moves through $15^\circ-18'$ relative to the node. Therefore if it is full moon when the sun is at the node, it should be within the solar ecliptic limits (18°-$30'$) during the previous and succeeding new moons and therefore there may occur a lunar and two solar eclipses during this interval. But if it is a new

moon at the time when the sun passes through a node the moon is beyond the lunar ecliptic limits ($12^\circ-5'$) at the proceeding and succeeding full moons and therefore there would be only one solar eclipse during this interval. Even if the full moon or the new-moon occurs within a degree or two on either side of the node, the same result will happen. As the distance between the two minor solar ecliptic limits near a node is $30^\circ-42'$, at least a single new moon must take place somewhere within this range, as the sun can move through only $30^\circ-36'$ relative to the node between two new moons, and since this new moon occurs within the minor solar ecliptic limits it should be a solar eclipse. Thus there should be at least one solar eclipse with or without any lunar eclipse or two solar eclipses and a lunar eclipse when the sun is near a node.

208. Maximum and minimum number of eclipses in a year.

If a lunar eclipse occurs when the sun is exactly at a node, the lunar eclipse near the other node will occur four days after the sun's passage across that node; for, six lunations take place in 177 days, while the sun comes to the second node 173 days after. Now at the second node it is impossible to have 3 eclipses which is the maximum at a node. If however the first lunar eclipse had happened two days before the sun arrived at that node, it is possible to have 3 eclipses at each of these nodes. For the solar ecliptic limit is 18°-$30'$ distant from the node and the sun is more than 16° from these limits and in half a lunation it could move only 15°-$18'$. Now six days after the second passage of the sun through the first node a lunar eclipse again occurs, since 12 lunations take 354 days and the synodic period is 346·6 days. Now this lunar eclipse is not followed by a solar eclipse at the next new moon, as the sun is then far beyond the solar ecliptic limit, but the previous new moon should be a solar eclipse. Hence at this node only two eclipses occur; and counting all the

eclipses at the first and second nodes and these two, we get 8 eclipses in $12\frac{1}{2}$ lunations; but $12\frac{1}{2}$ lunations take more than a year ($368\frac{3}{4}$ days). Hence if we take all the eclipses that could possibly occur in a year, either the first solar eclipse or the last lunar eclipse should be dropped, in which case we can have as a maximum only seven eclipses in a year, provided the first happens early in January; five of them may be solar and two lunar or four of them solar and three lunar.

It has been seen that the sun's passage at either of the two nodes may cause only a solar eclipse each with no lunar eclipse, if the two full moons at these nodes occurred $12° - 5'$ beyond them. It is also possible that the second passage of the sun at the first node may not be preceded by a full moon occurring inside the lunar minor ecliptic limit. Again, these 12 lunations giving 3 solar eclipses and covering a period of 354 days may not be included in one year, if the first solar eclipse occurred after the 11th January in any year. Thus the minimum number of eclipses possible in a year is two solar eclipses.

Though the absolute minimum is two solar eclipses there are many years when this number is not obtained for the simple reason that solar eclipses are phenomena for a narrow strip of the earth's surface and unless this portion of the surface is accessible to people, the eclipses will go unnoticed. In this way many eclipses go unobserved,

209. The Saros.

In ancient times, when the study of the sun's and the moon's motions were imperfect, people in China, India, Egypt and other places had a simple method of predicting the occurrence of the eclipses. This was from a knowledge of a period containing an exact number of lunations and also an exact number of synodic periods of revolution of the nodes,

One such period associated with the Chaldeans is a period of 6585 days (18 years and 11 days, or 18 years and 10 days according as the number of leap years is 4 or 5) comprising 223 lunations (6585·32 days) or 19 synodic revolutions of the nodes (6585·78 days). This enables one to give the eclipses that would occur in a Saros, if the corresponding eclipses of the previous Saros had been recorded.

Again the apse line of the lunar orbit performs a complete revolution with respect to the sun in 411·74 days, and 16 such synodic periods of the moon's line of apsides cover 6587·87 days which is only 2 days in excess of the Saros. The position of the moon with respect to the apse line determines its distance from the earth and therefore the nature of the eclipse. From the above it is clear that the moon's position is nearly the same with reference to the apse line after one Saros. Hence two solar eclipses separated by one Saros may be of the same nature (partial, total, or annular). A longer period enabling us to predict eclipses is one of 14558 days, as this covers 42 synodic revolutions of the nodes, and 493 lunations.

210. Geometrical method of determining the time, duration, and magnitude of a lunar eclipse.

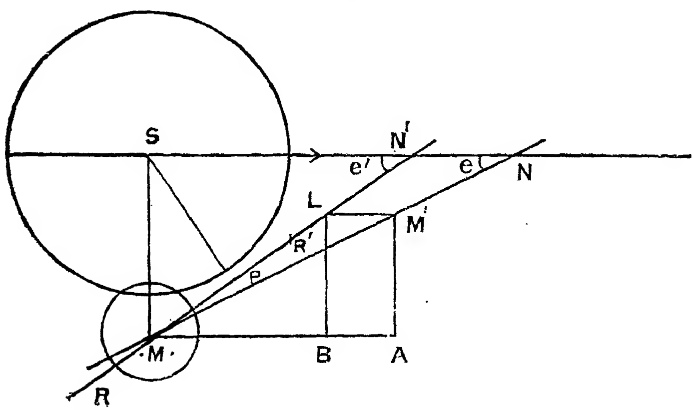


Fig. 125.

Let M N be the path of the moon, S N the path of the centre of the shadow cast by the earth, and N the node. (fig. 125). Let S be the centre of the earth's shadow, and the circle whose centre is S be the section of the shadow cone at the place where the moon enters the same; let S M or l be the moon's latitude at opposition, and c the sum of the radii of the two circles representing the moon and the shadow. Let m, p, and s be respectively the hourly changes in longitude and latitude of the moon and the change in longitude of the sun and be represented by M A, A M′ and A B in the figure. Then it is easily seen that M L N′ is the relative path of the moon, the earth's shadow being supposed to be stationary.

Draw S P $\perp_r$ to M N′ and a circle with centre S and radius c cutting M N′ at R, R′. Then R and R′ would represent the positions of the moon at the beginning and end of an eclipse, and P would be the point of nearest approach on the relative orbit. We can get the corresponding positions of the moon on the actual lunar orbit by drawing lines parallel to M A through these points and allowing them to cut the lunar path.

Let θ be the inclination of the lunar orbit to the ecliptic and θ' the angle between the ecliptic and the relative orbit.

Then $\cot \theta = \dfrac{m}{p}$, $\cot \theta' \; \dfrac{m-s}{p}$, and $SP = l \cos \theta'$

If $l \cos \theta'$ be equal to or greater than c, at any opposition, there will be no lunar eclipse during that opposition.

When the moon's disc just grazes the earth's shadow at the time of nearest approach we have $l \cos \theta' = c$, and the corresponding value of N S is given by $l \cot \theta$ or $c \sec \theta' \cot \theta$. Now the greatest and least values of this quantity are called the superior and inferior ecliptic limits.

If $l \cos \theta'$ be less than c, a partial or total eclipse will take place.

When the moon's centre is at P, $(c - l \cos \theta')$ would represent the breadth of the eclipsed moon. If this is less than the moon's diameter, the eclipse is partial; if this is equal to or greater than the moon's diameter, the eclipse would be total.

At time t after opposition the distance between the centres of the shadow and the moon's disc is given by $[(l-pt)^2+(m-s)^2 t^2]^{\frac{1}{2}}$. If this is equated to c, the values of t obtained from this quadratic equation are the times of the beginning and end of the eclipse, commencing from the instant of opposition.

In this equation, if the values of t be imaginary, or equal, there will be no eclipse.

If the expression for the distance between the centres be equated to the difference between the radii of the shadow and the moon, the two solutions of t would give the beginning and end of totality. If the values of t in this equation be imaginary, the eclipse will only be partial.

In either case, the middle of the eclipse is given by half the sum of the two roots of the equation viz $\dfrac{pl}{(m-s)^2+p^2}$

211. Analytical method of calculation of a lunar eclipse.

Let x be the increase in the hourly rate of change of R. A of moon over that of the centre of the earth's shadow, expressed as fraction of an hour at the moment of opposition in R. A. Let δ, δ' be the declinations of the moon and the centre of the earth's shadow and let y and y' be their hourly rates of change,

Now, in time t after opposition, the distance between the moon and the centre of the earth's shadow is given by $[(\delta+ty)-(\delta'+ty')]^2 + 54000^2 x^2 t^2 \cos^2 \frac{1}{2}(\delta+\delta')$ since this quantity is small and since one hour of R. A is equal to $54000''$.

If d be the distance between the centres, we have

$$d^2 = At^2 + 2Bt + C$$

where $A=(y-y')^2 + 54000^2 x^2 \cos^2 \dfrac{\delta+\delta'}{2}$

$B=(y-y')(\delta-\delta')$ and $C=(\delta-\delta')^2$.

From the above equation we get the beginning and end of an eclipse by putting for d, $d_1=(p+P-s)\dfrac{51}{50}+m$, and the beginning and end of totality by putting for d, $d_2=(p+P-s)\dfrac{51}{50}-m$.

Putting $d=d_1$ in the above equation, we find that there will be a lunar eclipse if $At^2+2Bt+c-d_1^2=0$...(1) has real roots. The two roots giving the beginning and end of the eclipse are $-\dfrac{B}{A}\mp\dfrac{(B^2-AC+Ad_1^2)^{\frac{1}{2}}}{A}$.

If the roots are to be real $B^2-AC+Ad_1^2$ should be positive i. e. $d_1 > \left(\dfrac{AC-B^2}{A}\right)^{\frac{1}{2}}$ ....(2)

This is then the condition for the occurrence of a partial eclipse.

The difference between the roots of the equation (1) gives the duration of the eclipse.

i. e. $t_2-t_1=2\dfrac{(B^2-AC+Ad_1^2)^{\frac{1}{2}}}{A}$ ....(3)

If we put d_2 for d_1 in equation (1) we get in the same manner the condition for the occurrence of a total eclipse and also the duration of totality.

At the middle of the eclipse the centres are nearest and so $At^2 + 2Bt + C$ has then the minimum value $\left(\frac{AC-B^2}{A}\right)^{\frac{1}{2}}$, and this occurs when $t = -\frac{B}{A}$ measured from the time of opposition in right ascension.

The magnitude of a partial eclipse is given by that fraction of the eclipsed diameter of the moon's disc pointing to the centre of the shadow when the distance between the centres is least. The radius of the umbral shadow is $(p+P-s)\frac{51}{50}$, and the moon's angular radius is m.

Hence the diameter of the moon eclipsed is given by $(p+P-s)\frac{51}{50}+m-\left(\frac{AC-B^2}{A}\right)^{\frac{1}{2}}$, and this expression divided by $2m$ gives the magnitude of the eclipse.

212. Point of the lunar disc where the eclipse begins.

Let M and C be the centres of the moon and earth's shadow at the beginning of the eclipse and P be the celestial pole. Let PM cut the moon's disc at N, the most northerly point of the moon's disc. The angle T where the eclipse commences is specified by the angle NMT measured anticlockwise from NM. Draw CD perpendicular to PM; then $MD = (\delta - \delta')$ nearly.

$$\therefore \cos NMT = \frac{\delta' - \delta}{CM} \text{ where } CM \text{ is } (p+P-s)\frac{51}{50} + m.$$

The position of the point where the eclipse ends can be similarly calculated.

213. Bessel's method of computing a solar eclipse.

The method of computing the circumstances of a solar eclipse given here is due to Bessel.

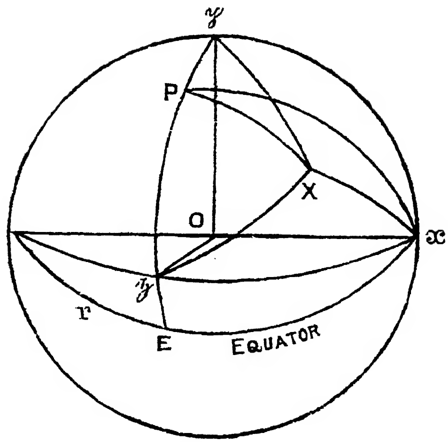


Fig. 126.

(1) *The Besselian elements.*

At any time, a line is supposed to be drawn from the centre of the earth parallel to the line joining the centres of the sun and the moon. Let this line meet the celestial sphere at z. This line is taken to be the z axis and the plane through the centre of the earth perpendicular to this axis is taken to be the $x\,y$ plane or the fundamental plane, the positive side of which is that portion on which the sun and the moon lie. If P be the north celestial pole, the plane through zP cuts the $x\,y$ plane along the y axis. Ox, the x axis is perpendicular to the plane $z\,o\,y$, x being the pole of yPz.

Let (α, δ) be the apparent R. A and Declination of the sun.

and (α_1, δ_1) be the apparent R. A and Declination of the moon.

(a, d) .... apparent R. A and Declination of the point z.

Let X represent the sun on the celestial sphere.

Let x, y, z be the co-ordinates of the sun at any time with respect to the axes chosen, and r, the geocentric distance of the sun taking the equatorial radius of the earth to be unity.

As x is the pole of yPz its declination is zero and its R. A. is $(90+a)$ and $X\hat{P}x=(90+a-\alpha)$

$\therefore$ From the triangle XPx

$$x=r\cos Xx=r\sin(\alpha-a).\cos\delta\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Again in $\triangle\, yXP$, $yP=d$, and $X\hat{P}y=y\hat{P}x+X\hat{P}x$

$$=90+90+a-\alpha$$
$$=180+a-\alpha$$

$$\therefore\quad y=r\cos Xy=r\,[\cos d\sin\delta-\sin d\cos\delta\cos(a-\alpha)]\ldots(2)$$

Also in triange PzX, $Pz=(90-d)$. $PX=90-\delta$ and $zPX=(\alpha-a)$

$$\therefore\quad z=r\cos zX=r\,[\sin d\sin\delta+\cos d\cos\delta\cos(a-\alpha)]\ldots(3)$$

If at any time x_1, y_1, z_1, be the rectangular co-ordinates of the moon with reference to these axes, and if r' be the moon's geocentric distance we can obtain in the same manner, the following equations.

$$x_1=r'\sin(\alpha_1-a)\cos\delta_1,$$

$$y_1=r'\,[\sin\delta_1\cos d-\cos\delta_1\sin d\cos(\alpha_1-a\,]$$

$$z_1=r'\,[\sin\delta_1\sin d+\cos\delta_1\cos d\cos(\alpha_1-a)\,]$$

Since the z axis is chosen parallel to the line joining the sun and the moon, $x=x_1$ and $y=y_1$ and x, y or x_1, y_1 are the co-ordinates of the centre of the shadow cast by the moon on the fundamental plane.

$$\therefore\ r\cos\delta\sin(\alpha-a)=r'\cos\delta_1\sin(\alpha_1-a)\ldots\ldots\ldots\ldots(4)$$

and $r\,[\sin\delta\cos d-\cos\delta\sin d\cos(\alpha-a)\,]$

$$=r'\,[\sin\delta_1\cos d-\cos\delta_1\sin d\cos(\alpha_1-a)\,]\ldots\ldots\ldots\ldots(5)$$

Equations (4) and (5) give the values of a and d at

any time, since r, r', α, δ, α_1 and δ_1 are known. Approximate solutions for a and d are obtained at intervals of ten minutes, since $(\alpha_1-\alpha)$ and $(\delta_1-\delta)$ are very small at about the time of the eclipse.

Let $\beta=\dfrac{r'}{r}=\dfrac{\sin P}{\sin p}$ where P and p are the solar and the lunar parallaxes.

Putting (α_1-a) in (4) as $[\alpha_1-\alpha+\alpha-a)]$ and expanding we get

$$\cos\delta\sin(\alpha-a)=\frac{r'}{r}\cos\delta_1[\sin(\alpha_1-\alpha)\cos(\alpha-a)$$
$$+\cos(\alpha_1-\alpha)\sin(\alpha-a)]$$

i. e. $\sin(\alpha-a)=\beta\sec\delta\cos\delta_1[\sin(\alpha_1-\alpha)\cos(\alpha-a)$
$+\cos(\alpha_1-\alpha)\sin(\alpha-a)]$

or $\sin(\alpha-a)[1-\beta\sec\delta\cos\delta_1\cos(\alpha_1-\alpha)]$
$=\beta\sec\delta\cos\delta_1\cos(\alpha-a)\sin(\alpha_1-\alpha)$

or $(\alpha-a)(1-\beta\sec\delta\cos\delta_1)=\beta\sec\delta\cos\delta_1\;(\alpha_1-\alpha)$ since $(\alpha-a)$ and $(\alpha_1-\alpha)$ are small quantities.

$$\text{i. e. } a=\alpha-\frac{\beta\sec\delta\cos\delta_1(\alpha_1-\alpha)}{1-\beta\sec\delta\cos\delta_1}$$

$$=\alpha-\frac{\beta\sec\delta\cos\delta_1}{1-\beta}(\alpha_1-\alpha)$$ since δ is nearly equal to δ_1.........(6)

Similarly from (5) $d=\delta-\dfrac{\beta}{1-\beta}(\delta_1-\delta)$..................(7)

The variations (x', y') per minute of the co-ordinates x, y of the centre of the shadow are obtained from (1) and (2) on substitution from (6) and (7) for a and d. These are given in the Nautical Almanac for every hour about the time of the eclipse.

Let μ be the hour angle of z for the meridian of Greenwich at sidereal time T.

Then $\mu = T - a =$ [a known quantity from (6).]
and μ', the variation of μ per minute can also be obtained.

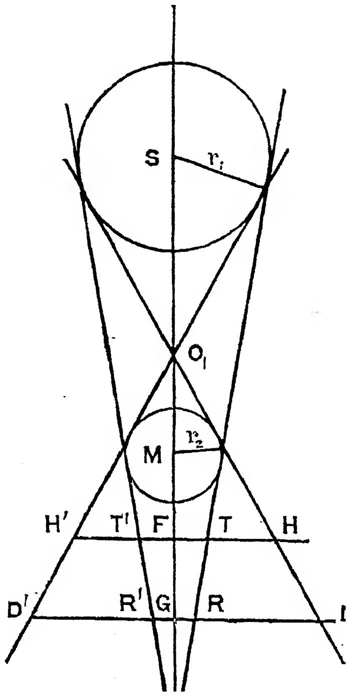


Fig. 127.

Let f_1 and f_2 denote the semi-vertical angles of the tangent cones formed at O_1 and O, S and M being the centres of the sun and the moon (fig. 127)

$$\sin f_1 = \frac{r_1 + r_2}{SM} = \frac{r_1 + r_2}{r\,(1-\beta)},$$ where r is the distance of the sun from the earth.........(8)

$$\sin f_2 = \frac{r_1 - r_2}{r\,(1-\beta)} \qquad \text{...(9)}$$

Equations (8) and (9) give the values of f_1 and f_2.

Let G represent the centre of the earth in figure 127 then $GM = z_1$ and $O_1 M = r_2 \operatorname{cosec} f_1$.

$\therefore c_1$, the z co-ordinate of O_1, the vertex of the penumbral cone $= z_1 + r_2 \operatorname{cosec} f_1$..............................(10)
and c_2, the z co-ordinate of O, the vertex of the umbral cone $= z_1 - r_2 \operatorname{cosec} f_2$..(11)

It should be noted that c_1 and c_2 are measured positively in the direction GM.

Let l_1 and l_2 be the radii of the sections of the penumbral and umbral cones made by the fundamental plane D′ R′ R D.

Then $l_1 = GD = c_1 \tan f_1 = z_1 \tan f_1 + r_2 \sec f_1$.........(12)

and $l_2 = GR = c_2 \tan f_2 = z_1 \tan f_2 - r_2 \sec f_2$.........(13)

From (12) and (13) values of l_1 and l_2 can be calculated.

In the Nautical Almanac, x, y, log sin d, log cos d, μ, l_1, and l_2 are calculated for intervals of 10 minutes and log x', log y', log μ', log tan f_1 and log tan f_2 are tabulated for intervals of an hour. These are called the Besselian elements which are to be used in predicting the circumstances of a solar eclipse at any place on the earth.

(2) *The application of the Besselian elements in eclipse calculation :*

Let F H denote the plane of the observer parallel to the fundamental plane and let (ξ, η, ζ) be the position of the observer with reference to the axes chosen. Then the plane F H is denoted by $z = \zeta$.

If L_1 and L_2 be the radii of the sections of the penumbral and umbral cones by the plane through the observer.

$L_1 = FH = (c_1 - \zeta) \tan f_1 = l_1 - \zeta \tan f_1$..................(14)

$L_2 = FT = (c_2 - \zeta) \tan f_2 = l_2 - \zeta \tan f_2$..................(15)

L_1 is always positive and L_2 is always negative when the vertex of the umbral cone is situated as in the fig. (127). Therefore for any one at a distance ζ from the plane G D to be within a total eclipse zone L_2 should be a negative quantity.

If the geocentric latitude and distance of the observer be φ and ρ and λ, his longitude west of Greenwich, and if in fig. (126) X is the geocentric zenith for the observer, we have

$$\xi = \rho \cos xX;\ \eta = \rho \cos yX;\ \zeta = \rho \cos zX.$$

Also P X being the meridian of the observer, and μ the Greenwich hour angle of z, $X\hat{P}z = \mu - \lambda$

$\therefore$ $XPx = 90^\circ - (\mu - \lambda)$ and $PX = 90^\circ - \varphi$ and $Px = 90^\circ$.

$\therefore$ From the triangle x P X,

$$\xi = \rho \cos xX = \rho \cos \varphi \sin (\mu - \lambda)$$

Similarly, from the triangle P X y

$$\eta = \rho\ (\sin \varphi \cos d - \cos \varphi \sin d \cos (\mu - \lambda)]$$

and from the triangle P X z

$$\zeta = \rho\ [\sin \varphi \sin \delta + \cos \varphi \cos \delta \cos (\mu - \lambda)]$$

From these, the variations ξ', η', ζ' per minute of ξ, η, ζ can be calculated;

for instance $\xi' = \mu' \rho \cos \varphi \cos (\mu - \lambda)$ where μ' is the variation of μ per minute.

The values of ξ, η, ζ are computed for the assumed time of the beginning of eclipse and can be calculated for intervals of ten minutes from the values ξ', η' and ζ'.

Thus L_1 and L_2 are derived when the above value of ζ is subtituted in (14) and (15)

When partial eclipse begins or ends for any observer, his distance from the centre of the shadow is L_1, and therefore at such a time,

$$(x-\xi)^2+(y-\eta)^2=L_1^2 \qquad \text{.......................................(16)}$$

At the beginning or end of totality the observer is on the umbral cone and therefore

$$(x-\xi)^2+(y-\eta)^2=L_2^2 \qquad \text{.......................................(17)}$$

The exact time of any of these phases of the eclipse is calculated as follows :—

Suppose we have to find out the beginning or end of totality at a place; let t be the Greenwich time of the instant, measured from any instant T near totality.

Let (x_0, y_0) be the values of (x, y) at time T

and (ξ_0, η_0) be the values of (ξ, η) at time T.

Then $x=x_0+x'\,t$, $y=y_0+y'\,t$;

$\xi=\xi_0+\xi'\,t$ and $\eta=\eta_0+\eta'\,t$.

$L_2=l_2-\zeta \tan f_2$, where l_2 and f_2 may be taken to be the same at time $T+t$ as they were at T, since the variations are small.

Hence the beginning or end of totality or annular phase of an eclipse is given by the quadratic equation obtained from (17) by substituting for x, y, ξ and η from the above.

i. e. $[x_0-\xi_0+t\,(x'-\xi')]^2+[y_0-\eta_0+t\,(y'-\eta')]^2=L_2^2$...(18)

This can be put in the form.

$$n^2\,t^2+2\,m\,n\,t\cos(M-N)+m^2-L_2^2=O \qquad \text{..............(19)}$$

$$\left.\begin{aligned} \text{where } x_0-\xi_0&=m\sin M,\ y_0-\eta_0=m\cos M\\ x'-\xi'&=n\sin N,\ y'-\eta'=n\cos N\end{aligned}\right\} \quad .(20)$$

In the above, the positive values of $\sqrt{(x_0-\xi_0)^2+(y_0-\eta_0)^2}$ and $\sqrt{(x'-\xi')^2+(y'-\eta')^2}$ are taken for m and n. Now m and M are the distance and the position angle of the centre of the shadow relative to the observer and n and N are the

magnitude and direction of its motion relative to the observer.

The beginning and end of total eclipse are given by the two roots of the equation (19)

$$\text{i. e. } t = -\frac{m}{n}\cos(\mathrm{M}-\mathrm{N}) \pm \frac{\mathrm{L}_2 \cos\psi}{n} \qquad \ldots\ldots(21)$$

$$\text{where } m \sin(\mathrm{M}-\mathrm{N}) = \mathrm{L}_2 \sin\psi.$$

The solutions obtained above may not be very accurate. and so these approximate solutions are used to give a new epoch of Greenwich time T_1 i. e. $(\mathrm{T}+t_1)$ from which we start reckoning time. From this, we again obtain another equation similar to (21) for the beginning or end of totality. To obtain the beginning and end of a partial eclipse at any station, the same procedure is adopted as above on choosing L_1 for L_2.

214. The points on the solar disc where the eclipse commences or ends.

The relative position of the observer at the beginning or end of the eclipse and therefore of the point of the solar disc where the generator of the tangent cone touches the solar disc, is given by

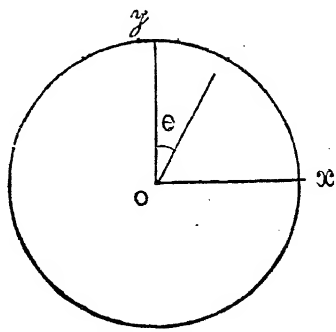


Fig. 128.

$$\mathrm{L}_1 \sin\theta = x_0 - \xi_0 + t\,(x' - \xi')$$
$$\text{and } \mathrm{L}_1 \cos\theta = y_0 - \eta_0 + t\,(y' - \eta')$$

where t is the time for the beginning or end of the eclipse. Since the north point lies on the zy plane, the point of the solar disc where the eclipse commences is given by $\theta°$ measured from the north point of the sun.

215. Importance of eclipses.

Total solar eclipses are of great astronomical importance, while lunar eclipses are comparatively less important. A total solar eclipse lasts only for a few minutes, but a lunar eclipse lasts for a few hours; the former is a phenomenon visible only to a few in a narrow strip of land, while the latter could be seen by the inhabitants of nearly half the globe.

The only time when the solar corona could be studied is during a total eclipse of the sun. The instants marking the beginning and end of totality are the rare occasions when the sun's chromosphere could be studied, by taking what is called the flash spectrum of it. It was only during the total solar eclipse of 1870, that the existence of the reversing layer was found out. Total solar eclipses are the only occasions when it is possible to test the correctness of the general theory of relativity by which the rays of light are supposed to be deflected by strong gravitational fields. Chronologically, the dates of many events of great importance could be settled, if they could be associated with eclipses of the sun. Again, the secular acceleration of the moon is determined by a comparison of the early observations of the total eclipses of the sun, with the modern tables of the moon.

There are yet many other physical problems connected with the sun whose solutions are awaited with a keen interest, as modern instruments are used in the observation of the sun at the time of a total eclipse.

216. Occultations.

The moon in the course of its sidereal revolution frequently passes in front of a star, and gives rise to what is called an *occultation*. The star suddenly disappears at the eastern edge of the moon's disc and a few minutes later reappears at the western limb. The times of

disappearance or reappearance of a star at the dark limb of the moon can be observed with great accuracy and these observations enable us to determine the moon's position accurately, if the star observed is known, and if the position of the observer is also known. On the other hand, if the co-ordinates of the moon be known accurately, and an occultation be observed from two stations on the earth's surface, the difference of longitudes between the two places can be accurately known.

The Nautical Almanac gives every year, a list of important occultations visible from Greenwich, containing the times of disappearance and re-appearance and the points of the moon's disc at which these take place. It is also possible to calculate from the data given, the circumstances of the occultation for any other point on the earth's surface, where it can be seen. These are to be worked out on the lines exactly similar to those for a solar eclipse, the star having however no motion, parallax or semi-diameter. An occulation of a star is visible over only a small portion of the earth's surface.

List of eclipses occurring in a Saros of 18 years and 11 days.

Year.	Ascending Node. Lunar.	Ascending Node. Solar.	Descending Node. Lunar.	Descending Node. Solar.
1932	March 22	March 7	Sept. 14	Aug. 31
1933	——	Feb. 24	——	Aug. 21
1934	Jan. 30	Feb. 14	July 26	Aug. 10
1935	Jan. 19	Jan. 5 Feb. 3 Dec. 25	July 16	June 30 July 30
1936	Jan. 8	Dec. 13	July 4	June 19
1937	Novem. 18	Dec. 2	——	June 6
1938	Novem. 7	Novem. 22	May 14	May 29
1939	Oct. 28	Oct. 12	May 3	April 19
1940	——	Oct. 1	——	April 7
1941	Sept. 5	Sept. 21	March 13	March 27
1942	Aug. 26	Aug. 12 Sept. 10	March 3	March 16
1943	Aug. 15	Aug. 1	Feb. 20	Feb. 4
1944	——	July 20	——	Jan. 25
1945	June 25	July 9	Dec. 19	Jan. 14
1946	June 14	May 30 June 29	Dec. 8	Jan. 3 Novem 23
1947	June 3	May 20	——	Novem. 12
1948	April 23	May 9	——	Novem. 1
1949	April 13	April 28	Oct. 7	Oct. 21
1950	April 2	March 18	Sept. 26	Sept. 12

CHAPTER XVII.

THE SUN.

217. Introduction.

The sun is the central figure of the solar system, of which the earth that we inhabit, is a member. The myriads of stars we see around us are each, probably, a system like the sun with a retinue of planets around it. These stars are so far away that the planets, if any, around them could not be detected even by the best of modern telescopes. The sun may thus be considered to be a typical star whose proximity to us enables us to study this object more elaborately than any of the other stars.

The mean distance of the earth from the sun can be derived from the solar parallax and a knowledge of the earth's radius (see chapter XII.)

The actual size of the sun is then determined, by measuring the angular diameter of the sun. This has a mean value of $32' - 4''$. From this the sun's diameter is found to be 865,000 miles. It is about 108 times the diameter of the earth.

The mass of the sun can be determined if the mass of the earth is assumed and if the distance between the sun and the earth is known.

Let G and g be the forces of attraction on a unit mass near the earth's surface due to the sun and the earth and let m and M be the masses of the earth and the sun respectively. Let the radius of the earth be r, and let R be the distance of the sun from the earth.

$$\text{Then, } \frac{G}{g} = \frac{\dfrac{M}{R^2}}{\dfrac{m}{r^2}}, \text{ giving } G = \frac{M}{R^2}\,\frac{r^2}{m}\,.\,g.$$

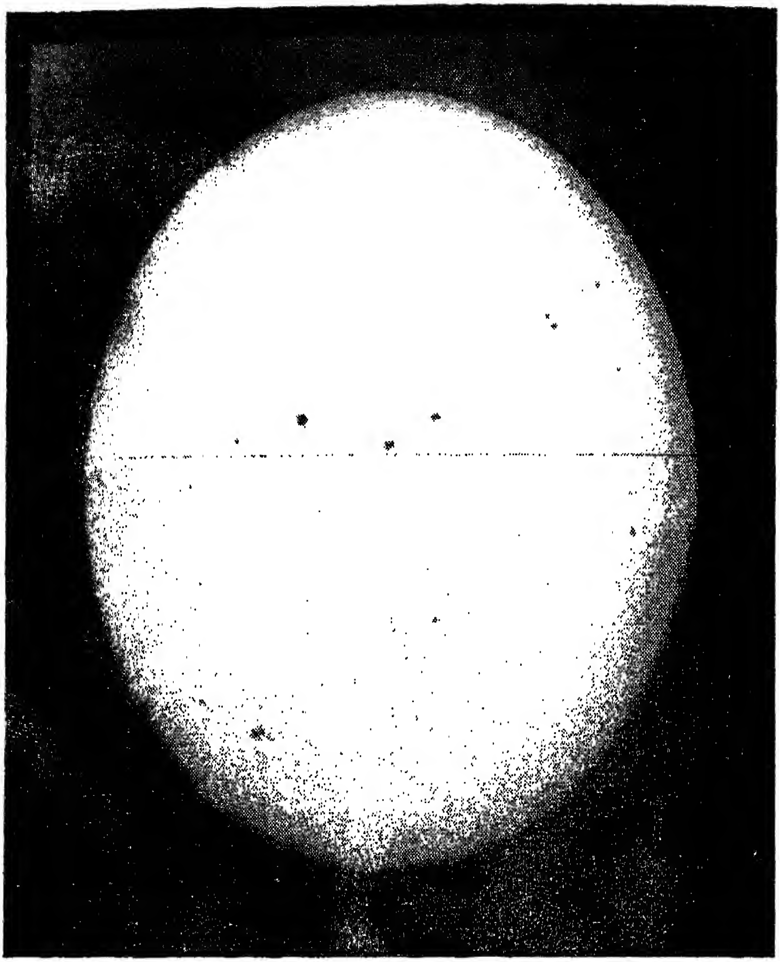


Plate No. 3.

A photograph of the sun taken on 19th February 1926. The solar disc is brighter in the centre, and gets darker towards the limb. The actual shape of the sun extends to several millions of miles beyond the definite spherical boundary shown by the photograph. Many spots are seen on the solar disc. Some of these spots are long lived and others exist only for a short time. From their apparent movements across the solar surface, the sun's period of rotation about its axis is inferred.

If V be the velocity of the earth, assuming it to be moving in a circular orbit of radius R, we have $\frac{V^2}{R}=G=\frac{M}{R^2}\cdot\frac{r^2}{m}g$, where V is given by $2\pi R=365\frac{1}{4}V$.

$\therefore \frac{V^2}{R}=G=$ a known quantity. Thus M is found to be 332,000 m.

From the volume of the sun and its mass, we get the mean density of the sun to be $332{,}000\div(108)^3$ times that of the earth. This comes to about a quarter of the earth's mean density, the latter being 5·5.

The gravitational force at a point on the sun's surface is obtained by assuming the whole mass of the sun to be concentrated at the centre and in terms of the corresponding value (g) on the earth's surface it comes to $\frac{332{,}000}{(108)^2}g$ or 27 g nearly.

218. Solar surface.

The surface of the sun as seen through a telescope or as photographed, is bright near the central portion of the disc and gets darker and darker towards the limb. This appearance is due to the fact that the rays from near the edge have to pass through a greater thickness of solar atmosphere and so are more absorbed than the rays coming from the central parts. The solar disc appears to be granulated, and what we see are different parts of a huge mass of fire sending towards us energy liberated from its interior which is in a terrible state of heat running up to millions of degrees of temperature. Inside the sun, matter is in an unknown gaseous condition, and what we see outside is the shape of the cloud formed by various substances in different stages of existence.

The sun's disc is generally sprinkled with spots and sometimes groups of many spots could be seen on it. Galileo, who used the telescope first to examine

celestial bodies, was also the first to observe these spots; and from their movements across the solar surface, the sun's rotation period was determined even long ago. The observation of spots indicates for the sun various speeds of rotation in different latitudes. This confirms the inference that the sun is not rotating as a solid body. The mean period of rotation is varying from 25 to 38 days. The axis of rotation is inclined at 7° to the axis of the ecliptic and the equatorial plane of the sun cuts the ecliptic at points of longitudes 75° and 255°. The sun comes to these points on the 6th of June and 6th of December and on these dates the sun's equator looks like a diameter on the solar disc.

219. The spectroscope and elements in the sun.

Before the application of the spectroscope to the study of the sun by Fraunhofer, Kirchtroff and Rowland, little was known about the composition of the sun. The spectroscope is an instrument which analyses any beam of light into minute wave lengths that constitute white light. What we see when a beam of sun-light is passed through a prism and when the beam emerges at the other end is only the seven main colours that combine to form white light; each of these colours consists of an infinite number of different light vibrations, and any form of spectroscope enables us to study from these vibrations the nature of the substance which emits a particular vibration or set of vibrations. Different chemical elements when allowed to glow in a vaporous condition, give rise to characteristic bright lines in their spectra. These lines form a basis for identifying the substance emitting such bright lines. Thus the bright lines shown by the photograph of a nebula taken by means of a spectroscope (spectrogram as it is called) give us an indication that the nebula is not only gaseous in composition, but also that it contains such and such definite chemical substances.

Now, when a glowing gas has behind it a hotter incandescent body, the lines emitted by the glowing vapour appear as dark lines on the continuous bright spectrum of seven colours given by the white-hot body. This is simply-due to the less intense character of the vibrations emitted by the gaseous substance giving rise to relatively darker lines compared with the more intense vibrations emitted by the hotter body behind. This is explained scientifically by the fact that light emitted by a source of higher temperature is absorbed while passing through a vapour in lower temperature in the wave lengths emitted by the vapour. Now these dark lines are called absorption lines, and they give us the composition of the vapour through which the light of the incandescent source passes. The dark lines of the solar spectrum are thus due to the absorption caused by the solar atmosphere and the bright continuous spectrum is what the solar disc gives. A good many of these absorption lines which are more than 20,000 in number have been identified with the lines of about fortyfive elements studied in the laboratory. The majority of these lines have not yet been identified and they are due to the absorption by the atmosphere in the sun containing either unknown elements or elements in unknown forms. Gradually more and more of these are being identified with known substances in one or other of their various states of existence.

220. Sunspots.

Mention has already been made of the occurrence of certain dark regions in the solar surface, some of which are short-lived and others continuing to exist for a longer time. These are called sunspots and they have a very dark interior called umbra and a less dark exterior called penumbra. These regions of the solar surface are comparatively cooler than the other parts, having a temperature of about 5000° C and the study of their spectra shows

that some compounds like titanium oxide and magnesium hydride are present there, indicating that the temperature there is lower than in other parts of the solar surface where these compounds get dissociated into their component elements. It is also noteworthy that some of the lines of the sunspot spectra are much darker than the corresponding lines of the photospheric spectrum. This corresponds to the darker lines of the furnace spectrum of a substance when compared with the arc spectrum of the same substance, where the substance is in a state of higher temperature.

It is also known that sunspots are regions of intense magnetic forces. This is inferred from the splitting of certain spectral lines into two or three when the source of light is surrounded by strong magnetic fields. Now certain lines of the spot spectrum undergo such splitting, and from the separation of the components it is possible to estimate the strength of the magnetic field.

These spots are also seen to be surrounded by hydrogen or calcium vapour in the form of vortices. Now the direction of rotation of these vortices is found to be generally clockwise in the southern hemisphere and counter-clockwise in the northern hemisphere. The magnetic character of these spots is attributed to the motion of negatively charged particles moving in these vortices.

The sunspots are signs of eruptions of a violent nature due to some deep-seated disturbance in the interior of the sun. When the ejected matter falls down again on the sun's surface, it gets cooler and appears darker on the sun's surface than the surrounding regions.

221. Sunspot period.

The spots are generally short-lived, though some are seen to remain for more than 50 days. They extend over the sun's surface from the equator to 30° north or south. Near the spots could be seen bright patches called faculae which are not necessarily related to spots

From a study of the daily records of the sunspots and their area, it has been concluded that there are occasions when the sun exhibits a maximum area for these spots as also a minimum area. This period is now known to be about $11\frac{1}{2}$ years. There is also a progressive change in the distribution of these spots in latitude. At the beginning of a cycle when the spots are few, they occur in high latitudes and later, when the number of spots increase, they gradually prefer lower latitudes and regions on either side of the solar equator.

222. Solar prominences.

During the very short duration of a total eclipse we can observe flames shooting out to enormous distances from the sun's limb towards outer space. These are indications of great disturbances existing in the sun causing violent outbursts. These flames called prominences shooting out from the sun could not easily be seen at all times due to the great amount of light given by the sun's disc. At the time of a total solar eclipse the sun's disc is covered by the moon's disc and the glowing exterior of the sun could be seen to extend in all directions to more than three or four times the diameter of the sun's disc. By using a spectroscope, it is possible to study the shape and extent of these prominences even at other times. These prominences contain a lot of glowing hydrogen and if a spectroscope is directed round the circumference of the solar disc, the red hydrogen line will suddenly become bright and if at that place the slit of the spectroscope is widened, one could see the whole of the prominence in red colour, showing the shape of the hydrogen flame in the prominence.

These prominences are found to occur in two distinct belts of the solar disc both north and south of the equator. They have also a cycle of variations coinciding with the sunspot cycle. The prominences in the low latitude

belts are occurring in the sunspot zones and their activity is shifting towards the solar equator from the beginning of a sunspot cycle to its end. The prominences in the higher zones decrease in activity at the time when the sunspot is minimum, and move towards the poles where they die out at the time when the sunspot is maximum.

One group of these prominences is usually associated with young and active spots. They occur in the form of rockets and arches of various shapes. Another group of prominences is massive and pyramidical in shape, and usually long-lived and is not associated with spots. These prominences after continuing to exist for about 30 or 40 days, suddenly break up rising to nearly half a million miles in height with a velocity of about 300 miles a second. These prominences are then called eruptive prominences.

223. The chromosphere.

The photosphere of the sun showing the bright solar disc is bounded by what is called the reversing layer giving rise to the absorption lines of Fraunhofer and this is further surrounded by a thin layer of glowing and less dense vapour called the chromosphere. This thin layer could be seen and studied by moving the slit of a spectroscope at any time tangential to the solar disc or at the instants before and after a total eclipse of the sun. Just before the bright disc of the sun is covered completely by the moon, the chromosphere gives in a spectroscope bright line spectrum, due to the several substances glowing in it. A study of these lines tells us what all elements are found in this outer layer of the sun. Here too are found most of the elements found in the interior of the sun. But there are differences and these are due to the difference in the physical conditions prevalent in the two regions. For instance, there is evidence of helium in the chromosphere but not in the spectrum of the solar disc. Again lines of hydrogen, titanium etc. are getting stronger

in intensity when seen in the chromospheric spectrum, while those due to iron, nickel, sodium etc. get weaker than in the ordinary solar spectrum. From these differences and from the laboratory study of the behaviour of these lines due to substances under different physical conditions, the temperature, pressure, density etc. of the chromosphere have been determined.

224. The corona.

The bright appendage of the sun seen at the time of a total eclipse is called the corona. Its light is faint compared to the light from the solar disc and so, could not be seen except when the solar disc is covered. Its light is somewhat brighter than that of the full moon and its shape is different at different times varying with the sunspot cycle. It is found to extend to several millions of miles streaming out in different directions away from the solar disc. It has no definite boundary and shows several filaments and curved arches near the solar disc. The coronal light is partially polarised and therefore shows preference for certain directions. A good amount of this light is due to the reflection of sunlight by extremely small particles.

225. The sun's temperature and the maintenance of solar energy.

The sun is sending out its energy by radiation and only a small fraction of it reaches the earth. A large part of this fraction is absorbed by the atmosphere surrounding the earth. The quantity of heat falling on a square centimetre of the earth's surface at its mean distance from the sun would enable us to determine the total amount of energy emitted by the sun. This is called the solar constant. Langley, Abbot and Foule have determined this constant, by which it is known that each square centimetre of the earth's surface receives in one minute 1·93 calories of heat energy from the sun. Now the surface of a sphere of radius 93 million miles is 46000 times the surface of the

sun, and so each square centimetre of the sun's surface sends out $46000 \times 1{\cdot}93$ calories of heat per minute, which is the rate at which a 9. H. P. engine will radiate energy. Now from this value of the solar constant, the effective temperature of the sun can be calculated assuming it to be a perfect radiator which will radiate energy proportional to the fourth power of its temperature. Thus the temperature of the solar surface is found to be 5600° C. On the photosphere the temperature should be a little more. The temperature at the interior should be much greater than this. The total radiation by the sun at the surface is $0{\cdot}58 \times 10^{24}$ horse powers and if this is supplied only by the stored up heat inside the sun, its temperature should fall down by nearly one degree a year. This would mean that the sun would become a cold body in a few thousand years. Geological studies of the earth give indications that organic life has been in existence on it for many millions of years; and if after the lapse of all these years the sun should have its present temperature, surely there should have existed during all these years some source of heat supply for the sun. Helmholtz has suggested that this supply of heat comes from the contraction process taking place in the sun, due to the attraction of the various particles. He calculated that if the sun's diameter is getting reduced by 150 metres per year, the heat liberated in that process would make up for the heat radiated by the sun. If such a contraction were taking place, the apparent angular diameter of the solar disc would not diminish by more than two seconds of arc in 30000 years. This theory is then not contradictory to the results of observation, which show that no appreciable diminution of the sun's apparent size has been noticed during all these centuries of observation. On the assumption that the sun began to radiate heat outwards from the time when it was a huge nebula, it would follow that the sun would have been a nebula 22 million years before the

present time. Supposing the earth was formed from the sun long time ago, it could not be earlier than this date. According to geologists, the earth should have been in existence for a longer period, and therefore the sun, its parent mass, should have been radiating heat for a correspondingly longer time. Therefore, another theory has been propounded to account for the maintenance of sun's heat. It is a well-known fact that a gram of radium is radiating heat at the rate of 138 calories per hour. Though this element has not been discovered in the sun, helium, one of its transformation products, has been found to exist in the sun. To account for the present rate of radiation, all that is necessary is to assume that each cubic metre of the sun contains about 3·6 grams of radium. Even assuming that the sun's source of energy is derived entirely from radio-active processes, the life of the sun as a heat-giver is not found to be as long as is given by geological considerations. Another hypothesis brought forward to account for the supply of heat inside the sun is the liberation of energy at that high temperature due to the mutual destruction of electrons by collision, resulting in the transformation of their mass into energy. It is some such sub-atomic process which occurs at such a high temperature, that accounts for the continuous radiation by the sun for a time considerably longer than 300 million years, which is the age of the earth fixed by geological considerations.

CHAPTER XVIII.

THE STELLAR UNIVERSE.

226. Introduction.

On any clear night one could see the sky illuminated by a very large number of tiny points of light called fixed stars. The actual number seen at any time by the naked eye is not beyond 3000, though for the first appearance their number appears to be countless. Actually, the number of stars with which space is filled is of the order of a thousand million; but only a small fraction of these are of sufficient brilliancy as to be seen by the naked eye. With the aid of even a small telescope the number of stars that could be seen becomes increased manyfold. This is due to the fact that the lens of a telescope gathers more light than the human eye and so makes even a faint object visible to us.

Among these myriads of bright celestial bodies, there are the so-called fixed stars, the planets, the comets, the meteors and the nebulae. The stars are each of them a huge mass of different elements, all in a state of very high temperature, just like the sun. The temperature at the central portion is of the order of a few million degrees, whereas the temperature at the surface may be of the order of a few thousands of degrees centigrade.

It is their enormous distance that causes them to look so small and apparently faint in lusture. These distances are so great that the earth's motion round the sun causes only a very small apparent shift of 0·75 of a second even in the case of the nearest star which is at about 260,000 times the sun's distance from us.

These stars are no doubt moving in space with velocities ranging from a few miles to a few hundreds of miles

a second; but their distances are so great that it is impossible to find any great and easily noticeable displacement in the sky. This is the reason why in ancient times the stars were considered to be fixed.

227. Stellar motion.

The motion of a star in space can be resolved into motion in the line of sight and motion perpendicular to the line of sight. The latter is called the proper motion of the star and is noticeable as an apparent displacement of the star in the sky. This proper motion of a star could be inferred from well determined star positions separated by known intervals of time or from photographic records of stars taken on dates separated by a known period.

The velocity of a star in the line of sight either towards us or away from us is determined from the observed shift of any lines in the stellar spectrum from their normal position as known in a terrestrial laboratory. If the distance of the star is known, the proper motion in seconds of arc can be converted into the linear velocity perpendicular to the line of sight. This, together with the radial velocity gives the actual velocity of the star in space. Calculations of this kind lead us to infer that stars have all ranges of velocities reaching up to a few hundreds of miles a second.

228. Stellar constellations.

The ancient astronomers divided the visible sky into groups of stars and called each group a constellation giving it either the name of some mythological person or object or animal. In very few cases only there is to be found any apparent resemblance between the object and the constellation bearing the name. Scorpio is one such case. The division of the sky into the different constellations is a convenient way of remembering the arrangement of the stars and should have served a good purpose at a time

when the accurate positions of the stars could not be determined as at present. Ptolemy prepared a star catalogue of forty-eight constellations including the twelve *zodiacal constellations,* which are the groups of stars lying along the ecliptic and along each of which the sun's course lies for a month. Ptolemy's list of constellations has been increased, especially as the southern constellations were not visible to men of his country. Now there are 88 constellations, dividing the stars in the sky into well-defined groups. Individual stars in each constellation are designated by the name of the constellation prefixed by the letters of the Greek Alphabet usually assigned in the order of their apparent brightness. (Examples. α. Bootes. β. Geminorum. γ Andromeda. θ Orionis etc.) This system of naming the stars was started by Bayer in 1603. When the letters of the Greek Alphabet are exhausted, the letters of the Roman Alphabet or numerical numbers are used. Some times stars are also designated by the numbers given to them in any special star catalogue. (Example: Lalande 45,580.) The best way of specifying faint stars is by giving its R. A and decl. at a definite epoch. Many bright stars have also their own names which are of Greek, Latin or Arabic origin. Thus α Bootes, α Tauri and α Canis Majoris are called Arcturus, Aldebaran, and Sirius respectively.

229. Stellar magnitude.

This is a term that is used to indicate the relative brightness of different stars. Hipparchus selected about 20 brightest stars in the sky and called them first magnitude stars and he called the faintest stars that could only just be seen as the sixth magnitude stars. He also gave to the stars of intermediate brightness, magnitudes ranging from 2 to 5. This rough classification was later on made more precise, by which it has been found that decrease in brightness in geometrical progression corresponded to

an increase in magnitude in arithmetical progression and a first magnitude star was found to be 100 times brighter than the faintest visible stars of the sixth magnitude. Some of the very bright stars are brighter than the first magnitude stars and their magnitudes are spoken of as being 0, —1, —2 etc. Thus the magnitude of Sirius is—1·4. These magnitudes only refer to the apparent luminosity of a star and not to its intrinsic luminosity. To denote the latter, a term called *absolute magnitude* is used. This is taken to be the brightness of a star when it is kept at a distance of ten parsecs from us.

230. Photographic magnitude.

The magnitude referred to in the previous article is the visual magnitude and it is determined by instruments used visually to observe stars; but with the development of photography, a method of determining stellar magnitudes from the images of stars on photographic plates has been devised. This is based upon the fact that brighter stars give stronger impressions on the plate than the fainter ones. The size of the image of a star is the criterion on which the magnitude determination depends. One fact however should be noted that two stars of equal visual magnitudes need not be of the same photographic magnitude, as the plates are more sensitive to the blue stars and less to red stars. A blue star may be photographically brighter than a red star of the same apparent brightness.

231. Double and multiple stars.

About a fifth of the stars that appear single to the naked eye are found to be double on telescopic or spectroscopic examination. Castor and Sirius are instances of double stars.

Resolution of a single star into two stars on telescopic examination may lead one to suspect that the star may

be composed of two bodies having some physical connection between them. But this may, in reality, be due to the fact that the one is nearly in the same line of sight as the other, though it may be at a considerable distance in front of or behind the other. Such stars are called *optical doubles.* If there is some physical connetion between the two stars, it should be evident in an indication that one star moves round the other in an ellipse, obeying gravitational laws. Such connected stars are called *binaries.* The binary character may be inferred by the actual displacement of the smaller star relative to the bigger (primary) star as seen through a telescope or by the relative displacements of the lines in the spectrum of the star or by a variation in the apparent magnitude of the star. The star is accordingly called a *telescopic binary* or *spectroscopic binary* as the case may be.

A spectroscopic binary or a star of variable brightness may not show its binary characteristic clearly owing to the close proximity of the two components and owing to the limit in the resolving power of the telescope. A star known to be a binary by spectral examination alone, sometimes shows itself to be a telescopic binary also on examination by a telescope of higher resolving power. Thus Sirius was known only to be a spectroscopic binary until the 40″ lens of the Yerkes Observatory was turned on to it, and it was then found to be a telescopic binary also.

The binary character of some stars are only revealed after many years of observation. If the angular separation of a pair in a telescope is 2″ of arc, to settle the question of relative motion, observations extending to a century may be required.

According to the gravitational law, each component of a binary system describes an ellipse round the common centre of gravity of the two stars. The relative orbit of the one round the other is similar to the other two and its

major axis is the sum of the major axes of the other two real ellipses. When we see a binary star, the apparent separation is only the projection of the real separation on the sky. Hence what we infer from observation is the relative orbit of the component star round the main or primary star. The projection of this orbit on the sky is also an ellipse.

232. Masses of double stars.

If m_1 and m_2 are the masses of the two components of a binary system taking the sun's mass as unity, P, the period of revolution of the secondary round the primary, 'a' the semi-major axis of the orbit and p, the parallax of the system, we have from Kepler's law $(m_1 + m_2) P^2 = \left(\frac{a}{p}\right)^3$. Hence the mass of the system is known if the parallax, the orbital dimensions, and period be known.

233. Triple and multiple stars.

Many stars, once known to be binaries are now found or inferred to be consisting of more than two stars. For instance, each of the stars of the visual binary Castor is known to be a spectroscopic binary having periods of 3 and 9 days and the period of the visual binary is about 300 years. The pole star (Polaris) known to be a binary is now understood to be a triple system. This was inferred from the variable radial velocity of the centre of mass of the apparent binary system indicating that the binary is attracted by a third star invisible to us. ε Lyrae is a system of four stars controlled mutually by gravitational laws.

234. Variable stars.

These are a class of stars whose brilliance undergoes variation either of a regular character or otherwise. A good many of these stars undergo changes in their apparent brightness in an irregular manner, while others have a

regular period of variation reaching a maximum and a minimum of brightness at times which could be predicted. The periods of light variations range from a few hours to hundreds of days. The variable stars are classified under the following groups possessing distinct characteristics:—

(1) *Eclipsing variables.*

These consist of a large number of stars which are now known to be binaries or triple systems, the variation of light being caused by one component which is either dark or fainter eclipsing the other. The range of variation in brightness will depend upon the relative sizes, their individual brightness and the inclination of their orbital plane to the line of sight. In fact, from a study of the curve showing the variation of light for such a star, definite conclusions as to the binary character or otherwise could be made. Further study of the light curve of a binary system discovered in this manner, enables one to know the orientation, the relative sizes of the bodies in the system, the ellipticity of the orbit described, the brightness of each of the components in terms of the sun, and the densities and the actual dimensions of the two components.

The star Algol (β Persei) known to be a variable to the Arabs from very ancient times, belongs to this class. From a study of its light curve, its period of variation is found to be more than 2½ days. At maximum brightness, the magnitude is 2·3 which is kept on for several hours; then the brightness falls suddenly by about 1·2 in 5 hours. After this, the magnitude rises again suddenly, reaching the original value in about five hours' time. About a day later, there is another very small drop in magnitude. Thus in each period, there are two minima occurring at equal intervals. This suggests that the bodies move in either a circular orbit or an elliptic orbit

whose axes are in the line of sight. As the minimum brightness is not kept up for any length of time, the light variation is due to a partial eclipse; as the two minima are different in values, one component is brighter than the other. Between the two minima, the light undergoes slight variation. This must be due either to the non-uniform surface brightness of the components or to their non-spherical nature. Further study of the light curve and measures of their radial velocity give the following additional information about Algol :—The orbit of the system is circular, and is inclined at about 8° to the line of sight. Both the bodies are somewhat elliptical in shape and the mean radius of the brighter component is 0·21 of the radius of the orbit and the corresponding value for the fainter star is 0·24. The brighter body emits 0·93 times the maximum light from the system. The mean density of the system is 0·07, assuming the density of the sun to be unity. The total light from Algol is about 200 times that of the sun.

The star β Lyrae is another typical variable star having a nearly circular orbit and a period of about 13 days. Its curve has two equal maxima and two unequal minima. The continuous variation of light is due to the ellipsoidal sizes of the components. Their mean radii are 0·27 and 0·68 of the orbital radius. The system consists of two bodies of which one is 9 times as bright as the other. Both are very low in density. The two are so near each other that their mutual tidal forces cause their ellipsoidal shapes.

This leads us to a possible explanation of the origin of binary systems in general. A parent spherical star which rotates becomes gradually more and more contracted and therefore flattened and the density increases to a critical value when fission takes place and the star becomes a binary. Each component of the binary which has its components at first in contact or actually very

near each other as in β Lyrae, becomes more and more separated from the other due to tidal friction and therefore the period of rotation also gradually increases. In this way we can imagine the different binaries to have developed into their present state from single stars. There are some difficulties however in accepting this theory; as for instance, the densities of many of the binaries are much less than that of the critical densities for fission and also the periods of a good many are many times what could be obtained from the action of tidal friction.

(2) *Cepheid variables.*

These stars have a short period of light variation ranging from one day to one or two months and their range of light variation is only about one magnitude. The variation of light is definitely known to be due not to their binary character but to a kind of periodic pulsation that takes place in the stars themselves. These pulsations cause changes in the rate of emission of light. The great use to which the cepheid variables are put, is in estimating the distances of the regions where these variable stars exist, which could not be estimated by any trigonometric means. There is a remarkable relationship between the absolute magnitude and length of period for any cepheid variable. From the observed period of a cepheid variable, which is known to exist in the Andromeda Nebula, the absolute magnitude of the variable is known, and from the apparent magnitude of the star which differs from the absolute magnitude by a constant depending upon the distance of the star, the distance of the nebula is calculated. The Andromeda Nebula is thus found to be so distant that light takes 900,000 years to reach us from there. The characteristic nature of the curve of light variations for Cepheid variables is that there is a rapid rise of the brightness and

that the fall is more gradual, after the maximum is reached. δ Cephei is the most typical of this class of stars.

(3) *Long period variables.*

These are stars having a period of variation in their light changes ranging from about 150 days to 450 days. The range of magnitude variation in the case of these stars is also large, from 3 to 8 magnitudes. Many of the stars are reddish in colour, thc redder ones having usually longer periods. The most typical star of this class is Mira (O. Ceti) having a period of about 330 days, and having a variable magnitude ranging from 2 to 9. The minimum and maximum magnitudes vary somewhat for different periods. The stars show spectroscopic evidence of the existence of bright flames of hydrogen gas periodically, suggesting that the variation of light is probably due to periodic outbursts as in the case of the sun's activity shown by the sunspot cycle.

(4) *Irregular variables.*

These are another type of variable stars for which no definite period can be given; their range of variation is also small, being not more than two magnitudes. The cause of variation may be due to some sort of crest forming on their surface, diminishing the light from them; when a certain stage is reached in this crest formation, the imprisoned gaseous matter under increased pressure bursts out with corresponding increase of brightness. Such occurences however cannot be said to happen at periodic intervals.

(5) *Novae or new stars.*

These stars suddenly burst forth in any region of the sky and are known by the name of the constellation where they occur first, such as Nova Cassiopeiae observed by Tycho Brahe in 1572 reaching a brightness greater than that of Venus or Nova Ophiuchi seen by Kepler in 1604 to be brighter than Sirius. Another bright nova that

appeared in this century was Nova Persei seen in 1901, when it shone brighter than α Bootes. (Arcturus). The light curves of these stars show a sudden increase of brightness and then a sudden decline of brightness for some time and later more gradual loss of light with irregular oscillations until they reach a steady value. After their decline to a steady value, none of them is known to get into active state again. No doubt, these stars were existing in the same constellation even before they burst out into prominence; only they were known to be either faint or not recognisable. Nova Aquilae iii before it burst into the same rank as Canopus on 9th June 1918, was a faint star of the 11th magnitude. On the 5th of June a photographic plate showed it to be of 10·5 magnitude; on the 7th June in another plate it appeared to be of the sixth magnitude; on the 9th it reached 0·5 magnitude. In the course of these four days it increased in brightness more than 2500 times. The brightness then began to decline. On June 17th it was of the second magnitude and by the end of June it reached a much fainter stage until it was reduced to the 10th magnitude in which state it is to be seen even now. Almost all the novae discovered are found to occur in the neighbourhood of the milky way. These novae are produced probably at a time when a feebly luminous star passes through some dark cloud of matter which exists in all regions of the milky way. On entering such a dark cloud of matter, the star is heated by friction to a greater brightness and the spectrum of such a star should show both bright and absorption lines and this is actually found to be the case with many novae. The absorption lines in the spectrum of a nova are due to expanding and cooling gases and these lines are found to have a displacement towards the violet end of the spectrum showing that the gases are approaching the observer.

235. The milky way or galaxy.

On any clear night one could see a luminous belt of cloudy matter encircling the heavens, containing within it the constellations of Cassiopeia, Perseus, Auriga and running between Orion and Gemini and dividing itself into two branches from Centaurus to Sagittarius where it is brightest and densest. Its breadth varies from about 3° to 45° in certain places. This belt is called the *milky way or galaxy*. Even though we are not able to see with the naked eye many distinct stars in certain regions of the milky way, a telescope or a photographic plate exposed to any part of the milky way reveals that the milky way is the most crowded region of stars. Some parts of the milky way (for instance part of Sagittarius) are so packed with stars that even the biggest telescope could not resolve the region into star points. The north pole of the milky way is situated at R. A. 90° and Decl. 28°. It is usual to denote the position of a celestial body with reference to the plane of the galaxy. The longitude is measured from the point where the milky way intersects the equator (R. A. 280°) and latitude is measured north of this plane. One advantage of galactic co-ordinates which makes them important in stellar investigations is the fact that they do not change by the precession of the equinoxes. Therefore any change in the galactic co-ordinates of a star determines the star's proper motion.

The galactic plane forms a plane of symmetry for our stellar universe. The stars all crowd towards it and get less and less in number per square degree as one goes away from this plane towards the poles on either side. Among the visible stars, the milky way contains per unit area nearly three times as many as could be seen near the galactic poles. Taking into consideration much fainter stars, the density of stars in the milky way is twenty times the density at the poles of the milky way.

Hence it is inferred that the universe of stars is flattened towards the milky way and is lens shaped and that the great star density in the milky way is due to the greater distance to which the stars extend in that direction. This is also verified by the existence of numerous star clouds of great density situated at enormous distances in the milky way. The most distant objects in the milky way are at a distance of over 6000 parsecs (1 parsec = 3·26 light years) as shown by the estimated distances of Cepheid variables occuring in them.

236. Star clusters.

In certain parts of the sky, there are aggregations of stars showing a regular and definite configuration. These are called *star clusters.* In the constellation of Hercules, there is a typical cluster, spherical in shape containing thousands of stars. The boundary of the cluster is not definitely marked, but the density clearly falls from the centre along the radius. Such clusters containing countless stars packed into a spherical or globular shape are called *globular clusters.* About a hundred of such clusters have been discovered; their distances are very great and are estimated by the known Cepheid variables occuring in them and by other indirect methods. These bodies are found to exist in space at distances ranging from 22,000 light years to 220,000 light years. From the known distance of a cluster and its apparent angular diameter, its actual size could be estimated. The diameter of a globular cluster is found to be of the order of several hundred light years.

A second type of star cluster (*open cluster*) is one like the cluster in Praesepe consisting of a group of stars with no regular boundary and no condensation at the centre of the cluster. Stars in this cluster are only very few in number when compared with the globular clusters.

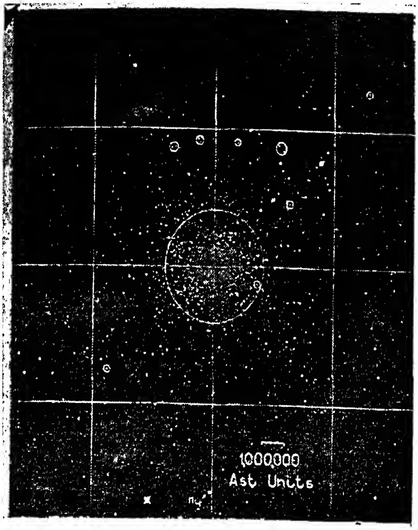


Plate No. 4.

M (3)—Globular cluster N. G. C. 5272 in Canes Venatici.

This beautiful cluster of stars would look like a nebula in a small telescope. A six inch telescope will resolve it into stellar points and in a big telescope, it is a wonderful object. Some of the stars marked are a special class of variables enabling us to estimate the distances of such clusters. These globular clusters are more than 20,000 light years away from us.

N. G. C. 6853.
The Dumb-bell nebula in Vulpecula.

This nebula is the largest of the planetary nebulae. It is clearly visible in a small finder and is an interesting object in an aperture over two inches. On each of the two opposite sides, a well defined curved outline corresponding to the grip of the dumb-bell is seen. With telescopes of large aperture, a few faint stars could be seen in the nebula.

Plate No. 5.

There is a third kind of cluster in which stars apparently are not so closely grouped together, but which show a definite physical connection in as much as they have a common motion of the same magnitude in the same direction. The stars in the Pleiades group come under this class. There is a similar cluster of about forty stars in Taurus, all near the sun and having a common motion. These open clusters and other clusters are situated at distances ranging from 400 to 16000 parsecs.

237. Nebulae.

A very important class of celestial objects occupying enormous space and at the same time looking like patches of cloud when seen through a telescope are known as *nebulae*. These objects are mainly gaseous in composition except for some condensation here and there and could not completely be resolved into distinct stars. That these are gaseous in composition, is inferred by the bright line spectra emitted by them, whereas stars like the sun give absorption lines due to the dense interior giving a continuous bright background. These objects fall into the following three subdivisions.

(i) *Spiral nebulae.*

These are the most numerous of all nebulae forming about a million objects in the sidereal universe. The most conspicuous spiral nebula is the one in the constellation Andromeda and could be seen as a faint patch of light on any clear moonless night by those gifted with a good vision. This was recognised as a nebula by observers, though not as a spiral nebula, even before the invention of the telescope. This like the other nebulae of the spiral type, consists of a bright central nucleus with spiral arms starting from opposite extremities of the nucleus and going round the nucleus more than one turn. There are some nebulae which are spiral in character but which do not show their

spiral character due to their being seen edge-on. These spiral nebulae are never found in or near the galaxy, and they are most numerous near the north galactic pole. The radial velocities of many of these spiral nebulae exceed 1000 *k. m.* per sec. and all these objects are receding from us with terrific speeds. Their distances, their velocities and their dimensions all go to confirm the hypothesis that the spiral nebulae are such that each forms a separate universe comparable to our universe of stars with the galaxy as the most crowded region in it.

Many spiral nebulaeshow evidence of angular motion relative to the nuclei and about an axis perpendicular to the plane of the spiral arms. From the length of the spiral arms and the velocity of rotation, the age and the period of rotation of the nebule, can be roughly estimated and the latter comes to be of the order of two hundred thousand years.

The spectra of this class of nebulae show absorption lines on a continuous background showing that these objects are not entirely gaseous; the lines are evidently due to the stars associated with these nebulae. But even the biggest telescope is not able to resolve a spiral nebula into individual stars. The distances of spiral nebulae are too great to be determined by trigonometric methods. Comparing the linear velocity of rotation of a spiral nebula with the angular velocity of rotation of a nebula that could be viewed broad-side on, the distance could be estimated. In the case of the spiral M. 101 this method gives a distance of 25000 light years. Another way of estimating their distances is from observations of new stars that appear in them. From their apparent magnitudes and the assumption that their absolute maximum brightness is the same as that of the new stars that occur in the galaxy, the distances of spirals can be obtained.

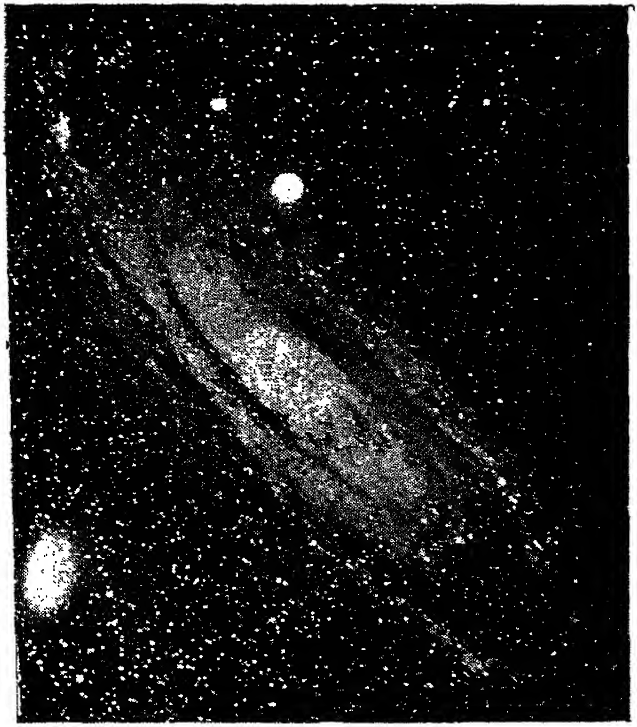


Plate No. 6.

M (31)—The great nebula in Andromeda.

This object is visible to the naked eye on moonless nights; small telescopes reveal an oval hazy mass. The wonderful spiral character and the size of the central nucleus could be seen in a large telescope with a high power. The diameter of the nucleus is 15 light years in length. The nebula is rotating with a period of about 80,000 years, and it is approaching us with a velocity of 180 miles a second. It is at an enormous distance of 900,000 light years from us. Its mass is $3{\cdot}97 \times 10^{41}$ grams and is 20 million times the mass of the sun.

(ii) *Planetary nebulae.*

A second class of nebulae that are of regular shape and outline having in many cases discs like planets are given the name planetary nebulae. About 150 members of this class have been known. Many of these nebulae show central condensation like the ring nebula in Lyra. These are found almost in the region of the milky way. Their distances are found by trigonometric methods, by observation of the central nucleus and are found to be of the order 50 parsecs. Unlike the spirals, planetary nebulae are comparatively near objects. The densities of these nebulae are of the order of a hundred times that of Neptune. Their masses vary from 4 to 150 times that of the sun. In many cases of this class of celestial objects, there is evidence of internal motion or rotation.

The spectrum of the planetary nebula resembles greatly that of a star of the Wolf-Rayet class, which is generally a stellar nucleus with a thin nebulous cloud around. Comparison of various planetary nebulae leads to a possible series of evolution of a planetary nebula without a distinct nucleus to one with a nucleus which then develops to a star with a thin nebulosity and finally to a Wolf-Rayet star of no nebulosity even.

(iii) *Irregular nebulae.*

These form a third class of bright or dark objects of no definite boundary or shape. These occur mainly in the milky way. The nebula in Orion and the star cloud in Sagittarius and the dark cloud in the Southern Cross are good examples of this class. In the milky way there are side by side with very crowded regions of stars, great rifts or holes of absolute darkness. These are due to the presence of dark nebulae between us and the remote stars of the milky way. The bright stellar points that occur in an irregular nebula form points of observation for

estimating the distances of these objects. Their distances are small compared with the spiral nebulae. The Orion nebula has a parallax of 0·005″ only.

The irregular nebulae contain plenty of hydrogen and helium, as they show the bright spectroscopic lines due to these elements. These objects have a small radial velocity.

238. Comets.

Comets are certain bright celestial objects with tails of different dimensions moving under the sun's gravitational influence. When they come near our earth, they become visible to us, increasing in brightness and also developing their tails as they approach us. These become fainter as they recede from us, and their tails also diminish not only apparently but also in actual size. Comets are perhaps masses moving in random directions in inter-stellar space, and when they come under the gravitational influence of any star such as the sun, they change their course to conic sections depending upon their initial velocities when they come under the influence of the star. Such a star will thereafter occupy the focus of the cometary orbit. The majority of the comets move in elliptic orbits of different eccentricities, some of the orbits being very elongated; their periods are also varying from a few years to hundreds of years. In physical constitution they are different from planets in as much as they are partly self-luminous and are comparatively very much less massive.

Families of comets.

The small density of comets makes them easily susceptible to the influence of the bigger planets in the solar system whenever they happen to be near enough to any such planet. In such a case, the planet controls the comet in such a way that the aphelion of its orbit is always near the orbit of the planet. Such a comet is then said to have become associated with the particular planet.

N. G. C. 3034. N. G. C. 4449.

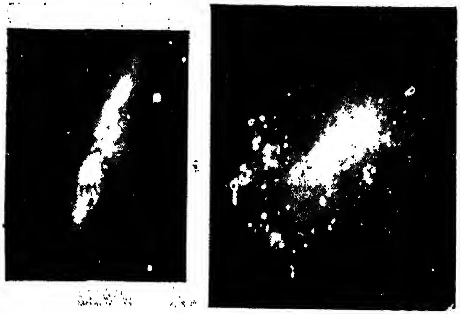


Plate No. 7.

These are typical objects of the class of irregular nebulae. They have no definite shape and extend to large regions of space.

(a) (b)

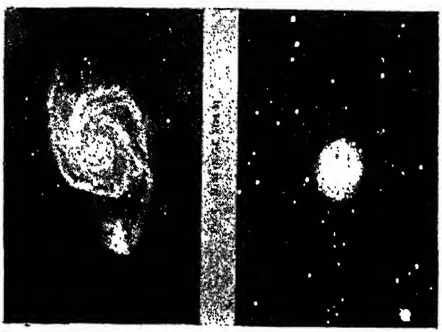


Plate No. 8.

(a) M (51) 5194-5 in Canes Venatici.

This 'whirl pool' nebula was the first to be recognised as a spiral. The spiral arms develop from opposite extremities of the central part rotating with a period of about 45 000 years.

(b) N. G. C. 7217.

This nebula shows a ring of stellar condensations with a central nucleus like Saturn, and is in a stage of development between a spiral and a planetary nebula. This is not in the ordinary course of evolution of rotating nebulae subject to tidal action. These will in course of time evolve to the stage of spirals.

There are in the solar system cometary orbits having their aphelia near the orbits of Jupiter, Saturn, Uranus and Neptune, the first having more than thirty members and the last about six. Halley's comet having a period of about 75 years belongs to the Neptune family.

Parts of a comet.

The chief parts of a comet are the head, the nucleus, and the tail. The head is a cloud of nebulous matter which becomes first visible to us, even when the comet is at an enormous distance. Though its shape is not very defined, it is either round or oval. The central portion of the head of a comet is much brighter than the rest and is called the nucleus. It is by observing this part that the position of the comet is determined. The third part is the tail of the comet streaming out of the head and keeping always away from the sun. The tail fades in brightness as it goes further and further away from the head. From photographic studies of comets it is understood that the tail grows in size as the comet approaches the sun. The tail seems to be consisting of small particles of matter ejected by the nucleus due to radiation pressure from the sun. As the pressure of radiation increases with the comet's approach to the sun, the dimensions of the tail increase. The comets are of varying dimensions, some very large and occupying a large amount of space and others comparatively smaller in size. The heads of comets sometimes are hundreds of times the size of the biggest planet. The lengths of the tails of some large comets are a few millions of miles and their volumes have magnitudes more than that of the sun. The masses of the comets are very small. No comet has perhaps more than $\frac{1}{100,000}$ of the earth's mass. This is the reason why no comet is able to cause any perturbation to the planets or their satellites though they pass very close to them in their course. The

density of comets should then be very small. In fact, stars can be seen through a comet's head very near the nucleus. The average density of matter in the head of a comet is smaller than that of the residual air in an air chamber exhausted by a good air pump. The average density of matter in the tail is still smaller. Hence even when the tail of a comet strikes against us, we will not be able to notice the least effect, though it is suggested by some that the gases in the tail may be poisonous.

Substances in comets.

A spectroscopic study of comets has shown that they consist of metals like sodium, magnesium and iron. There are also carbon compounds like cyanogen existing in the comets. The brightness of a comet is due partly to reflected sunlight and partly to electrical phenomenon similar to what takes place in a discharge tube.

239. Meteors.

Of all astronomical observations, the observation of meteors requires the greatest alertness, in as much as the whole phenomenon is seen in a few seconds The meteors come without any law or warning; but one fortunate thing about them is that they are so frequent in their occurrence that any one observing for about an hour in any part of the sky on a clear might could see about a dozen or more easily. These meteors are bodies of masses varying from a few milligrams to a few grams, which shoot across inter-stellar space and which become visible to us when they are made red hot as a result of friction with the atmosphere which they encounter in their rapid motion. It is only after their entering the earth's atmosphere that they begin to shine. Very small ones are consumed by the heat generated before they get very far down in the atmosphere. The larger masses, which appear not like shooting stars, but like fire falls, penetrate much farther and light

Plate No. 9.

The photograph is that of a meteoric trail. The camera is exposed towards the celestial pole and the circular lines are the apparent paths of stars, illustrating the rotation of the earth about the polar axis. The bright trail near the centre is that due to the pole star which is nearly $1^{\circ}-4'$ away from the pole.

up the whole landscape around them until their speed is reduced by the resistance of the atmosphere to such an extent as not to keep them glowing. Occasionally these bodies reach the earth as meteorites and the largest one so far obtained is a piece which weighs about 70 tons. It consists of about 17 % of nickel and 82 % of iron. Meteors also contain calcium, as has been shown specroscopically.

When the apparent path of a meteor has been recorded by more than one observer situated in different places, the actual length of its path, the height at which it was first seen and the height at which it disappeared can be calculated with precision. If the time of flight is also known, the speed relative to the earth is easily calculated, and from the known speed of the earth relative to the sun, the speed of the meteor relative to the sun is known. Such calculations have shown that on an average, shooting stars are seen from a height of 70 miles up to about 50 miles above the surface of the earth.

The average speed of these meteors which one can find in any direction at random, is found to be about 45 miles a second. This speed is such that a mass of this velocity cannot move in an elliptic orbit round the sun and therefore such meteors come from interstellar space and are moving in hyperbolic orbits.

Meteor showers.

There are other meteors which show themselves at definite seasons and at definite parts of the sky. All these belong to the solar system and move with smaller velocities, and they are found to be distributed along the orbits of comets known to us. This naturally leads us to the conclusion that they originally belonged to the comet in whose path they are at present found moving like a host of bodies. Now all these move in parallel directions. If one examines a few of them at a time when the

earth in the course of its annual revolution crosses their path, they show themselves in a bright meteoric shower. The appearence of such a shower for any observer would be the same as if all these are moving in arcs of great circles.

240. The zodiacal light.

This is a faint hazy band of light extending from the sun for more than 100° and seen on a clear night for a long time after sunset especially in February, March and April, as the ecliptic is then nearly perpendicular to the western horizon. It is also seen before sunrise in the Autumn. The portion of the zodiacal light near the sun is relatively brighter. The origin of this light is probably due to the diffusion of sunlight by small particles around the sun.. The spectrum of the zodiacal light shows some of the prominent lines of the solar spectrum. The total mass of this ring of matter should be very small indeed and its mean density should be considerably less than the density of the atmosphere at a height of 100 miles. If the density had been higher, comets could never get past the sun; on the other hand, they would be reduced to nothing due to the heat generated while passing through such a cloud of matter.

The Aurora Borealis.

Occasionally. the sun sends out streams of electrified particles, which on reaching the earth give rise to magnetic storms and displays of Aurora Borealis. This phenomenon is due to the glowing of some rare gases high up in the atmosphere. They are very common in high latitudes.

Plate No. 10.

Plate No. 11.

The zodiacal light.

This photograph of the zodiacal light is taken by a special lens of great light-gathering power but defective in other ways. The distorted images are those due to the stars.

CHAPTER XIX.

THIRTY CONSTELLATIONS

241. Aries.

Aries, the Ram lies northeast of the constellation Pisces. It was the leading sign of the zodiac, as the intersection of the ecliptic and the equator once occupied this sign; but owing to precession this point is now well advanced into the sign Pisces.

In mythology, Aries represented the fabled Ram with the golden fleece, which at the behest of god Mercury carried Phryxus and Helle from their angry mother over land and sea. On the way, Helle lost her hold and fell into the sea while Phryxus safely arrived at Colchis on the eastern side of the Black Sea. To show his gratitude, Phryxus sacrificed the Ram and gave the golden fleece to the king of the country.

The Babylonians adopted this constellation into the zodiac when its stars commenced to mark the vernal equinox. The Jewish people were delivered from the bondage of Egypt when the sun was in this sign and so the Jewish month Nisan was associated with this constellation. The Egyptians named Aries "The Lord of the Head" and for the civilised nations of the East, the year commenced when the sun and moon entered Aries.

The brightest star Alpha of this constellation (Hamal) is at a distance of 74 light years. It has an apparent magnitude of 2·2 and an annual proper motion of 0·242 sec., and is moving towards us with a velocity of 14·3 *k. m.* per second. Beta Aries is 34·3 light years distant, and has a magnitude of 2·7. It is a spectroscopic binary, having a period of 107 days. Gamma Aries is a binary having magnitudes of 4·2 and 4·4. Both the components

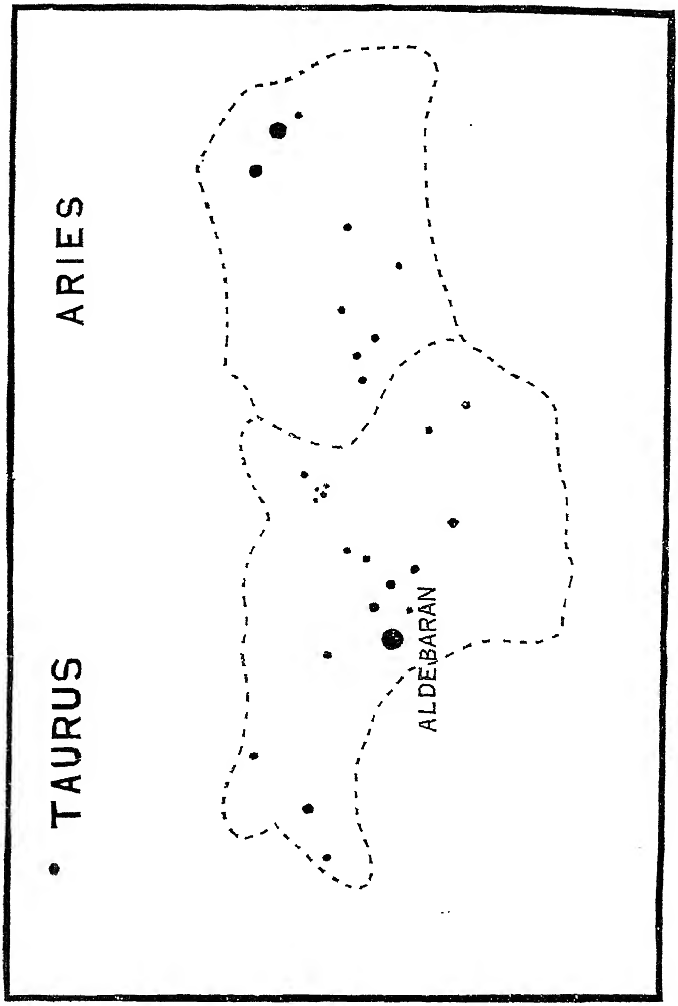


Fig. 129.

have the same proper motion (0·136 sec.), and the same radial velocity of approach (1·2 *k. m.* per sec.).

242. Taurus.

Taurus, the Bull, is the second sign of the zodiac. It is bounded by Gemini on the east, Aries on the west,

Auriga on the north and Orion on the south. It should have marked the vernal equinox from about 4000 B. C. to 1700 B. C. On the walls of a sepulchre excavated at Thebes, Taurus is shown as the first sign of the zodiac. Apis, the bull of Egypt was regarded as the living representation of the zodiacal bull.

In Greek mythology, this is the bull that carried away Europa, daughter of Agenor, from Phoenicia to Crete. Jupiter, struck by the beauty of Europa, assumed the shape of a snow-white bull and attracted Europa to sit on its back.

Alpha Tauri or Aldebaran is a first magnitude star and it is the 'Al Dabaran' or 'the leader' of the Pleiades. Its spectrum shows the existence of hydrogen, sodium, magnesium, calcium, iron, bismuth, tellurium, antimony and bismuth. It is 43 light years distant and has a diameter of 33,000,000 miles. Its mass is ten times greater than that of our sun and thirty-six times as bright.

Zeta Tauri is a spectroscopic binary having a period of 138 days. Lambda Tauri is a variable (magnitude 3·3 to 4·2) of period 3 days 22 hours 9 minutes.

The Crab Nebula is an irregular nebula in this constellation having a length of 5 minutes. It is situated a little to the north-west of the southern horn of the Bull.

Hyades and Pleiades, the two remarkable star clusters and N. G. C. 1647, another small cluster lie in this constellation. The Hyades cluster is shaped like the letter V, having Aldebaran at the top of the left hand branch. According to Greek mythology, the Hyades is supposed to represent the daughters of Atlas. Their brother was killed by a wild boar and being unable to restrain their grief they pined away and died. They were changed into stars and placed in Taurus.

Hyades has been classed in recent days, in the group of what are called 'moving clusters'. All the members are travelling towards some common region with equal velocities along parallel paths. It has been discovered at the Yerkes Observatory that they are receding from our system. About 39 stars are found in the cluster. Of these, fourteen have been classed as stars of the 5th magnitude while 20 others were found to be spectroscopic binary systems. All these stars are much brighter than the sun.

The name 'Pleiades' is derived from the Greek word 'plein' to sail. Among the Greeks, the opening and ending of the season of navigation were marked respectively by the heliacal rising and setting of this cluster. No other group of stars is of more scientific and historic interest than this. In the Greek mythology the Pleiades were the seven daughters of Atlas, the giant. The legend goes to say that these seven maidens along with their sisters, the Hyades, were transformed into stars on account of their virtues and mutual affection. Their names were Alcyone, Merope, Maia, Electra, Taygeta, Sterope and Celaeno. Of these, all except Merope have some of the gods for their suitors, and Merope married the king of Corinth. Hence Merope alone is dim and appears less bright than the rest.

The cluster is one degree in length east and west and two-thirds of a degree in breadth. A three inch refractor will show as many as eighty or ninety stars. Professor Bailey of the Harvard Observatory counted nearly 4000 stars within a small area of two degrees square, upon a negative exposed to the Pleiades group. But all these stars should not be taken to be physically connected with the Pleiades cluster. It is quite possible that most of them are situated at large distances beyond it.

243. Gemini.

Gemini, the Twins comes third in the zodiac. Castor and Pollux are the two chief stars. They are called the Twins. In mythology, they were the twin brothers, sons of Jupiter by Leda, the wife of the King of Sparta.

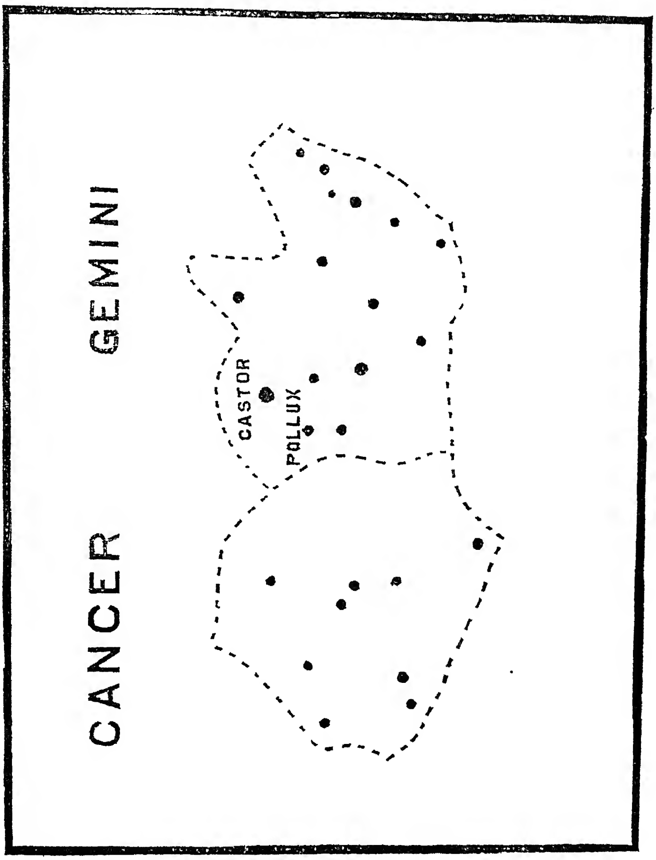


Fig. 130.

They sailed with Jason in quest of the golden fleece, at which they distinguished themselves by their heroism. In appreciation of their heroic deeds, Jupiter changed them both into a constellation.

α Gemini or Castor is one of the most interesting of the binary stars in the northern sky. The magnitudes of the components are 2·5 and 3·5 respectively. Their annual proper motion is 0·203 sec. and they revolve about their common centres of gravity in a period of 347 years. As a result of the spectroscopic researches carried out by the Russian Astronomer Belopolski, and Professor Curtis, Castor is found to be one of the most remarkable of quaternary systems. The two components of Castor are each accompanied by one invisible companion.

β Gemini or Pollux has a magnitude of 1·2 and is at an apparent distance of 4 degrees and 30 minutes from Castor. This distance was taken by the Arabs as a unit of length and was called the 'Ell.' It has an annual proper motion of 0·624 sec. and has an actual diameter of 13,000,000 miles. Its distance is 34·3 light years and its mass is nearly five times that of the sun.

There are two clusters in the constellation. N. G. C. 2168 has an angular diameter of about 50 minutes and is at a distance of 24 light years. N. G. C. 2392, which is a planetary nebula has an angular diameter of 25 seconds and is at a distance of 270 light years.

244. Cancer.

Cancer, the Crab is the fourth sign of the zodiac. This is considered to be the crab that caught hold of the foot of Hercules during his fight with the snake in the Lernaean Marsh. In commemoration of this, Juno placed it in the sky.

α Cancer marks the southern claw of the crab. Its parallax is 0·03 secs. and this corresponds to a distance of 108 light years.

Zeta Cancer is a quaternary system. The primary and its companion are of magnitudes 5 and 5·7 respectively and revolve round their common centre of gravity in a period of 60 years. The third retrogrades round the

first and the second in a period of 600 years. Their combined mass comes to nearly nine times that of the sun.

N. G. C. 2632 is a beautiful cluster lying on the head of the Crab, surrounded by four stars—Gamma, Delta, Eta and Theta. This cluster appears as a hazy mass of light to the unaided eye, but powerful telescopes resolve it into more than 500 stars.

245. Leo.

This constellation is the fifth sign of the zodiac. According to the Roman Fable, this was the lion which

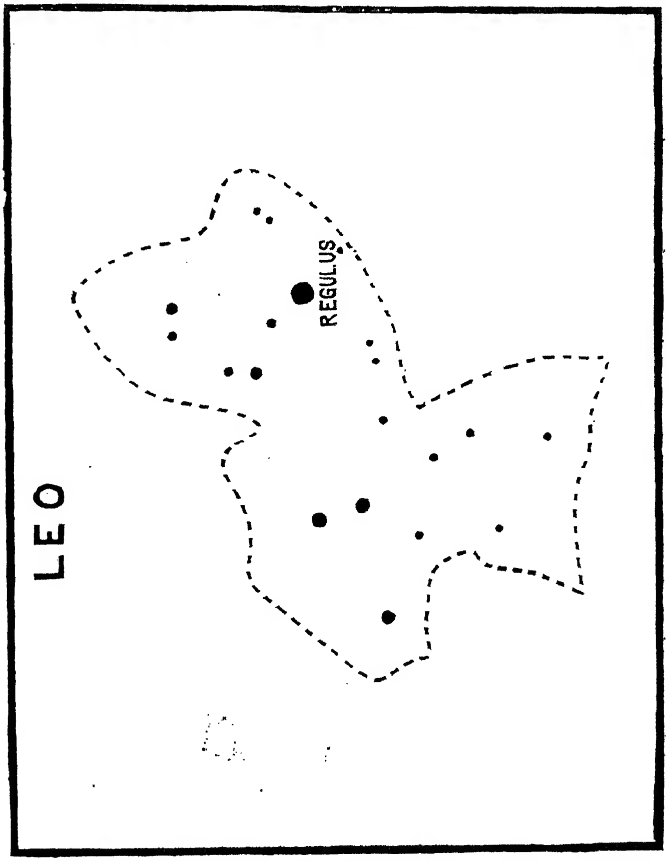


Fig. 131.

haunted the forest of Nemea and which was finally killed by Hercules.

Alpha Leo is called Regulus. It was one of the four royal stars of the Persian Monarch, the four guardians of the heavens. They are Regulus, Aldebaran, Antares and Fomalhaut. Regulus has a visual magnitude of 1·3 and it has a companion of magnitude 7·6 moving with it. Its diameter is 5 times that of the sun and it emits 250 times as much light as the sun. Its parallax is ·033 sec and it corresponds to a distance of 99 light years.

Gamma Leo is one of the finest binaries. The components have the magnitudes 2·6 and 3·8 respectively and their angular separation is 3·5 seconds.

246. Virgo.

It is the sixth sign of the zodiac. It is a long constellation bounded by Libra on the east, Leo on the west, Bootes on the north and Corvus on the south.

According to mythology, virgo, the virgin was Astraea the daughter of the king of Arcadia. She lived upon the earth in its golden age but was forced to flee away from the earth, by the wickedness of mankind. After her return to heaven she was placed in the heavens as one of the zodiacal constellations.

Alpha Virgo is called Spica. It has a magnitude of 1·2 It is a spectroscopic binary. It has a parallax of 0·014 sec. which corresponds to a distance of 232 light years.

Gamma Virgo is also a remarkable binary. The magnitude of each component is 3·6 and their angular separation varies from 0·5 sec. to 6 secs.

There are a large number of nebulae situated in the head and breast of the virgin and so this constellation is sometimes called the 'field of the nebulae.' The three prominent nebulae are N. G. C. 4526, N. G. C. 4594, and N. G. C. 4649.

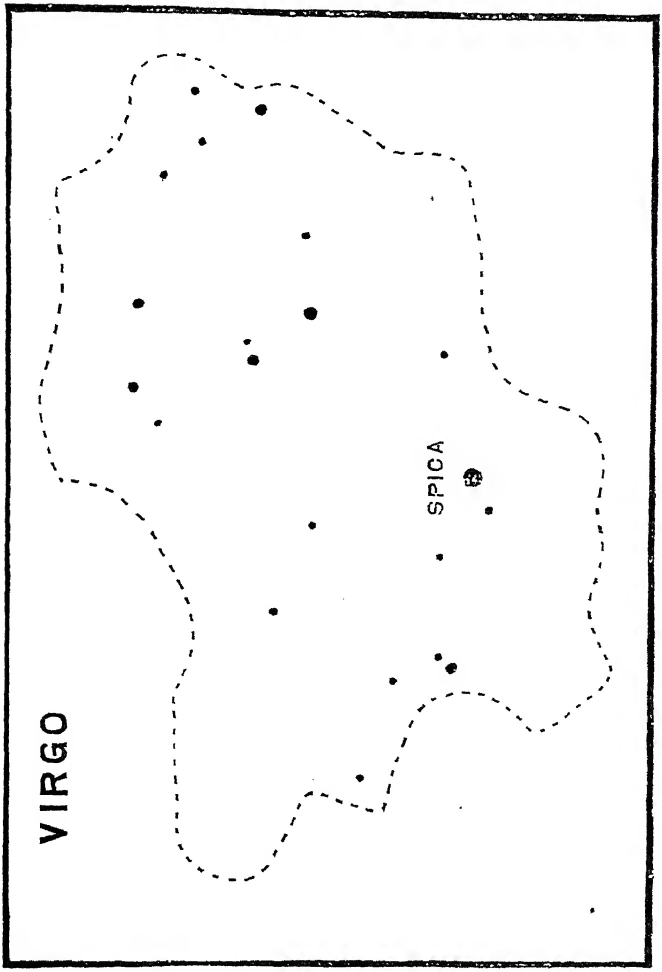


Fig. 132.

247. Libra.

It is the seventh of the zodiacal signs. Originally, it represented the balance of the sun at the horizon between the upper and under world. It is the first autumnal sign and so it also denotes the equality of the days and nights at the equinox.

Alpha Libra is a double star, the magnitudes of the components being 2·9 and 5·1 respectively. Its distance is calculated to be about 72 light years.

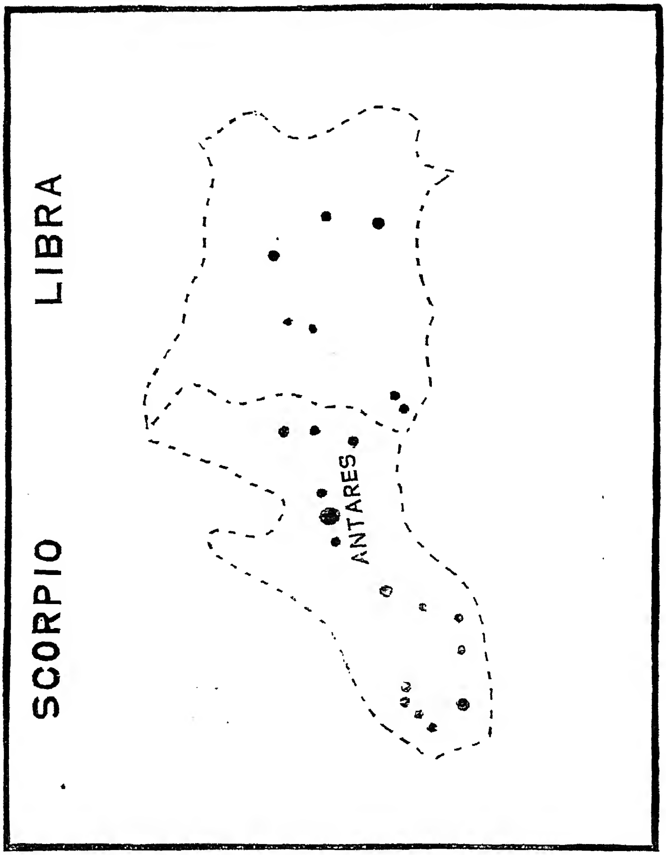


Fig. 133.

N. G. C. 5904 (M5) is a beautiful globular cluster lying just above the beam of the balance not far from Beta. Its apparent diameter is 15 minutes. Its parallax corresponds to an actual distance of 41,000 light years.

248. Scorpio.

Scorpio is the eighth sign of the zodiac. It is one of the few constellations that bear a resemblance to the object

after which it is named. A curved line of bright stars marks the head and claws of the Scorpion. According to Ovid, the Scorpion came out of the earth at the command of Juno and killed the mighty hunter Orion.

Alpha Scorpi or Antares is a red star resembling Mars. It is also a binary having components whose magnitudes are 1·2 and 7 respectively. The angular separation is 3·5 secs. Its distance is estimated to be 251 light years. Beta Scorpi is also a spectroscopic binary.

N. G. C. 6093 is a bright globular cluster lying halfway between Alpha and Beta. Its actual distance is 65,180 light years.

N. G. C. 6231 is another open cluster lying between Mu and Zeta Scorpi. This cluster is composed of about 200 stars, having magnitudes ranging from 6 to 8. Its distance is 2,037 light years.

249. Sagittarius.

Sagittarius, the Archer is the ninth constellation in the zodiac. There are eight stars forming two quadrilateral figures.

Beta is a double star, having components of magnitudes 4·2 and 4·4 respectively. Delta is also a double star, having magnitudes 2·8 and 14·5 respectively.

In this constellation, there are several milky way clouds containing gaseous nebulae and star clusters. N. G. C. 6822 is a nebula of length 20 minutes and width 10 minutes. There are about 750 stars, and the distance of the nebula is estimated to be about 700,000 light years.

N. G. C. 6523, which is called the 'Lagoon Nebula' is a splendid object, visible even to the naked eye. Its distance is estimated to be about 1600 light years.

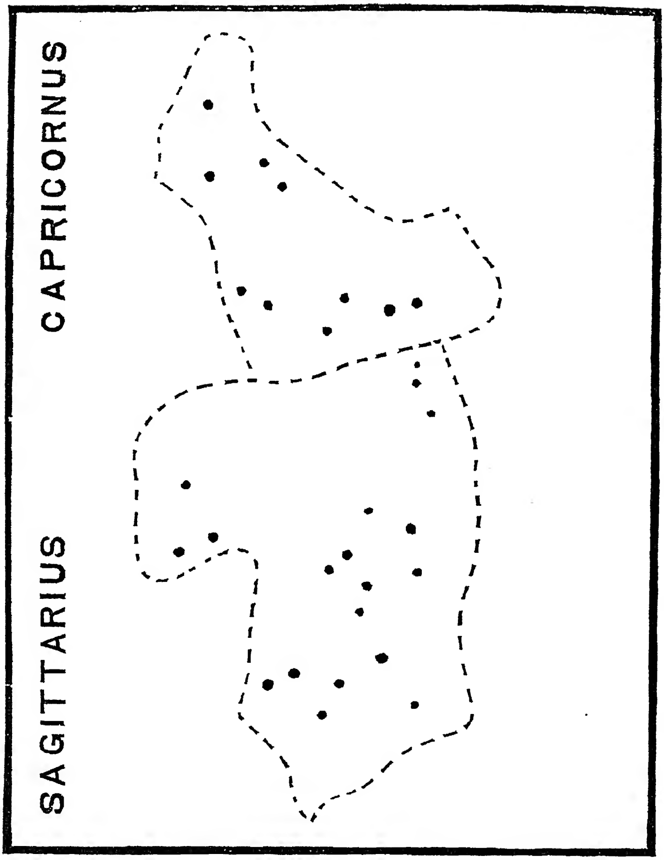


Fig. 134.

N. G. C. 6864 is a large, bright globular cluster, strongly condensed in the centre and falling off towards the edge. This cluster is estimated to be at a distance of 146,575 light years.

250. Capricornus.

This is the tenth sign of the zodiac. Alpha is a wide double star. Alpha 1 is a double star having magnitudes 3·2 and 4·2 and Alpha 2 is found to be a triple, having magnitudes 3,11·5 and 11·5 respectively.

N. G. C. 7099 and N. G. C. 6981 are two clusters having diameters 4 minutes 6 seconds and 2 minutes 4 seconds respectively.

251. Aquarius.

Aquarius is the eleventh of the zodiacal constellations. Since the sun passes through it during the rainy season,

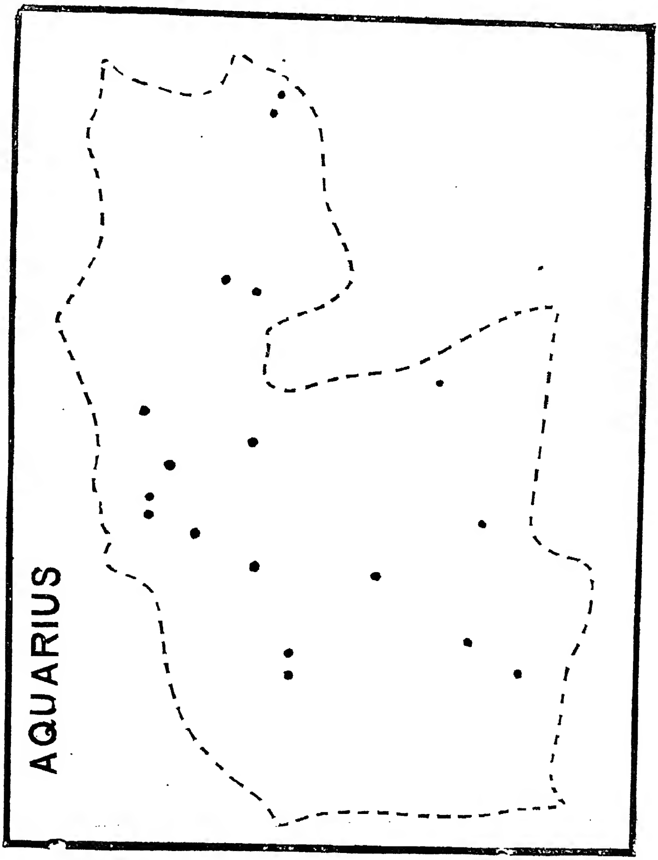


Fig. 135.

it is named Aquarius or the waterman. The Egyptians believed that the flooding of the Nile was caused by the water-bearer sinking his vessel into the fountains of the

river to refill it. So Aquarius meant for the Egyptians, the rainy season.

Alpha is a pale yellow star of magnitude 3·2, having a companion of the 11th magnitude. Its distance is estimated to be about 326 light years.

Zeta is a binary whose magnitudes are 4 and 4·1, Their angular separation is 3·3 seconds.

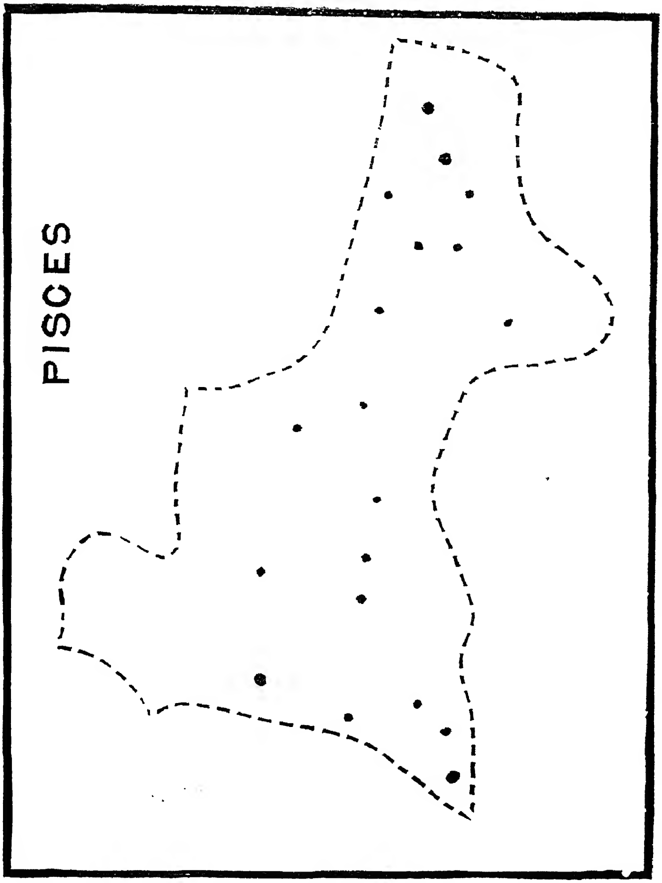
N. G. C. 7009 is the Saturn Nebula found a little to the west of Nu Aquari. Its diameter is about 20 seconds. N. G. C. 7089 is a globular cluster having an apparent diameter of 7 minutes. Its actual distance is estimated to be about 50,856 light years.

It was in this constellation, that Mayer observed in 1756, what he took to be a fixed star. A few years later William Herschel took the star to be a comet, but soon he discovered that it was a new planet which was named Uranus afterwards.

252. Pisces.

This is the last of the zodiacal signs. Though it does not contain any bright star, it is quite important, as the Vernal Equinox lies in it. According to Greek mythology, the story goes to say that when Venus and Cupid were walking along the banks of the river Euphrates, Typhon, the giant frightened them by suddenly appearing before them. To escape from him, Venus and her son jumped into the river and assumed the form of fishes. These two fishes were placed among the stars by Minerva, just to commemorate the event.

Alpha is a fine binary star of magnitudes 4 and 5·5 respectively. Their angular separation is 3 seconds. Its distance is estimated to be 84 light years.



Fig, 136.

253. Aquila.

This is situated near Capricornus and Aquarius.

Alpha or Altair is a white star of magnitude 0·89. Its distance is 14·5 light years and its parallax is 0·232 sec.

Nova Aquilae appeared on the 8th June 1918 in this constellation. In March 1919 it appeared as an ordinary star of the 6th magnitude and five months later, it was found to have developed into a planetary nebula.

N. G. C. 6705 is an open cluster of diameter about five minutes. It consists of nearly 200 stars and is visible to the naked eye on a clear moonless night.

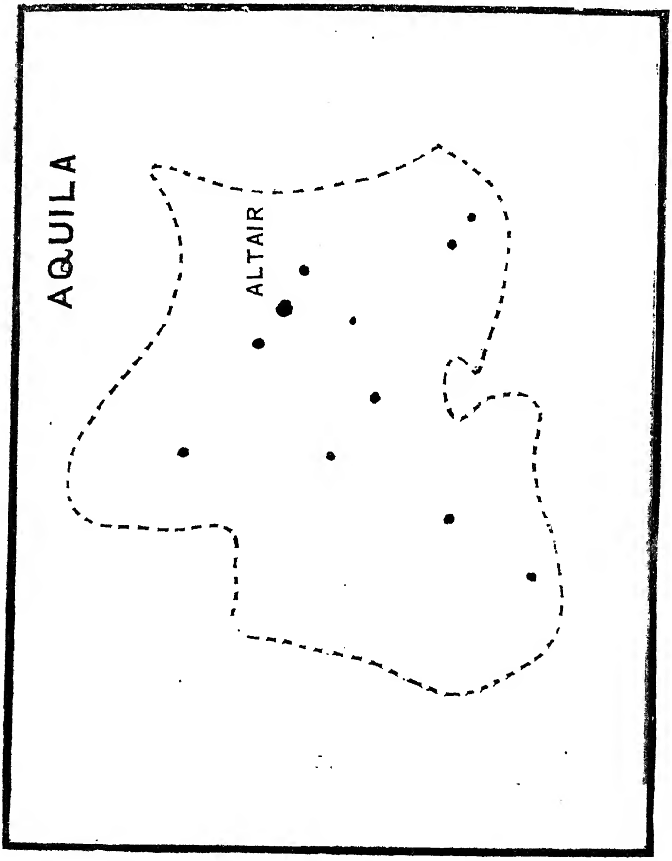


Fig. 137.

254. Auriga.

Auriga is situated midway between Perseus and Ursa Major. Auriga 'the Charioteer' is represented as a young man seated on the milky way carrying a goat on his shoulder and two kids on his left hand. The bright star Capella is the goat and the two stars Eta and Zeta are her kids.

Capella with Arcturus and Vega form the most brilliant trio in the northern hemisphere. Alpha or Capella has an apparent magnitude of 0·2 and has a parallax of 0·071 sec. corresponding to an actual distance of 46 light years. The spectrum of the star is similar to that of the

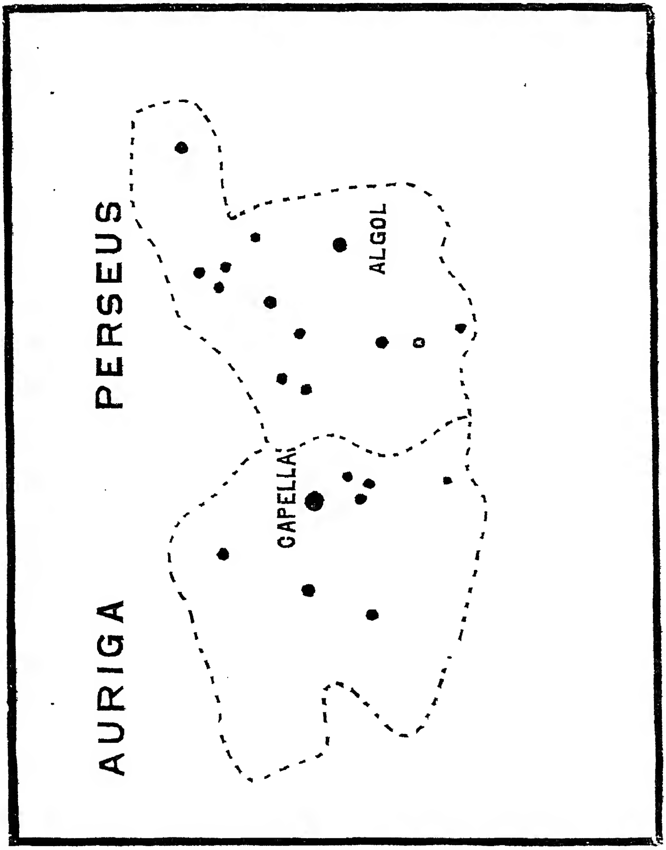


Fig. 138.

sun and so it is supposed to be identical with the sun in physical constitution. It is a spectroscopic binary having a revolution period of 104 days.

N. G. C. 2099 is a beautiful cluster of stars. Its angular diameter is 37 minutes. When seen through a

powerful telescope, the cluster is resolved into hundreds of stars of magnitudes varying from 10 to 14.

255. Perseus.

Perseus lies in the brightest part of the milky way. The curved line of stars extending from Cassiopeia to Capella is known as the 'Segment of Perseus'.

Alpha has a magnitude of 1·9 and has a parallax of 0·017 sec. corresponding to a distance of 192 light years. It is a spectroscopic binary, having a period of 4 days.

Beta or Algol is an eclipsing variable star. The star remains at magnitude 2·3 for two and half days, and in the next four and a half hours declines to magnitude 3·5. In the next four and a half hours it returns to its former lustre. These light fluctuations are due to the presence of a satellite which periodically comes between the star and the earth and cuts off a part of its light. It was found that the interval between successive light minima were not constant. Further investigations as to the cause of this irregularity revealed that there was a third body in the system which was revolving with Algol and its companion in a circular orbit.

In the morning of 22nd February 1901, a bright nova was discovered in this part of the sky by Dr. Thomas Anderson. On February 23rd it attained its maximum brightness and then it began slowly to decline in brightness until at last it became invisible to the naked eye on August 1st.

N. G. C. 869 and 884 are two magnificient clusters, visible to the naked eye as misty patches. The apparent diameter of each is 45 minutes.

256. Cygnus.

Cygnus lies in the milky way surrounded by the constellations Draco, Cepheus, Pegasus and Lyra.

Alpha is called Deneb meaning the 'hen's tail'. It is a white star of magnitude 1·3 and is at a distance of 543 light years.

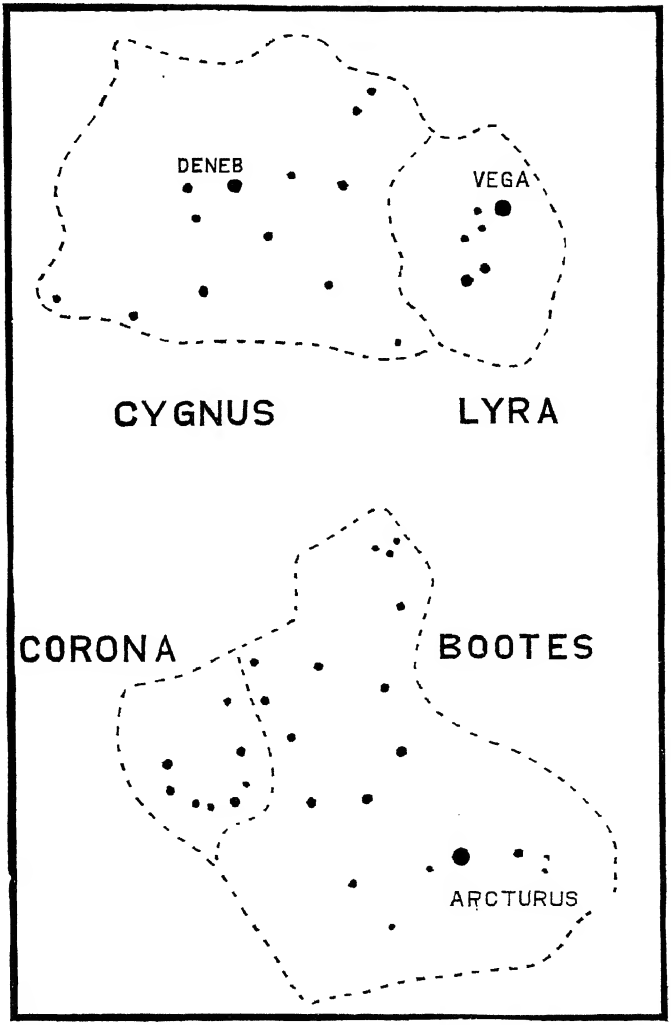


Fig. 139.

Beta is a beautiful double star. One is pale blue while its companion is bright orange and on account of

this contrast in colour it is an interesting object to be seen through a telescope.

61 cygni is an interesting binary. It is at a distance of 10 light years from us. The magnitudes of the components are 5·3 and 5·9 respectively.

257. Lyra.

Lyra lies between the constellations Hercules and Cygnus.

Alpha Lyra is called Vega. It has a bluish tinge and it is the lucida of the constellation. Its magnitude is 0.1. It has a companion of the tenth magnitude, which is also of the same bluish colour as Vega.

The wonderful Ring Nebula in Lyra is situated at about one third the distance from Beta to Gamma. It is elliptic in appearance, having a faint star in the centre surrounded by a mass of nebular matter. Its distance is estimated to be about 1100 light years.

258. Corona Borealis.

It is called the 'Northern Crown'. It appears in the form of a semi-circle of six bright stars, a little to the north-east of the bright star Arcturus.

Alpha has a magnitude of 2·3 and has a parallax of 0·062 sec. corresponding to a distance of 53 light years. This is a spectroscopic binary, having a period of 17·36 days.

Beta is also a spectroscopic binary having a period of 490 days.

259. Bootes.

Owing to its proximity to the Great Bear, this constellation is often alluded to as the "Bear-keeper".

Alpha Bootes or Arcturus is one of the most brilliant stars in the northern hemisphere. It has a magnitude of 0·2, and is at a distance of 43 light years. It is a spectroscopic binary having a period of 211·9 days.

Xi was one of the first binary stars discovered by Herschel in 1780. The magnitudes of the components are 4·8 and 6·8 respectively. Its distance is estimated to be equal to 14.2 light years.

260. Ursa Major.

Uraa Major or the 'Great Bear' is the most conspicuous of all the constellations in the northern hemisphere. It is more commonly called by the name 'The plough'. Three stars in a line form the handle and four stars form the remaining portion of the plough. A line drawn through Alpha and Beta passes through the pole star and hence these stars are called 'the pointers.'

Alpha is a binary whose components have magnitudes 2 and 11 respectively. Beta is a spectroscopic binary.

Zeta, the middle star in the tail of Ursa Major is one of the most remarkable binary systems. It has a naked eye companion also. When a telescope is directed towards the two stars, four stars are seen in the field of view.

261. Ursa Minor.

This is similar in shape to the 'Great Bear'.

Alpha or Polaris marks the north celestial pole. This star is not exactly at the north celestial pole but it is nearly 1 degree and 6 minutes away from the pole. Owing to precession, Polaris will be changing its position relative to the north pole and a few centuries hence, it may not at all be near the north pole. Polaris is found to be a ternary system of three stars. It is at a distance of 79 light years.

Gamma is a double star whose components are of magnitudes 3·1 and 5·8 respectively. The angular separation of the components is 57 minutes.

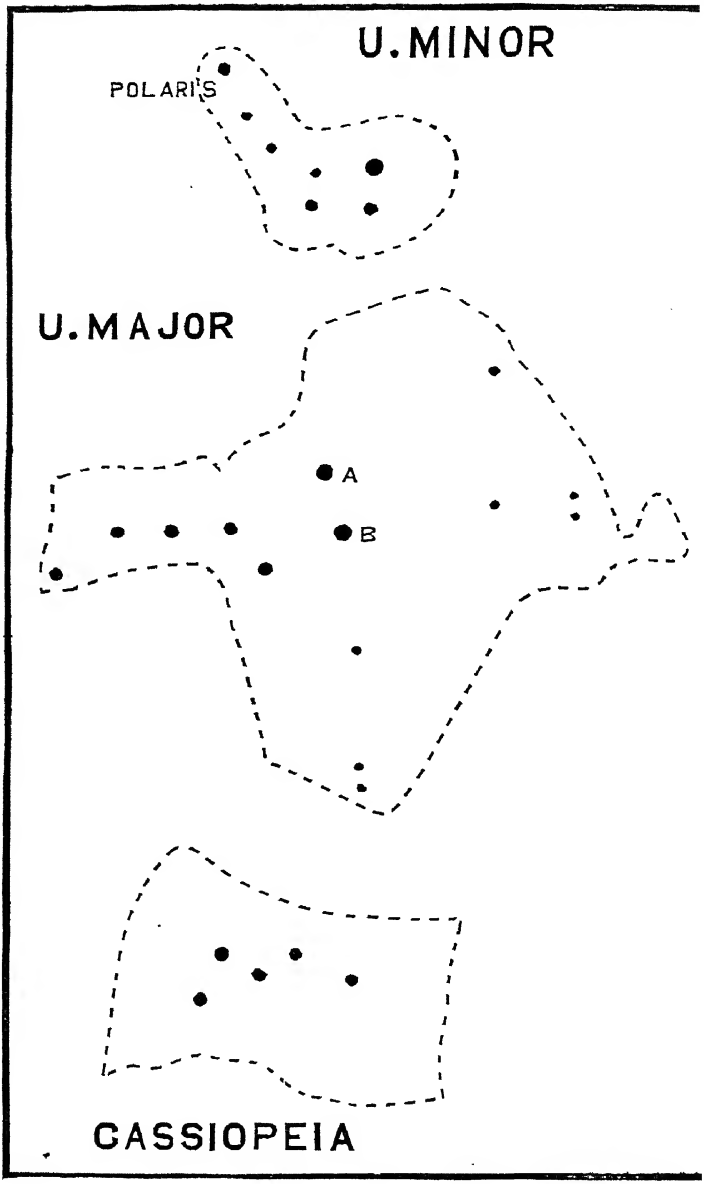


Fig. 140.

262. Cassiopeia.

This constellation presents the appearance of a distorted W. Leaving one faint star out of consideration, the whole constellation will look like a chair. It was known to the Romans as "The woman in the chair." In mythology, Cassiopeia, the mother of Andromeda is supposed to be chained to her seat to prevent her from falling out in the process of circling round the pole.

Alpha of this constellation marks the front leg of the chair. It varies in brightness from magnitude 2·2 to 2·8. It is estimated to be at a distance of 81 light years.

Beta marks the back of the chair. It has a magnitude of 2·4 and is at a distance of 44 light years. Gamma is a binary whose magnitudes are 2 and 11 respectively. Its parallax is ·022 sec. which corresponds to a distance of 148 light years.

A brilliant nova which appeared in this constellation on 11th November 1572 was observed by Tycho Brahe. It reached a brightness equal to that of Venus at her greatest brilliancy and then gradually began to decline until at last in March 1574, it disappeared.

N. G. C. 7789 is a large cloud of minute stars situated near Delta Cassiopeia. Its angular diameter is 20 minutes and its distance is estimated to be 9758 light years.

263. Pegasus.

This is a fairly large constellation. It is very easy to locate in the sky the square of Pegasus, having in one of its corners Alpha Andromeda.

Mythologically, Pegasus is the horse which sprang from the blood of Medusa, when her head was cut off by Perseus. It was placed among the constellations by Jupiter.

Alpha has a magnitude 2·5 and is at a distance of 90 light years. Kappa is a fine binary whose components have magnitudes 4·8 and 10·8 respectively. One of its components is a spectroscopic binary, having a period of six days.

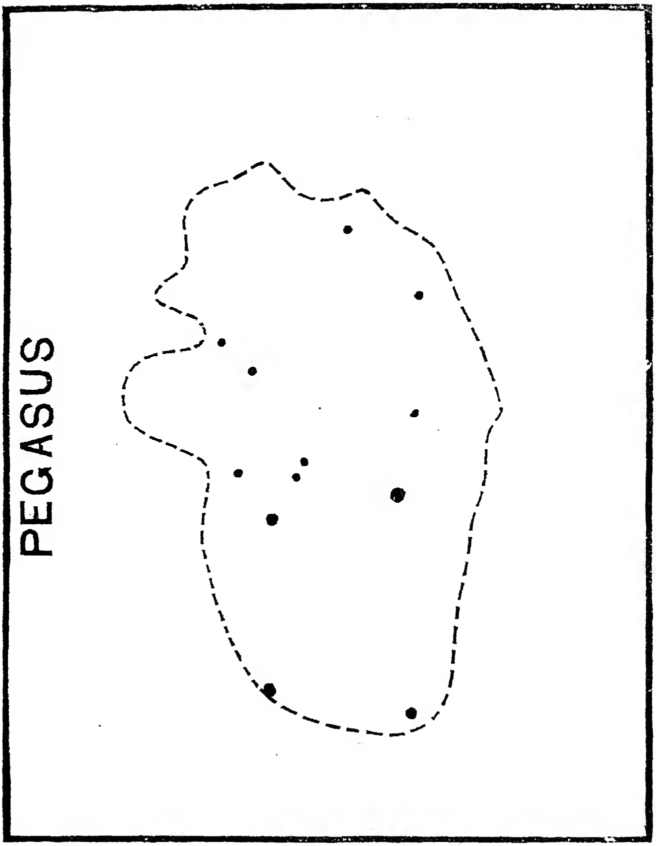


Fig. 141.

N. G. C. 7078 is a globular cluster having an apparent diameter of 6 minutes. Its distance is estimated to be 47,922 light years.

264. Crux.

This constellation, more commonly known as the 'Southern Cross' is a small group of stars lying near the

south celestial pole. It consists of four bright stars looking like a cross. The two stars which are at the top and foot of the cross have almost the same right ascension and so when these stars come to the meridian the line joining the other two will be perpendicular to the meridian. So, knowing the times at which this line looks perpendicular to the meridian during the various seasons, it is easy to estimate the time by observing the position of this constellation. Therefore it is sometimes referred to as the "Southern Celestial Clock".

Alpha is a triple star whose magnitudes are 1·5, 2, and 6 respectively. The angular separation of the two large stars is 5 seconds while the third one is at a distance of 60 seconds from the brightest.

265. Centaurus.

This is one of the largest constellations having a length of about 60°.

Alpha is the fourth brightest star in the heavens, and the second nearest neighbour in the stellar system. It is composed of two brilliant suns, one having a magnitude 1 and the other 1·5. Its distance is only 4·3 light years.

Beta has a magnitude of 0·9, and is found to be at a distance of 78 light years.

N. G. C. 5139 (Omega centauri) is a big globular cluster, though it is visible as a single star to the naked eye. The angular diameter of the cluster is 31 minutes, and its distance is estimated to be 21190 light years.

266. Piscis Australis.

This constellation lies south of Capricornus and Aquarius It is important, as it contains the bright star 'Fomalhaut' which is considered to be one of 'the four guardian stars of the heavens'. Alpha Piscis or Fomalhaut has a magnitude of 1·3.

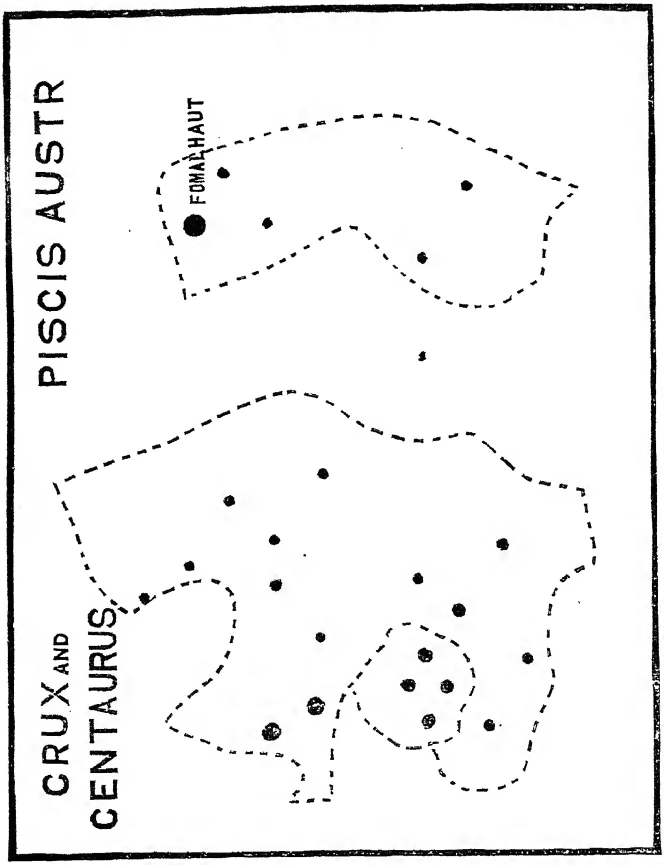


Fig. 142.

267. Canis Major.

This is a constellation lying to the southeast of Orion. It is important as it contains 'Sirius', the brightest star in the heavens. In mythology, it is supposed to represent the dog that Laelaps presented to Cephalus. Jupiter was so pleased with the swiftness of the dog that he transferred it to the sky.

Alpha Canis Major or Sirius is three times as bright as a first magnitude star. Sirius has been found

to be a binary star, having a companion of the 9th magnitude. Though Sirius was suspected to be a binary even from early times, the fact that it was a binary was

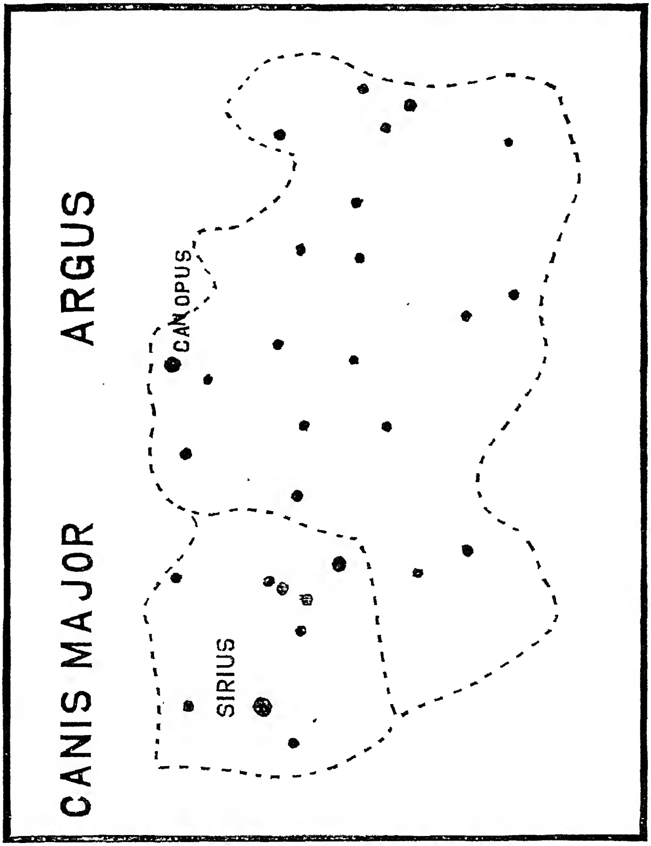


Fig. 143.

openly revealed only in February 1862, when the 18 inch telescope which was being built for the Chicago Observatory was turned on to it casually.

268. Argus.

This is a large constellation extending over a length of about 75° of the heavens. It is supposed to represent the

ship 'Argo' which Jason used in his expedition for recovering the golden fleece.

Alpha Argus or Canopus is the second brightest star in the sky. It has a magnitude of 0·86. This great sun is also accompanied by another of magnitude 3·5.

Eta Argus is one of the most remarkable variable stars in the sky. In 1677 it was a star of the 4th magnitude. In 1837, it was as bright as Sirius and afterwards it declined in brightness to a star of magnitude 7·8.

N. G. C. 2447, N. G. C. 2808, and N. G. C. 3532 are three bright clusters having angular diameters of 30 minutes, 5 minutes, and 65 minutes respectively.

N. G. C. 3372 is a great nebula surrounding Eta. It has been called the 'Keyhole Nebula' owing to its significant features.

269. Orion.

Orion is one of the most brilliant constellations in the heavens. This constellation is pictured as a giant. The belt of the giant is marked by three stars lying in a row at equal distances of about a degree. A line drawn through them passes through Sirius. Below the belt, there is a fine sword which is also marked by a small row of three stars. The great nebula in Orion is situated near the central star in this row.

Alpha or Betelgeuse has a magnitude of 1·2 and is at a distance of 181 light years. This was the first star whose diameter was measured by the interferometer. This star is found to be a spectroscopic binary, having a period of 6 years.

Beta or Rigel is a bluish white star, having a magnitude of 0·3. Its distance is estimated to be 465 light years. Besides being a double star, it is also a spectroscopic binary.

N. G. C. 1976 is the famous "Nebula in Orion" and it is one of the most wonderful objects in the heavens. A telescope would reveal four stars forming a trapezium in

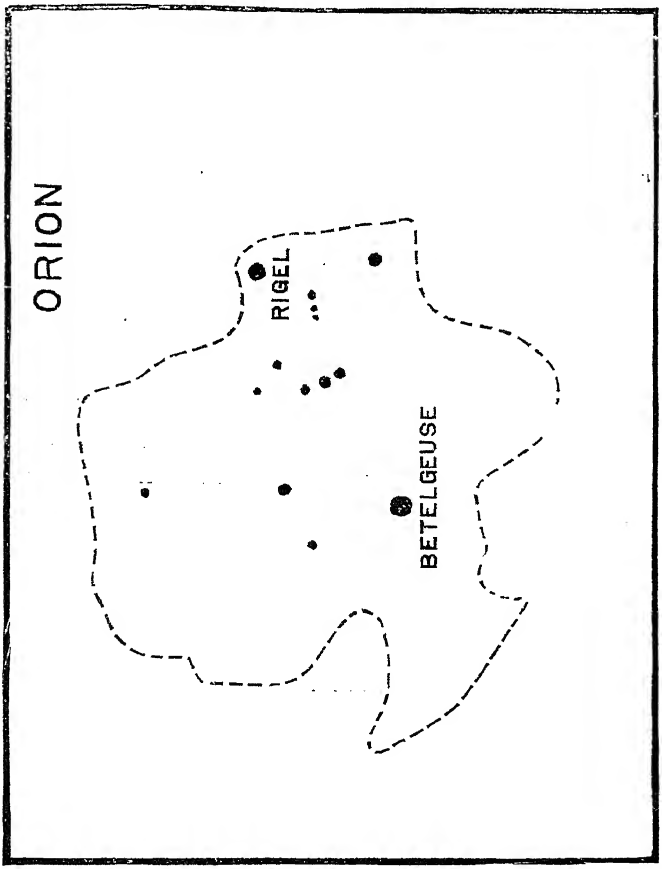


Fig. 144.

the middle of this nebula. The nebula is nearly two-thirds of a degree in diameter, and its distance is estimated to be nearly 1000 light years. A spectrosopic study of the nebula has shown that it is composed of various gases, shining by inherent light.

270. Eridanus.

Eridanus is a big constellation lying south of Taurus and extending over a length of sixty degrees from Orion in the east to Cetus in the west. This was one of the most ancient constellations and the Egyptians considered this to typify their sacred Nile.

Alpha or Achernar is a brilliant star, having a visual magnitude of 0·6. Its distance is estimated to be 58 light years.

Beta has a magnitude of 2·9 and is at a distance of 69 light years.

Omicron forms a beautiful ternary system. The magnitudes of the components are 4·5, 9·4 and 10·8 respectively. The first two are separated by a distance of 84 seconds. The companion stars are travelling round the primary with a retrograde motion in a period of 180 years. The parallax of this star corresponds to an actual distance of 17 light years.

CHAPTER XX.

ASTRONOMICAL INSTRUMENTS.

271. The Astronomical Clock.

A clock is a device for measuring time, and without such a contrivance, no progress in observational astronomy could have been possible. The principle used in measuring time by a clock is the fact that the time of oscillation of a pendulum is constant so long as its length is unaltered. The use of the pendulum as a regulator of the clock mechanism was suggested by Huyghens in 1657.

An astronomical clock is not different in its essential details from any other good clock, except in that its time can be relied on with greater surety. The pendulum of such a clock is compensated for changes of temperature and pressure and the clock has also a very accurate escapement.

If the pendulum is only a single rod terminated by the bob, the rate of the clock will change when the length of the pendulum changes due to the variations of temperature. Many devices have been adopted to prevent the change of rate of the clock due to temperature variations. The device in the Graham's mercurial pendulum consists in fitting the bottom end of the rod of the pendulum with a vessel of mercury, so that when the steel rod of the pendulum expands and thus lowers the centre of gravity of the pendulum, the mercury in the vessel of the pendulum expands upwards thereby raising the centre of gravity. The quantity of mercury in the vessel is so adjusted as to keep the centre of gravity of the pendulum at a constant distance from the centre of suspension. The device in the Harrison's grid-iron pendulum is to make it a combination of zinc and steel rods, arranged in such a way that the change in the

length of the pendulum brought about by the expansion of steel rods in one direction is exactly counteracted by the change in length brought about by the expansion of the zinc rods in the opposite direction. All these devices introduce small errors due to the lag in the adjustment when temperature changes rapidly. Many of the modern astronomical clocks are provided with pendulums of invar which is an alloy that does not change its length appreciably when variations in temperature take place.

The effect of atmospheric pressure on the rate of a clock consists in its changing the resistance offered to the swing of the pendulum and thereby causing a change in the period. One way of compensating this is to enclose the clock in a partially exhausted case.

The error of a clock is the amount of time to be added to the time given by the clock to get the true time.

The rate of a clock is the amount of its gain or loss in a day. If the clock loses, its rate is said to be *positive.* A good clock should not show variations in its rate.

272. The Chronometer

This is simply a very reliable form of timepiece used in the place of an astronomical clock when the latter is not available or is not properly mounted. Its superiority over ordinary watches or timepieces consists in that its balance wheel is so constructed that it keeps a constant time of oscillation. For this purpose, Harrison suggested that the rim of its wheel should be made up of several disconnected arcs, each arc being made of two metals (brass and steel). By such a contrivance, the moment of inertia of the wheel is kept constant even when the metals expand or contract. When this is the case, the hairspring will act evenly in reversing the motion of the wheel. The arcs of the wheel also carry small screw-weights which allow finer adjustment to

compensate for further differences which may still be caused in the time of oscillation. In any timepiece or watch, the mainspring when wound up keeps up the oscillation of the wheel and the periodic time of oscillation will not be uniform when the spring is continuously acting on the wheel throughout its periods of oscillation. A chronometer is provided with the detached escapement by which the balancewheel is acted on by the spring only during a small portion of each of its oscillation, and therefore the time of oscillation is uniform. The chronometer is usually suspended in a framework so that it keeps a horizontal position. This framework would not communicate to the instrument any kind of shake or disturbance given to it from outside. This is the reason why chronometers are used as time-keepers in all ships.

Generally, chronometers show Greenwich Mean Time. All good chronometers are tested at the Greenwich Observatory, where they are subject to variations of temperature just to ascertain whether they show the expected compensation for usual temperature changes and they are daily compared with the standard clock and their rates are determined. After this, they are certified to be fit for service. At any place, when the local time is known by astronomical observations, the chronometer time gives the longitude of the place, the latter being the difference between the two times.

Error and rate of a chronometer.

The amount by which a chronometer is slow over Greenwich Mean Time is called its error and the daily change in its error is called its rate. A good chronometer should have a uniform rate. Whenever a ship is in port, its chronometers are rated by comparison with the standard clock available there.

273. The Sundial.

This is a simple device by which the hour angle of the sun or the apparent solar time could be known by observations of a shadow cast by a rod fixed in such a manner as to be parallel to the axis of the earth. Such a

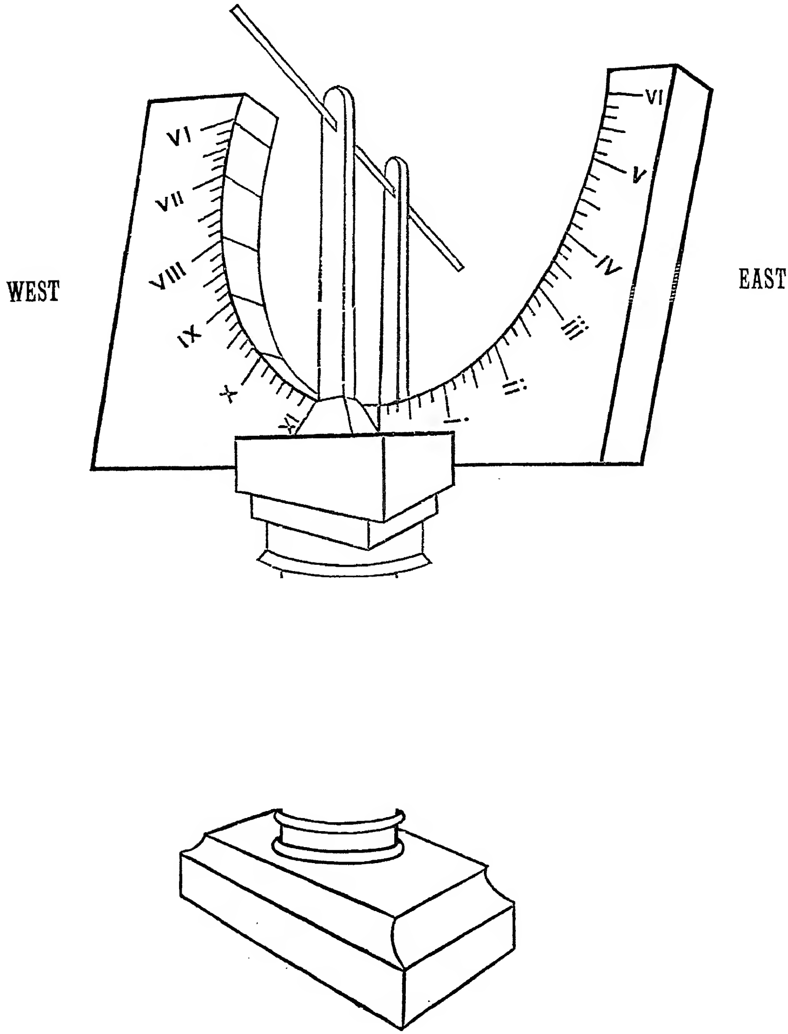


Fig. 145.

form of apparatus was being used by the ancient Hindus and Greeks. In using this instrument for getting the apparent time, the sun is supposed not to change its declination in the course of a day. The shadow cast by the rod on a horizontal, vertical or any other plane can be made to give us the apparent solar time by properly marking the hour-lines on such a plane.

The Equatorial Sundial.

A very simple form of the sundial is the Equatorial Sundial (See Fig.145) where the dial plane in which the hour-lines are to be marked is parallel to the plane of the equator and therefore the rod is perpendicular to it. The shadow of the rod will move in such a plane at the rate of 15° per hour, and therefore on a circle in that plane through whose centre the rod passes, the hours corresponding to 1, 2, 3. etc. of solar time will be marked at angular distances of 15°, 30°, 45° etc. from the point lying on the meridian. Such a type of sundial is shown in figure 145.

Horizontal Sundial.

Another type of sundial is the one in which the hours are marked on a horizontal plane. Obviously, the hour-lines here are given by the projections of the hour-lines in the previous type of dial. Figure 146 explains the mode of graduating such a form of sundial.

Let O P be the rod of the dial which is parallel to the axis of the earth and which is fixed on a horizontal plane on which the hour-lines are to be marked. Let OO′ be the meridian line on this plane, this being determined by the use of a gnomon. At apparent noon, the shadow cast by the rod will fall along O O′. Now, since the hour angle of the sun is changing at the rate of 1 hour per hour, if O1, O2, O3 etc. be the shadows cast by the rod on the dial plane when the hour angles of the sun are 1^h, 2^h, 3^h etc. then these lines determine the hour-lines corresponding to 1'o clock, 2'o clock, 3'o clock etc.

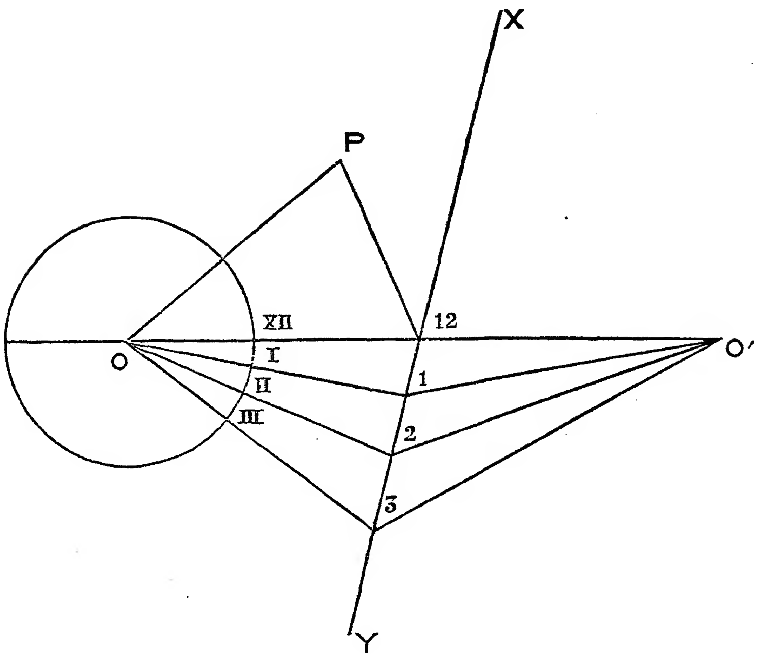


Fig. 146.

The following gives a simple geometrical construction for marking the lines O1, O2, O3 etc.

Let a plane through P $\perp^r$ to the rod O P intersect the dial plane along X Y and the hour-lines in the points, 12, 1, 2, 3 etc. Then it is evident that the angles 12 P 1, 1 P 2, 2 P 3 etc. are each equal to 15°. Now if the length of O P be equal to l, P 12 $= l \tan \varphi$ and O 12 $= l \sec \varphi$, where φ is the latitude of the place. Imagine that the plane P Y X is rotated about X Y till it is brought down to the plane of the dial, the point P of the plane being brought to O′ and the lines P 1, P 2 etc. to O′1, O′2 etc. Then the angles 12 O′1, 1 O′2 etc. are each equal to 15°.

Hence, in order to graduate the sundial, along the meridian line OO′ on the dial plane measure off a length

$O\,12 = l \sec \varphi$. Produce O 12 to O′ so that $12\,O' = l \tan \varphi$. Through the point 12 draw X Y $\perp^r$ to OO′. At O′ make angles 12 O′1, 1 O′2, 2 O′3 etc. each equal to 15° meeting the line X Y at the points 1, 2, 3 etc. Then the lines O1 O2, O3 etc. are the hour-lines corresponding to 1'o clock, 2'o clock, 3'o clock etc. The hours can be marked either on the line X Y itself or on a circle with O as centre as shown in the figure.

It should be noted that the graduations on this dial are not uniform, as the equal angles giving different hour-marks in the equatorial sundial do not project into equal angles on any other plane.

274. The Telescope.

One of the most important of astronomical observations is the determination of the position of a celestial object and for this purpose a suitably mounted telescope is indispensable.

The object glass of the telescope forms an image of the celestial object at its focal plane, which is again magnified by the eyepiece and this magnified image is observed. If in the place of the eyepiece a photographic equipment is suitably fitted, the image can be photographed also. The special advantage of the telescope over the human eye lies in its great light-gathering power, which is proportional to the surface area of the object glass.

Refractors and Reflectors.

The object glass of a telescope can be either a lens or a combination of lenses, or a concave reflecting surface which forms a clear image of the object at its focus. The image at the focus in both the types of telescopes is examined after magnification by an eyepiece. which is mainly a magnifying glass. The telescope in which the image of the celestial object is formed by a lens or combination of lenses is called a refractor, as the rays from

the object undergo refraction before they form the image. If it is formed by the reflection of such rays it is called a reflector. In a reflector, the light after falling on a concave reflecting surface, gets reflected and forms an image of the object at the focus. The reflecting telescope was invented by Newton in 1670 to avoid the chromatic effects shown by the single lens of any refractor. As the refractive indices for the different colours of light are not the same, a single lens will not form a sharp image of a star. A blue image of the star for instance will be formed nearer the lens than where the red image of the same star will be formed. This phenomenon is known as chromatic aberration. The difference in the refractive powers for any two colours varies with the type of glass. Therefore, by choosing two different types of glass and making with them different types of lenses, it is possible to make a combination giving a fairly sharp image of a star. The particular manner in which chromatic aberration is corrected depends upon the use to which the telescope is put. If the refractor is used for photographic purposes, lenses that will bring the blue and violet rays to nearly the same focus are chosen. If the telescope is intended for visual observation, lenses making the yellow and the green rays converge to the same place are used. Again more than two lenses have to be used for the object glass to give a large flat field of view. The cost of making a combination of lenses for a refractor, free from chromatic aberration and other defects is much more than that of making a reflector of the same size.

Different types of reflectors.

There are several types of reflecting telescopes the main types among them being the Newtonian type, the Gregorian type and the Cassegrain type. In the Newtonian reflector, the beam of reflected light is intercepted by a small plane mirror placed on the axis of the

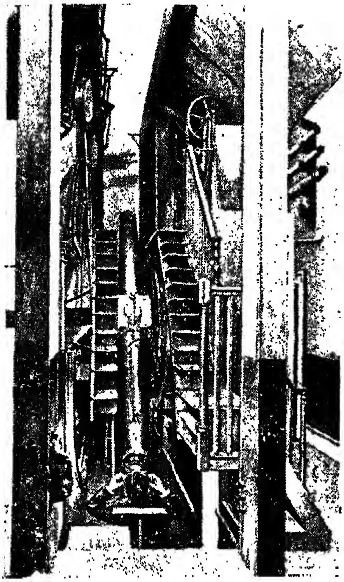


Plate No. 12.

**The Transit Telescope,
Royal Observatory, Greenwich.**

**The Thompson Telescope in
the Royal Observatory, Greenwich.**

This Equatorial telescope of 26″ aperture is fitted with a large camera, and is chiefly employed for the determination of stellar distances. A small visual telescope is also mounted side by side with the camera to help guiding the instrument during the time of exposure.

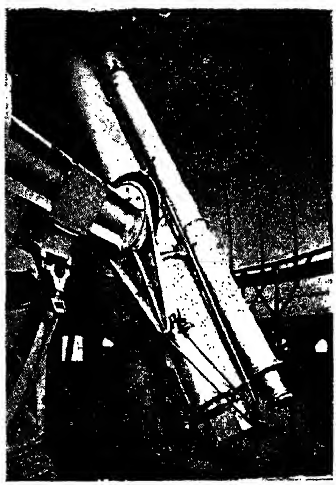


Plate No. 13.

telescope and inclined at 45° to it. This mirror changes the course of the convergent beam which is brought to a focus at a side of the tube where the eyepiece is placed for convenient observation. In the Gregorian type, the mirror is bored at the centre and the light from the mirror is reflected through the hole by a small concave mirror kept on the axis of the mirror and a little away from the focus. By this device, the focal length of the mirror is increased. In the Cassegrain type, the beam from the object glass is reflected back either through the central hole by a small convex mirror placed on the axis of the telescope a little nearer than the primary focus or to a side of the tube by again interposing a small plane mirror in front of the object glass to divert the beam reflected by the convex mirror. Here the equivalent focal length is further increased and we get a flatter field of view than in either of the previous types.

275. Comparison of reflectors and refractors.

Reflectors are to be preferred for certain kinds of work and refractors are to be chosen for other kinds of work. One thing that is to be remembered is the comparative cheapness of a reflector. The lenses that go to make the object glass of a good refractor should be absolutely homogeneous. To cast and prepare a big glass disc of the necessary quality is very difficult, and then to grind and polish the same to perfect lenses of the required curvature so as to make a combination free from chromatic aberration would cause much trouble and require great skill. But in making a reflector, one need pay attention only to get ready one surface to the required shape and optical quality. Moreover, the disc need not be homogeneous. So it is easier to make comparatively large size reflectors than refractors of the same size.

The phenomenon of spherical aberration is also an equally important factor that has to be attended to, in

making a good telescope. In the case of a reflector, elimination of this error could be done very easily by making the concave surface paraboloidal, but in the case of a refractor, the error can be allowed for, only by altering the curvature at different parts of the lens by local grinding which is an elaborate and difficult process.

The perfect achromatism of a reflector is another advantage for it over the refractor, which can never be perfectly so. The advantage of a refractor is that there is not so much loss of light when the rays reach the focal plane as in a reflector of the same aperture. Up to about 50 inches aperture, refractors are better for visual work, but reflectors are better for photographic work. When the aperture is larger than 50 inches reflectors are superior both for visual and photographic purposes. A reflector requires resilvering its surface whenever its surface gets tarnished, while a refractor kept carefully will never deteriorate in quality, with lapse of time. The focal length of a reflector is affected less than that of a refractor by temperature differences. The refractor can be made to give a flatter field than a reflector.

276. The Meridian circle or the Transit circle.

The transit circle already referred to, is the fundamental instrument of observation in an observatory. With its help, the R. A. and declination of celestial bodies are determined, and also the latitude of the observatory.

It consists of a graduated circle C (See fig. 147) rigidly attached to an axis X through its centre and perpendicular to it. This axis is also the axis of rotation of a telescope rigidly attached to it in such a way that the optical axis of the telescope is perpendicular to it. The extremities of this axis end in cylindrical pivots, E. W, which are capable of rotating freely on two bearings supported by two pillars of very strong foundation. The axis E W should be horizontal and should be in the east to west

direction. Usually, this axis is capable of being reversed from the east to west direction to the west to east direction. When the instrument is in proper adjustment, the plane of the graduated circle and the optical axis of the telescope should be parallel to the meridian. In the focal plane of the telescope, there is a frame work (see fig. 148) carrying a number of fixed meridional wires

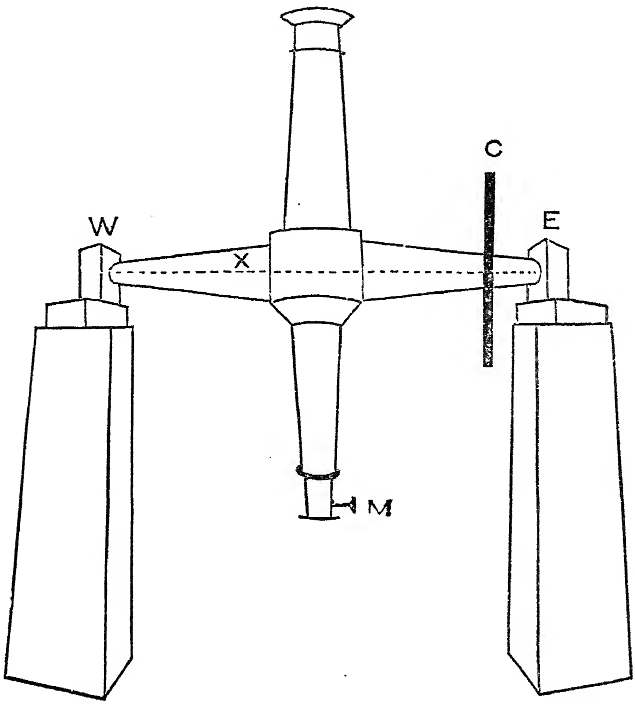


Fig. 147.

(usually 7 or 9) all parallel to the plane of the meridian and another wire or a pair of them perpendicular to these passing through the centre of the field of view. There is also a frame carrying a wire that could be moved perpendicular to the axis of the telescope. This movement is made by a micrometer screw M with a graduated head, so that the distance through

which the movable wire has been displaced is given in terms of the revolutions of the screw-head. These lines are called the spider lines. When the image of a star is at the intersection of the middle meridional wire and the horizontal wire, the corresponding sidereal time gives the R. A. of the star, and the reading on

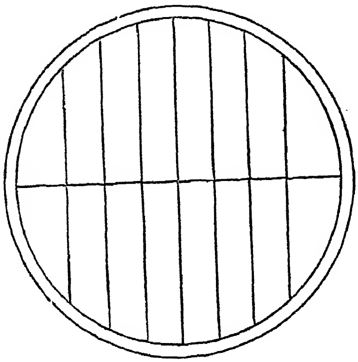


Fig. 148.

the graduated circle C, gives the star's zenith distance or altitude, it being assumed that the reading on the graduated circle is known when the telescope is vertical or horizontal. If the latitude of the place of observation is known, the observed zenith distance gives the declination of the star. It is in this way that the R. A. and declination of a number of stars are determined fundamentally. An instrument without the graduated circle could be used to observe the time of transit of a star and thus to get its R. A. only. Such an instrument is called a transit instrument.

By the term *the optical axis* of the telescope, we mean the line joining the centre of the object glass and the centre of the frame of spider lines in the focal plane.

A meridian circle in perfect adjustment should make its optical axis describe the meridian. As this is not always possible, the optical axis of such an instrument

may not be in the meridian plane, at the time when a transit is observed. In such a case, corrections for the errors of the instrument will have to be applied, so that the exact time of transit may be obtained.

A meridian circle is not in proper adjustment when any of the following conditions are not satisfied :—

1. The optical axis of the telescope should be perpendicular to the axis of rotation.

2. The axis of rotation should be absolutely horizontal.

3. The direction of the axis of rotation should be exactly east to west.

The deviation of the instrument from the above conditions gives rise to the following three errors :—(1) collimation error, due to the optical axis of the telescope not being perpendicular to the axis of rotation; (2) level error, due to the axis of rotation not being horizontal; (3) the deviation error or azimuth error, due to the axis of rotation not being due east and west.

These errors are inevitable in any meridian circle and therefore due allowance must be made for these whenever a transit is observed. In a transit circle properly mounted, these errors will be very small and in considering their effects, the squares and higher powers and products of these quantities may be omitted.

Let a transit circle have C, L and K as collimation error, level error and azimuth error respectively. The correction to be applied to the observed time of transit owing to these errors is obtained as follows :—

Let P be the north celestial pole, Z the zenith and N, the nole of the graduated circle of the instrument. If S be the position of a star of declination δ when it appears to be on the optical axis of the instrument, then the correction to be applied to the apparent time of transit to get

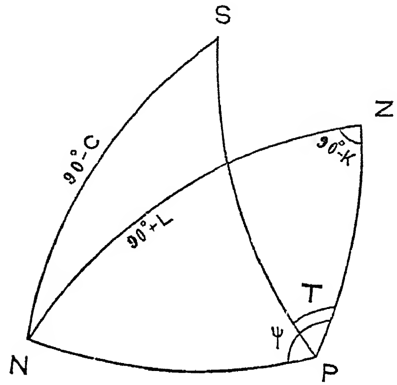


Fig. 149.

the true time of transit at a place of latitude φ, is obviously given by the angle S P Z (=T).

Let $N\hat{P}Z$ be equal to ψ and N P be equal to p. Then, from the triangle P Z N,

$$\cos p = -\sin L \sin \varphi + \cos L \cos \varphi \sin K \text{ ....(1)}$$

$$\sin p \sin \psi = \cos L \cos K \text{.................................(2)}$$

$$\sin p \cos \psi = -\sin L \cos \varphi - \cos L \sin \varphi \sin K \text{........(3)}$$

From the triangle S P N,

$$\sin C = \cos p \sin \delta + \sin p \cos \delta \cos (\psi - T)$$

$$= \cos p \sin \delta + \sin p \cos \psi \cos \delta \cos T$$

$$+ \sin p \sin \psi \cos \delta \sin T \text{............(4)}$$

Substituting in (4) for cos p, sin p sin ψ and sin p cos ψ, from (1), (2) and (3), we have

$$\sin C + \sin \varphi \sin L \sin \delta - \cos \varphi \cos L \sin K \sin \delta - \cos L \cos K \cos \delta \sin T + \cos \delta \cos T (\cos \varphi \sin L + \sin \varphi \cos L. \sin K) = 0 \text{.........(A)}$$

Since, T, L, K and C are very small, their squares can be neglected and we therefore get from (A)

$$T = C \sec \delta + L \cos (\varphi - \delta) \sec \delta + K \sin (\varphi - \delta) \sec \delta \text{....(5)}$$

This is Mayer's formula giving the correction for the observed time of transit.

Putting $m = \text{L} \cos \varphi + \text{K} \sin \varphi$ and

$$n = \text{L} \sin \varphi - \text{K} \cos \varphi,$$

we get another form of the same formula given by

$$\text{T} = m + n \tan \delta + \text{C} \sec \delta \quad \text{......(6)}$$

Here m and n are independent of the co-ordinates of the stars and can therefore be used for all stars when once the errors are known.

We shall next show how these errors of the transit circle can be determined.

277. Determination of the collimation error.

In finding this error, two distant points diametrically opposite have to be observed by the transit circle. These points of observation are obtained by a pair of small telescopes called collimating telescopes having cross wires in their focal planes.

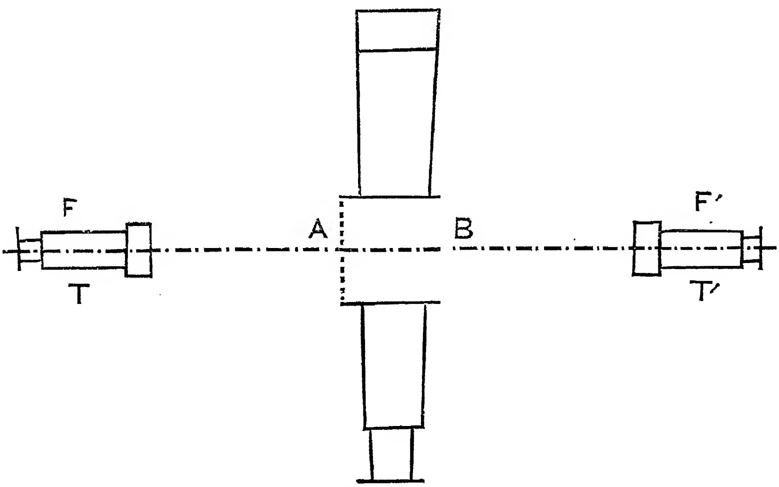


Fig. 150.

The central part of the transit tube has a cylindrical hole A B which allows rays of light from either of the small

telescopes placed on the north and south of the transit instrument to reach the other. These telescopes are provided with crosswires capable of movement with micrometer screw-heads. If light be admitted to the north collimator T, the rays from the intersection of the crosswires F in its focal plane will emerge from it as a parallel beam and will form an image in the focal plane of T′. By means of the micrometer of T′ the intersection of its crosswires F′ can be brought into coincidence with the image of F. When this has been done, we get the axes of the two telescopes to be along the same direction and thus F and F′ give us two points on the celestial sphere diametrically opposite to each other.

To determine the collimation error of the transit circle, the telescope is first directed to the collimator T, and the image of F is brought into coincidence with the intersection of the horizontal wire of the transit circle and its movable wire by means of the micrometer head, and the reading x_1 on its head is taken. Then the transit circle is turned round to the south collimator and the reading x_2 on the micrometer is taken for bringing the image of F′ on the intersection of the horizontal wire with the movable wire. These two observations give us a method of determining the collimation error as follows:—

Let h, δ be the east hour angle and declination respectively of an object seen to transit through a meridian circle of collimation error C. From the triangle PSN of fig. 149 we have

$$\sin C = \cos p \sin \delta + \sin p \cos \delta \cos (\psi - h)$$

Now, by moving the vertical wire through a distance x_1 from the fixed middle wire, we have changed the collimation error to $(C + x_1)$ and directed the telescope to F. If then the hour angle and declination of the point F be h and δ we have,

$$\sin (C + x_1) = \cos p \sin \delta + \sin p \cos \delta \cos (\psi - h).$$

Now, the co-ordinates of the point F′ are $(180+h)$ and δ.

$\therefore \quad \sin(C+x_2) = -\cos p \sin \delta - \sin p \cos \delta \cos(\psi - h)$.

$\therefore \quad \sin(C+x_1)+\sin(C+x_2)=0$

Since C, x_1, x_2 are small

$$C+x_1+C+x_2=0.$$

$$\text{i. e. } C = -\frac{x_1+x_2}{2}$$

278. Determination of the level error.

If a basin of mercury be placed beneath the meridian circle, and the telescope be turned upon it, we can see the crosswires of the telescope and their images side by side due to the reflection from the surface of the mercury. The rays from the crosswires in the focal plane of the telescope reach the basin of mercury as a parallel beam and get reflected back as a parallel beam falling on the object glass and converge in the focal plane as images of the wires. If there had been no level error and collimation error in the telescope, these images would have exactly coincided with the original wires themselves. Let C′ be the distance between the image of the movable wire and the fixed middle vertical wire Then, C′ is given by the reading of the micrometer screw when it is turned to bring the middle wire into coincidence with the image of the movable wire. The new collimation error of the telescope is (C+C′) and the new axis of the telescope is pointing towards the nadir, whose hour angle is 180° and declination is $-\varphi$, where φ is the latitude of the place. Putting T=180° and $\delta=\varphi$ in equation (A) of page 382 we get,

$$\sin(C+C') = \sin L$$

$$\text{i. e. } L = C+C'$$

When the collimation error C is known, the above equation gives the level error L.

The formula above written gives the collimation error when the level error is known otherwise, and the level error can be determined by the use of a long spirit level being made to rest on the bearings on which the axis of the telescope rests.

279. Determination of the azimuth error.

Let α_1, δ_1; α_2, δ_2 be the right ascensions and declinations of two known stars observed during a short interval of time during which the error of the clock remains constant.

Now, from the equation giving the correction to the time of transit due to the errors of the transit circle we get

$$\alpha_1 = T_1 + \Delta T + K \sin(\varphi - \delta_1) \sec \delta_1 \quad \text{......(1)},$$

where T_1 is the observed time of transit of one of the stars corrected for collimation and level errors, and ΔT, is any error, the clock might show.

Similarly for the second star, we have

$$\alpha_2 = T_2 + \Delta T + K \sin(\varphi - \delta_2) \sec \delta_2 \quad \text{......(2)}$$

Equations (1) and (2) determine both ΔT and K

i.e. $K = [(\alpha_1 - \alpha_2) - (T_1 - T_2)] \cos \delta_1 \cos \delta_2 \sec \varphi.$
$\operatorname{cosec} (\delta_2 - \delta_1)$

Here T_1 and T_2 are affected by the errors of observation, and therefore it is better to choose the multipliers of $T_1 - T_2$ to be as small as possible. Therefore $\operatorname{cosec}(\delta_2 - \delta_1)$ should be as small as possible, as $\cos \delta_1$ and $\cos \delta_2$ have only variations between 0 and 1. Hence one of the stars chosen should have its declination near 90° and the other near 0°. It is also seen that in determining the azimuth error of the meridian circle, the data for getting the error of the clock are also obtained. Besides these errors of the meridian circle, there are other errors such as (1) the errors of graduation on the circle attached to the axis and (2) the error due to the centre of the circle of graduation not coinciding with the axis of rotation. These

errors are reduced to a minimum by using four or six reading microscopes placed at equal intervals against the circumference of the graduated circle so that the mean given by the readings from the different microscopes is taken as the true reading for any particular position of the telescope.

280. Determination of the time of transit.

From a star's right ascension and declination, and the latitude of a place, we can know the exact time of transit of the star and also the altitude to which the transit instrument should be directed. Having fixed the telescope to point at the right altitude, watch through it one or two minutes before the time of transit. As soon as the star is seen to enter the field, bring the star to the horizontal wire and make it go along it from one vertical wire to the next until all the vertical wires are passed through. Even some time before the first vertical wire is crossed through by the star, the minute of the sidereal clock is noted, and the seconds are counted so that the time of the star's transit over the first wire could be noted correct to a second. This process of counting time and noting the times of transit over the remaining wires is continued till the star has crossed all the meridional wires. The average of these times corrected for instrumental errors, gives the time of transit. This will also be the exact time of transit over the middle wire if that wire is exactly in the mean position of all the others. This method of taking the transit is what is called *the eye and ear method.* In this method, some error is likely to occur due to the observer's personal equation in either estimating time or in noting the exact time of transit over each wire.

These errors are reduced by the use of a *chronograph* for recording time whenever transits are taken. This instrument consists of a cylindrical barrel round which a sheet of paper is wrapped. The barrel is rotated by a

driving mechanism at a uniform rate of half or one revolution per minute. A pen is fixed to the armature of an electromagnet and as the barrel revolves, this pen is made to travel in a lengthwise direction by a screw, so that the trace of the pen on the paper round the drum is a helical curve. The pen is electrically connected to the sidereal clock, from which a momentary current comes through the magnet every second giving a kick to the pen, thereby causing a jerk to the line marked by the pen. Whenever a fresh minute begins the momentary character of the kick is slightly changed so that the corresponding break on the line is different, indicating thereby the particular place on the paper where a fresh minute begins. The observer carries with him a tapper which is also electrically connected to the pen and by tapping it when a star crosses one of the meridional wires of the transit circle, he is giving an additional kick to the pen, which is also marked in the corresponding position round the drum. The exact fraction of a second when this occurs can be measured accurately from the markings on the chronograph sheet.

281. The Equatorial.

As the transit instrument is one which describes only the meridian plane, celestial objects can be seen through it only when they are on or very near the meridian. But, for the study of Physical Astronomy or for the continuous observation of any celestial body, it is necessary to have a telescope which can be pointed in any desired direction and moved so as to keep the same body always in the field of view. The equatorial is a telescope so mounted as to satisfy the above conditions. In this instrument, a framework carrying the telescope turns about an axis AB called the primary axis, which is parallel to the celestial axis. A graduated circle R is attached rigidly to this axis and perpendicular to it. This is called the hour circle and it is fitted with a vernier

and pointer-microscope for taking the readings. Perpendicular to A B there is another axis called the declination axis, about which the telescope T is free to turn. This axis also carries with it a graduated circle perpendicular to it, called the declination circle D and it is also fitted with a vernier and microscope for taking readings. The primary

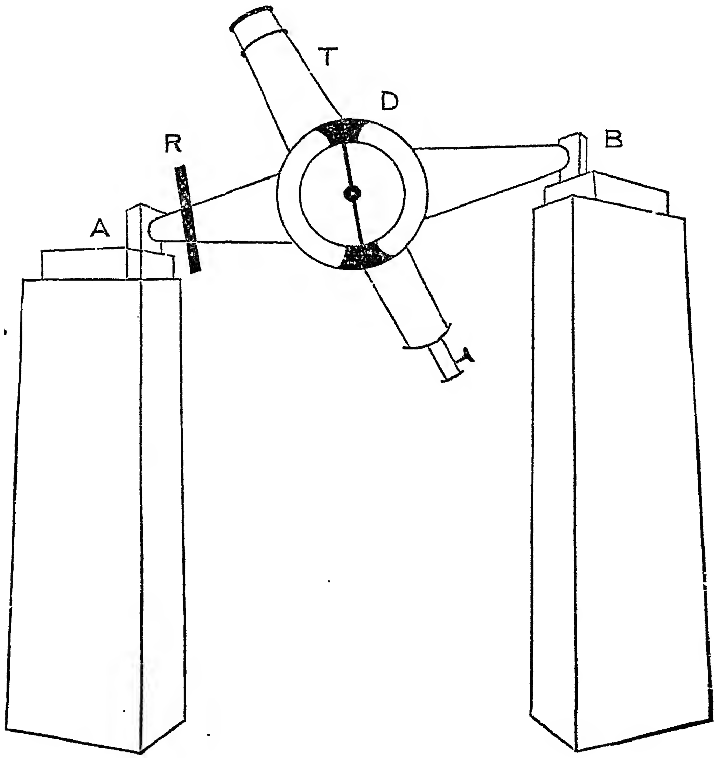


Fig. 151.

axis with the whole instrument is either supported by two pillars at its extremities or by a single pillar at its middle portion. Now, when the hour circle and the declination circle read zero, the telescope should be on the meridian and should point to a star on the equator whose hour-angle is zero. When the telescope is pointed in any other direction, the corresponding readings on the two

circles give the hour angle and the declination of the body seen through it.

If it is required to observe any body continuously, what is done is to attach the hour circle to a clamp which is worked by a drive in such a way that the axis is turned round exactly to counteract the rotation of the earth. When this is done, the declination circle can be clamped, provided the declination (as in the case of a star) remains unchanged throughout the observation.

A small telescope, called the *finder* is fixed to the large telescope of the equatorial in such a way that the optical axes of both are parallel. If an object is in the centre of the field of the finder, it will be seen at the same time nearly in the centre of the field of the telescope. Since the finder has a larger field of view, it is easier to bring a celestial object in the field of view of the finder than in that of the main telescope.

282. The adjustments of the equatorial.

These are six in number, and they are the following :—

(1) The inclination of the primary or polar axis should be equal to the latitude of the place where the equatorial is to be mounted.

(2) The declination circle should read zero when the telescope is at right angles to the polar axis.

(3) The polar axis should be placed along the meridian of the place.

(4) The optical axis of the telescope should be exactly at right angles to the declination axis.

(5) The declination axis should be at right angles to the polar axis.

(6) The hour circle should read zero when the telescope is on the meridian.

283. Fitting up an equatorial at any place.

First adjustment.

As the polar axis of the instrument should be parallel to the celestial axis, adjustments (1) and (3) are obviously necessary. By means of a gnomon or by a compass and a clinometer the axis is placed approximately in position.

Observe a star of known declination (say Capella whose declination is $45°-56'{\cdot}1$ N) with the telescope west of the pillar and also east of the pillar, and note the readings of the declination circle in both the positions. The mean of the two readings gives the declination of the star from the equator of the instrument, and its difference from the true declination gives the error in the inclination of the equator of the instrument, and hence in that of the polar axis. This error is corrected by the fine adjustment provision, for changing the inclination of the polar axis.

Example.

	°	′	″
Observed declination of Capella with the telescope west	43	48	30
Do. with the telescope east	46	57	0
2	90	45	30
Mean.	45	22	45
Refraction correction			− 33
Instrumental declination	45	22	12
True declination	45	56	6
Error in the inclination of polar axis	0	33	54

Second adjustment.

The difference of the two instrumental declinations of the star when the telescope is on either side of the supporting pillar is double the index error of the declination circle. By moving the verniers with the help of the adjustment screws, the position of the true zero of the declination circle is obtained.

		°	′	″
The difference of the declination readings in the above case	=	3	8	30
∴ Index error	=	1	34	15

Third adjustment.

Observe any star of known declination when it has an hour angle 6 hrs. east or 6 hrs. west of the meridian. The difference between the observed and true declinations divided by the cosine of the latitude is the azimuth error of the polar axis. To correct this, set the telescope to the true declination of the star corrected for the component of refraction in declination, and alter the azimuth of the pillar till the star is obtained on the crosswire of the eye piece which is parallel to the declination circles.

Fourth adjustment.

This is done by observing a star on or nearly on the equator with the telescope on each side of the pillar, the difference in time of the two observations being noted, and the hour circle read after each observation. The first reading subtracted from the second (Plus 24 hours if necessary) minus the interval of time between the two observations should give exactly 12 hours. If greater than 12 hours, the object glass end of the telescope forms an acute angle with the declination axis, and if less this angle is obtuse. There are suitable screws to make the necessary corrections.

Example.

γ Virginus of decl. $1° - 5'{\cdot}9$ S is the star observed.

Time by clock.			*Reading of hour circle.*			
H	M.	Sec.	H.	M.	Sec.	
11	21	50	11	52	30	Telescope east.
11	29	55	24	6	20	Do. west.
0	8	5	12	13	50	
				8	5	
			12	5	45	
			12	0	0	
			2 |	5	45	
				2	52·5	

Collimation error at declination $1°-5'.9$ $\Big\}$ $= 2' \ 52''.5$.

The angle between the object glass and the declination axis is acute.

Fifth adjustment.

This is made by placing the declination axis horizontal with the help of a striding level fixed to it, and reading off the time on the hour circle. Then move the telescope round to the other side of the pillar, so that the second reading of the hour circle is different from the first by 12 hours and if the bubble is not then at the zero reading, bring it to zero by moving the axis. Half this movement must be applied to one of the readings of the hour circle, and the bubble of the level brought to zero by altering the declination axis by the adjustment screws.

Example.

Let the hour circle reading when the declination axis is horizontal be the following :—

	H.	M.	Sec.	
	11	58	7	Telescope east.
	23	59	57	Do. west.
	12	1	50	
	12			
Difference	0	1	50	

Sixth adjustment.

The sixth and the last adjustment is made by bringing the declination axis horizontal by means of a level and then setting the hour circle to 0 *h*-0 *m*-0 sec.

The equatorial is now in perfect adjustment, and when any celestial object is seen through it, the readings on the hour circle and on the declination circle give the hour angle and the declination respectively of the object.

284. The Transit Theodolite or the Altazimuth.

These are instruments which could be conveniently used to find the altitude and azimuth of a heavenly body.

In each there is a horizontal circle turning upon a vertical axis and read by a vernier and reading microscopes. Upon this circle and turning with it are the supports of a horizontal axis with a vertical graduated circle and reading microscopes. A small telescope is capable of turning about this horizontal axis and its altitude is read by verniers on the vertical circle. The reading on the horizontal circle when the telescope points to any object gives the azimuth, if the theodolite is so adjusted initially that when the vertical circle and the telescope are in the meridian plane, the vernier of the horizontal circle reads 0° or 180°. The whole instrument rests on a stand supported by levelling screws, by which the instrument can be adjusted so that the horizontal circle is truly horizontal. This is ensured by the proper adjustment of the levelling screws while looking at the bubble of the spirit level which is kept parallel to the horizontal circle.

285. The Heliometer.

This consists of a combination of two half-lenses of the same size, which are capable of sliding movements along the direction of their common diameter by means of a

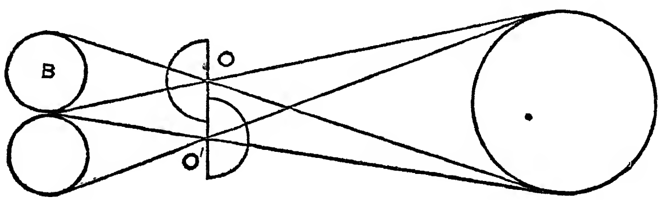


Fig. 152.

graduated screw-head. It is used in finding the angular distances between objects or in finding the angular diameters of bodies like the sun or the moon. Suppose it is required to find the angular diameter of the moon. When the two halves of the object glass are separated, we get two images of the moon, one image being formed by each

half. But when the two halves are together, only one image will be formed in their focal plane. Now the two halves are separated to such an extent as to make the two images of the moon just touch. At this position it is easily seen from figure (152) that the angle subtended at the point of contact B, by the two optical centres O and O′ of the two halves is equal to the angular diameter of the moon. The distance O O′ between the two centres is read from the turns given to the screw-head. Since the focal length of either of the lenses is known, the angle subtended at B by O O′ can be calculated.

286. The Sextant.

The sextant is another instrument used to measure the angular distance between two objects, or to find the angular diameter of objects like the sun or the moon.

It consists chiefly of a framework in the form of the sector of a circle containing an angle of about 60° (Fig. 153). The limb A B is graduated, and an arm is moving about the centre O carrying with it a smaller mirror called the index glass. This arm reads on the scale A B the inclination of the mirror O at any position to what it is at any standard position. There is another mirror, half of which is alone silvered, placed at X on O A at an inclination of about 60° to O A. A telescope is fixed on O B so that it may receive both reflected and transmitted rays from X. On looking through the telescope we could see direct images of objects like K, coming straight through the unsilvered part of X and also reflected image of an object like Y after undergoing two reflections, one at O and another at X. Initially when both the mirrors at O and X are parallel, which is the standard or zero position for the instrument, the direct and reflected images of a distant object K should coincide. The reflected image of another object like Y will be then away from the direction of K. But by moving

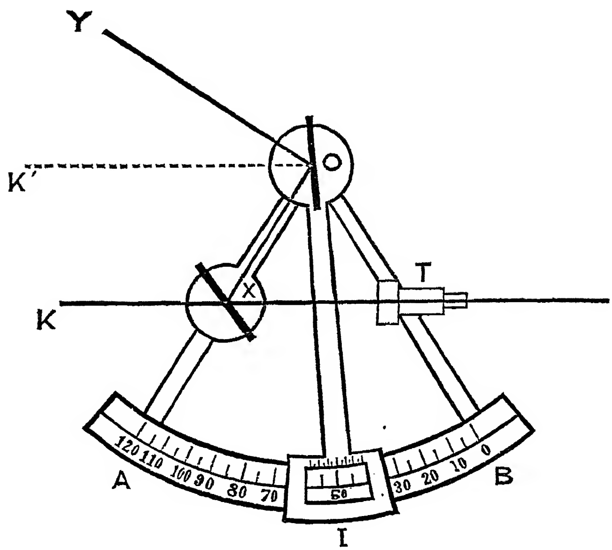


Fig. 153.

the index arm along B A, the reflected image of Y can be brought into coincidence with the direct image of K. From the diagram it is clear that when this is done, $I\hat{O}B = \frac{1}{2} Y\hat{O}K'$ where O K′ is parallel to X K, since when a mirror turns through a certain angle, the direction of the reflected ray corresponding to a fixed incident ray from any object turns through double that angle. Thus the angle between Y and K is obtained by a direct reading of the scale A B given by the pointer at I. To facilitate the direct reading of angles, figures marked on A B give double their actual values. The arm O I has a vernier and microscope attached at I, so that an accurate reading of the scale may be made.

When the sextant is used to measure the angular diameter of the sun, dark glasses are used to reduce the intensity of the sun's rays, and they are fitted up on the arm O A to be used at the time of observation.

Errors of the sextant.

If the pointer I does not read zero when the index-glass and the horizon-glass X are parallel, the sextant is said to have an *index error*, which should be added to all the readings given by the sextant. Generally, the graduation of the sextant on the limb is carried back for a degree or two beyond the zero reading. This is only to facilitate the finding of the index error of the instrument. Suppose the diameter of the sun's disc is found to be given by the reading r of the sextant, when the direct and reflected images are brought into coincidence. If e be the index error, the true diameter of the disc is $(r+e)$. Let the index be moved beyond the zero point in the other direction until the images again touch. If r' be the reading now, the diameter is given by $r'-e$. From the two observations the value of e is obtained to be $\frac{r'-r}{2}$.

There are also other possible errors to be looked for in a sextant, such as error of eccentricity due to the centre of the limb not coinciding with the centre of rotation of the index, errors of graduation of the limb etc.

Besides being very handy, the special advantage of the sextant for observations on board a ship is the fact that any displacement brought about by the unsteady holding of the instrument, will not spoil the reading given by it, as the coincidence of the two images is not affected by such a movement.

Some of the principal uses of the sextant are the following:—

(1) *To find the altitude of a star.*

The sextant is adjusted so that the reflected image of the star is made to lie on the visible horizon seen through the plane glass. When the plane of the sextant is slightly turned, the image of the star should just graze the

horizon without going below it, since the altitude of the star is its distance from the nearest point of the horizon. After applying the correction for dip and refraction, the star's true altitude is obtained from the reading of the sextant.

(2) *To find the angular diameter or the altitude of the sun or the moon.*

Here the altitudes of the upper and lower limbs are taken and their mean value is obtained. This value, corrected for refraction and dip, gives the true altitude. The difference between the two altitudes thus taken, would give the angular diameter of the sun or the moon, In finding the altitude of the sun or moon in this manner, allowance should be made for the motion of the body during the interval of observations.

In the absence of a well-defined horizon, an artificial horizon given by a basin of mercury may be used. The angle between the object and its image on the mercury surface is read by the sextant. Half this value corrected for refraction gives the true altitude.

287. The gyroscope.

The gyroscope or gyrostat is a heavy revolving disc capable of rotating about an axis perpendicular to the plane of the disc, the axis being supported by a frame work in such a manner that it can be pointed in any direction. This disc can be made to rotate very fast for a long time. When such a body is revolving rapidly and no external force or couple acts on it, its axis of rotation should point to a fixed direction. The disc and its axis are so mounted and are capable of movements only about axes that pass through the centre of gravity of the disc. If at the commencement of the spin of a gyroscope, its axis points to a certain star, it follows the star in its diurnal rotation, showing that its position is changing relative to the

surrounding objects of the earth. This is due to the rotation of the earth, since the axis should remain fixed in direction. Foucault used this apparatus to prove that the earth's rotation caused the diurnal motion of the stars.

288. The Filar Micrometer.

A micrometer is an instrument used for measuring the small angular distance between two objects seen simultaneously in the field of view of any telescope. With an equatorial, it may be often required to measure the angular diameters of the discs of planets or the separation of the components of binaries and the direction of the line joining them. For these purposes, the 'Screw and Position Micrometer' or the 'Filar Micrometer' is fitted to the eye-end of the equatorial telescope. Such a form of micrometer consists of a box of three frames carrying spider wires very nearly at the focus of the object glass of the telescope. One of these frames carrying two parallel wires is fixed, and the other two carrying each a single wire perpendicular to the two parallel wires could be moved along the direction of the parallel wires by screws and micrometer heads, fixed at the two extremities of the box. There is also a graduated circle behind the box. to read the angle through which the fixed frame of parallel wires is rotated in a plane perpendicular to the axis of the telescope.

In order to take observations on a binary, the micrometer is first turned as whole, so that the two components of the binary are along the direction of the fixed parallel wires. Then the movable wires are brought on to the two stars by turning the micrometer heads at the ends of the box and their readings are taken. Again the movable wires are made to cross each other, and adjusted so that each is again on the other star, and the readings of their second positions are taken. From these readings, the distance between the components is known in terms

of the revolutions of the micrometer screws. The value of one revolution of the micrometer in angular measure is inferred from the measured movement of a close circumpolar star in a known time when the telescope is fixed. From these, the angular distance between the components is calculated.

The reading of the graduated circle behind the box when the two stars are along the parallel wires, enables the observer to get the direction of the line joining the components. The angle that this direction makes with the declination circle through the primary star is called the *position angle.* Now any star is carried by the rotation of the celestial sphere in a direction perpendicular to the declination circle through the star, and therefore the position angle of such a direction is 90°. The reading of the graduated circle can easily be obtained for this position angle, by adjusting the micrometer in such a way that any star passes parallel to the fixed pair of wires when the telescope is at rest. The position angle corresponding to any other position of the fixed parallel wires when the componets of a binary are brought between them, is therefore known, and this gives the position angle of the binary.

Examples.

The following symbols are generally assumed in these examples :—

Terrestrial latitude	(φ)	Right Ascension	(α)
Celestial latitude	(λ)	Declination	(δ)
Celestial longitude	(β)	Hour angle	(h)
Obliquity	(ω)	Parallactic angle	(η)
Azimuth	(A)	Altitude	(a)

Zenith distance (z)

1. Determine the side a of the spherical triangle ABC, in which $A=117°-11'$; $C=154°-11'$ and the side $b=108°-31'$.

2. Find the side a and the angle B of a spherical triangle, given that $b=57°-43'$; $c=19°-18'$ and angle $A=120°-13'$.

3. Describe the changes that will be noticed in the apparent daily motion of the stars, by a man travelling along a meridian from the equator to the north pole.

4. Find the limit of declination for stars that are circumpolar at a place of latitude 75°. Find the least meridian altitude of a circumpolar star southing in a place of latitude 51°.

5. Show how the ecliptic could be drawn at any time for a given place. Mark the position of the sun on the ecliptic at 5 P. M. on September 12th, as seen by an observer at Trivandrum. (latitude $8\frac{1}{2}°$.)

6. At what time approximately will Regulus (R. A. $10^h-4^m-51^s$) cross the meridian on February 1st. Also

find the time of the year when Sirius (R. A. 6^h-42^m. Decl. $16°-37'$ S) will cross the meridian of Madras (lat. $13°-4'$ N) at 8 P. M. What will be its meridian altitude then?

7. Determine when an equatorial star of 10 hrs. R. A. will rise and set on the 10th of June. Where will the star be at noon?

8. Show that a star's altitude is the least arc that could be drawn from it to any point of the horizon.

9. Define morning and evening stars. Show when the star Spica (R. A. $13^h-21^m-43^s$ and decl. $10°-49'$ S) will be a morning or an evening star.

10. Indicate in a diagram of the celestial sphere for Madras the positions of the sun, the moon (3 days old) and the star Vega at 9 A. M. on the 23rd September, R. A. and declination of Vega being 18^h-34^m and $38°-42'$ N respectively.

11. When would you expect Fomalhaut (R. A. 22^h-54^m and decl. $29°-59'$ S) to be due south at about 8 P. M.

12. At winter solstice, the sun's meridian altitude at a place is 58°. Find the latitude of the place. What are the altitudes of the sun at noon on summer solstice and on the equinoxes?

13. A star crosses the meridian of a place at 2-12^m P. M. on the 21st March. When will it appear as a morning star?

14. Two stars S and S′ have their declinations $12°-24'-45''$ and $24°, 15'-40''$ and their R. A. differ by $42°-38'-41''$. Find the distance SS′.

15. If α_1, δ_1 and α_2, δ_2 be the poles of two great circles and ω their inclination to each other, show that $\cos\omega = \sin\delta_1 \sin\delta_2 + \cos\delta_1 \cos\delta_2 \cos(\alpha_1-\alpha_2)$ and hence

show that if α_1, δ_1 and α_2, δ_2 be the co-ordinates of two stars with respect to one system of co-ordinates and α_1', δ_1' and α_2', δ_2' be the co-ordinates of the same two stars with respect to a second system,

$$\sin\delta_1 \sin\delta_2 + \cos\delta_1 \cos\delta_2 \cos(\alpha_1 - \alpha_2)$$
$$= \sin\delta_1' \sin\delta_2' + \cos\delta_1' \cos\delta_2' \cos(\alpha_1' - \alpha_2')$$

16. Establish the following equations to obtain the longitude and latitude of a celestial body, from its R. A. and decl. :—

$$\sin\lambda = \cos\omega \sin\delta - \sin\omega \cos\delta \sin\alpha$$
$$\cos\lambda \sin\beta = \sin\omega \sin\delta + \cos\omega \cos\delta \sin\alpha.$$
$$\cos\beta \cos\lambda = \cos\delta \cos\alpha.$$

17. At any time find the altitude and azimuth of a body whose latitude and longitude are known.

18. If the R. A. of a star be equal to its latitude, show that its declination is equal to its longitude.

19. Find the azimuth and sidereal time when a star of R. A. (α) and decl (δ) rises in a place of latitude φ

20. When the longitude (l) minus R. A. (α) of a star is a maximum show shat $\sin(l-\alpha) = \tan^2 \frac{\omega}{2}$ and $\sin l = \cos\alpha = \frac{1}{\sqrt{2}} \sec \frac{\omega}{2}$ where ω is the obliquity of the ecliptic.

21. On June 1st, the time of transit of Vega (R. A. 18^h-34^m-31^s) was 18^h-33^m-40^s sidereal time. On the next day the sidereal time of transit of the same star was observed to be 18^h-33^m-28^s. Find the R. A. of a star which transited across the meridian on June 4th at 14^h-11^m-10^s sidereal time, the clock having the same error.

22. (i) Show that the velocity in azimuth at rising is the same for all stars at a given place.

(ii) Show that the cosine of the angle which the path of a star makes with the prime vertical is equal to the cosine of the latitude multiplied by the secant of the declination of the star.

23. A, B and C are the tops of three equal posts arranged in order, two miles apart along a straight canal. If the straight line A B passes 5 feet 4 inches above C, find the radius of the earth, assuming it to be spherical.

24. A steamer is moving north east at 15 miles an hour in latitude 45°. Find the rate at which it is changing its longitude.

25. The equatorial and the polar radii of the earth assumed to be a spheroid being r_1 and r_2, find the greatest difference between the geocentric and geographical latitudes of any place.

26. If the lengths of a degree of the meridian in latitudes 30° and 45° be s_1 and s_2 respectively, find the ellipticity of the earth.

27. C being the centre of the earth and O, a point on its surface whose geographical and geocentric latitudes are φ and φ' respectively, show that the reduction of the latitude is approximately equal to $c \sin 2\varphi$, where c is the ellipticity $\left(c=\dfrac{1}{300}\right)$ and that C O is approximately equal to $a\,(1-c \sin^2 \varphi)$, where a is the equatorial radius and b the polar radius. Also compute the geocentric latitude of A, whose geographical latitude is $42°-22'-45''$.

28. Assuming the earth to be spherical and neglecting refraction, find approximately the height of a peak of a mountain seen from another peak of height a at an altitude c from the horizon and at a distance d.

29. The setting sun slopes down to the horizon at an angle θ; prove that in latitude φ an observer on the top of a hill of height h will continue to receive the rays of the setting sun for $\dfrac{12 \operatorname{cosec} \theta \sec \delta}{\pi}\sqrt{\dfrac{2h}{a}}$ hours more than

another at the base of the bill, a being the radius of the earth and δ the sun's declination. Find to the nearest minute the acceleration in the time of sunrise on the 21st June for an observer on the top of a peak 10,000 ft. high in latitude 60°.

30. If s be the sun's semi-diameter in minutes of arc, show that the time taken by the sun to rise at a place of latitude φ is $\frac{1}{15} s (\cos^2 \varphi - \sin^2 \delta)^{-\frac{1}{2}}$ min. where δ is the declination of the sun; show also that sunrise takes place quickest at the equinoxes and slowest at the solstices.

31. Show that for a place on the arctic circle the daily displacement of the point of sunrise is equal to the daily change in longitude of the sun.

32. Show that for an observer in the Arctic circle the sidereal time of sunrise from December 22 to June 21 and the corresponding time of sunset from June 21 to December 22, are the same. Find the azimuth of the rising point, if l be the sun's longitude on any day.

33. What would be the character of the seasons if the earth's axis were at right angles to the ecliptic.

34. Find the inclination of the ecliptic to the horizon for an observer in latitude 15° S at 18 hrs. sidereal time. Indicate its position on the celestial sphere at noon on the 10th of August.

35. At what time approximately on September 1 would the ecliptic pass through the east and west points of Trivandrum (lat. 8°–30′) and what would be its inclination to the horizon?

36. Show that the points of intersection of the ecliptic and the horizon of some places oscillate on either side of the east and the west points in the course of a sidereal day, and that the inclination of the ecliptic to the horizon varies from $(90-\varphi-\omega)$ to $(90-\varphi+\omega)$

37. At a place in the temperate zone, show that the greatest and the least possible values for the azimuth of the sun at sunrise are given by $\cos^2 A = \sin^2 \omega \sec^2 \varphi$.

38. Show that the east hour angle of a rising body of declination (δ) is given by the equation

$$2 \cos^2 \frac{h}{2} = \sec \varphi \sec \delta \cos (\varphi + \delta)$$

39. If the declination of the sun between sunrise and sunset change from δ to $\delta + \theta''$, and if φ be the latitude, show that from winter solstice to summer solstice the afternoon will be longer than the forenoon by $\frac{\sec \delta \sin \varphi}{\sqrt{[\cos(\varphi+\delta)\cos(\varphi-\delta)]}} \cdot \frac{\theta}{15}$ seconds of time, and shorter by the same quantity during the remainder of the year.

40. Prove that the distances from the east and west points of C_1 and C_2, the points of intersection of the ecliptic with the horizon, are the values of θ given by the equation, $\cot \theta = \cot \omega \cos \varphi \sec 15\,t + \sin \varphi \tan 15\,t$, where φ is the latitude, t the sidereal time, and ω the obliquity.

41. Show that in N. lat. outside the Arctic circle, the points C_1 and C_2 of the previous question oscillate about the east and west points, and find the ratio of times taken to go from the most northernly to the most southernly positions and back again. How are these results modified for positions with in the Arctic circle ?

42. If at two places, the same two stars rise together, show that the places have the same latitude; if at one place they rise together and at the other place they set together, the places will be having the same polar distance from the opposite poles of the earth.

43. At a place whose latitude is 45°, the east hour angle of a star at the time of rising is 120°. Find its hour angle when it is due east.

44. Show that at a place of latitude 45°, the interval between the time when any star is due east and the time of its setting is constant.

45. Show that the change of zenith distance of a star by diurnal motion is always proportional to its change in hour angle, provided the declination of the star and the latitude of the place are each equal to zero.

46. If the latitude of a place be less than the declination of a star, the maximum azimuth of a star will be $\sin^{-1}\left(\frac{\cos\delta}{\cos\varphi}\right)$ and its hour angle is then given by $\cos^{-1}(\tan\varphi\cot\delta)$

47. Find the zenith distance and parallactic angle of a star of declination $38°-9'$ N and east hour angle 45°, the latitude of the observer being $53°-23'$.

($34°-10'$ and $48°-41'$)

48. If the altitude of a star is equal to the latitude of the place of observation, show that the hour angle and azimuth are respectively, $\cos^{-1}\left\{\tan\varphi\tan\left(\frac{\pi}{4}-\frac{\delta}{2}\right)\right\}$ and $2\sin^{-1}\left\{\sec\varphi\sin\left(\frac{\pi}{4}-\frac{\delta}{2}\right)\right\}$.

49. At a certain instant the same star is found to be at the zenith of a place of latitude φ_1, to be rising at a place of latitude φ_2, and to be on the prime vertical of a place of latitude φ_3, show that the longitude differences of the places are given by $\cos^{-1}(\tan\varphi_1\tan\varphi_2)$ and $\cos^{-1}(\tan\varphi_1\cot\varphi_3)$

50. Prove that at any place the interval between one of the transits of any star across a vertical of azimuth A and one of its transits across the other vertical equally inclined to the meridian is equal to $\frac{\cot^{-1}(\sin\varphi\tan A)}{\pi}$ of a sidereal day. (M. U.)

51. If at any place two stars whose declinations are δ and δ' rise together and the former comes to the meridian when the latter sets, show that $\tan\varphi \tan\delta = 1-2\tan^2\varphi \tan^2\delta'$

52. Two stars of declinations δ_1 and δ_2 are due east at the same time and they also set at the same time. Find the latitude of the place of observation and the interval between their times of rising.

53. If α_1, δ_1 and α_2, δ_2 be the R. A. and declinations of two stars having the same longitude, show that their right ascensions differ by $\sin^{-1}[(\tan\delta_1 \cos\alpha_2 - \tan\delta_2 \cos\alpha_1) \tan\omega.]$

54. If η be the parallactic angle of a star, show that the rate of change of its azimuth with respect to the hour angle is given by $\cos\eta \cos\delta \operatorname{cosec} z$, and hence show that for a star rising to the north of east, the azimuth changes at the same rate when it is due east as when it rises. Find the azimuth of the star when this rate is a minimum.

55. If δ be the star's declination and A its maximum azimuth, show that in t seconds of time from the moment when the azimuth is A, the azimuth has changed by $\frac{1}{2} 15^2 t^2 \sin 1'' \sin^2\delta \tan A$ seconds of arc. (M. U.)

56. If the latitude of a place be less than the declination of a star, show that the maximum rate of change of the zenith distance of the star is equal to the cosine of the star's declination. If the latitude of the place be greater than the declination, find the corresponding value.

57. The greatest azimuth attained by a circumpolar star in latitude 45° is also 45°. Find the star's declination.

58. Show that the greatest and the least values of the hour angle of the sun, when it sets at a place in the temperate zone are $\cos^{-1}(-\tan\varphi \tan\omega)$ and $\cos^{-1}(\tan\varphi \tan\omega)$.

59. If the sun's declination be 15° N, find the range of latitude on the earth's surface where the sun will not set that day. Find also the latitude of a place where there will be twilight throughout the night.

60. Show how the duration of twilight is shorter at a place in the Torrid zone, than at a place in the Temperate zone.

61. Prove that if the point where the sun rises when its declination is δ is at distances x and y from the extreme points, $\sin \frac{x-y}{2} = \sin \delta \sec \varphi$.

62. Why should December be one of the hottest months in South Africa (Lat. between 20°S and 35°S).

63. Show that in Cambridge (Lat. 52°-$13'$) twilight lasts for some days throughout the night.

64. Assuming the earth's orbit round the sun to be circular, show that the perpetual day at any place in the Arctic circle lasts so long as $\tan \varphi \tan \delta > 1$ and that its duration is given by $\frac{365\frac{1}{4}}{\pi} \cos^{-1}(\cos \varphi \operatorname{cosec} \omega)$ days where δ is the sun's declination at any time.

65. Show that for places within the Torrid zone, the ecliptic will be vertical twice a day at intervals separated by $2 \sin^{-1}(\tan \varphi \cot \omega)$ sidereal hours. (A. U.)

66. Show that near the equator the phenomenon known as the 'Harvest Moon' will be less marked than in the Temperate zones and that it will be noticed at every equinox.

67. If at the vernal equinox, the sun's R. A. increases daily at the rate of 3 min. 38 sec., find the minimum retardation of the sidereal time of sunrise at a place of latitude 52°-$13'$, taking ω to be 23°-$27'$.

68. If the sun whose declination is δ sets at a point distant c and d from the extreme points of setting in the course of the year, show that $\dfrac{\tan c/2}{\tan d/2}=\dfrac{\tan(\omega+\delta)/2}{\tan(\omega-\delta)/2}$

69. In north latitude φ a full moon rises at the autumnal equinox. If θ be the distance the moon goes ahead of the sun in one day, prove that on the following day the moon will rise later by

$$\frac{720}{\pi}\left\{\tan^{-1}(\cos\omega\cot\theta)-\sin^{-1}\left(\frac{\tan\varphi\sin\theta\sin\omega}{\sqrt{1-\sin^2\theta\sin^2\omega}}\right)\right\}$$

minutes, the angles being computed in circular measure. Explain what would happen in south latitude φ. (M. U.)

70. Assuming that the sun moves uniformly in the ecliptic in 365 days, show that in latitude φ, the number of nights in which there is twilight althrough the night is either the integral part or the integer next greater than $\dfrac{73}{36}\cos^{-1}\left\{\dfrac{\cos(\varphi+18^\circ)}{\sin\omega}\right\}$ where ω is the obliquity of the ecliptic and 18° is the sun's greatest angular distance below the horizon for twilight to be possible. (A. U.)

71. Show that when the difference between the longitude of any point of the ecliptic and its right ascension is a maximum, $\tan\beta=\sqrt{\sec\omega}$ and $\tan\alpha=\sqrt{\cos\omega}$.

72. Find the R. A. of a star of declination 16°-$34'$-$2''$ S and the errors of the sidereal clock in March and September from the following observations, assuming the rate of the clock to be constant.

	25th March	18th Sept.	19th Sept.
Decl. of the sun at noon	1° - 48′ - 56″	1° - 53′ - 0″	1° - 29′ - 43″
Time of transit of the sun	H M S 0 - 15 - 6	H M S 11 - 42 - 42	H M S 11 - 46 - 17
Time of transit of the star	6 - 36 - 10	6 - 40 - 25	6 - 40 - 25

73. Show how from the following observations, the R. A. of the sun on the 30th of March can be calculated.

	Decl. of sun.	Time of transit of sun h.	m.	s.	Time of transit of star h.	m.	s.
30th March	4°- 0′- 8″	0	1	4·47	15	1	54·76
11th Sept.	5°-20′-59″	0	1	4·09	4	19	11·38
12th Sept.	3°-58′- 3″	0	1	4·07	4	15	49·35

74. A few days after the Vernal Equinox, the declination of the sun is observed, and the difference of the R. A. of the sun and Vega is observed to be 17 h. 36 m. 38 s. Some days before the Autumnal Equinox when the sun's declination is the same as that at the first observation, the difference between the R. A. of the sun and Vega is 7 h. 32 m. 8 s. Find the R. A. of Vega.

75. Fix the position of the first point of Aries from the following data:—

	Decl. of sun at noon	Time of transit of sun h.	m.	s.	Time of transit of Canopus h.	m.	s.
25th March	1°-39′-12″	0	20	16	6	27	29
18th Sept.	2°- 2′-16″	11	39	11	6	20	29
19th Sept.	1°-39′- 1″	11	42	46	6	20	29

76. Find the R.A. of Altair from the following table:—

	Sun's Declination	Difference of R. A. of sun and Altair h. m. s.
21st March noon	0° — 2′ — 50″ S	19 — 47 — 38
22nd March noon	0° — 22′ — 52″ N	19 — 44 — 0

77. The sun's declination at transit on the 22nd of September was 17′-2″·8 N and on the next day, it was 6′-21″·6 S. The sidereal interval of time between the two transits was 24 h. 3 min. 35 sec. Find the sun's R. A. at

the time of the first observation. Discuss how far the above observations can be used to fix the position of the first point of Aries.

78. If on a certain day near the summer solstice the sun's R. A. is found to be $90° - x$, and the declination is δ, establish the following formula to determine the obliquity of the ecliptic. $(\omega - \delta)'' = \dfrac{\tan^2 \frac{x}{2}}{\sin 1''} \sin 2\,\omega - \dfrac{\tan^4 \frac{x}{2}}{\sin 2''} \sin 4\,\omega,$ and show how it is to be used in calculating the obliquity.

79. If a be the mean distance of a planet in astronomical units from the sun and T its sidereal period, show that the mean anomaly is given by $m = \dfrac{2\,\pi\,t}{a^{\frac{3}{2}}\,\mathrm{T}}$.

80. If the velocity of mercury be taken to be 30 miles per second in its orbit, find the velocity of saturn, given the distances of mercury and saturn from the sun to be respectively 0·39 and 9·54 of the earth's distance from the sun.

81. If e be the eccentricity of the orbit of the earth round the sun, and if u and v are the known velocities of the earth at perihelion and aphelion, the eccentricity is given by $\dfrac{u-v}{u+v}$.

82. Show that the velocity of a planet at any time can be resolved into two components one of which is perpendicular to the radius vector to the sun and the other is perpendicular to the major axis of its orbit round the sun, and find their magnitudes.

83. Show that the number of times an inferior planet crosses the meridian of a place is the same as that of the sun and that it is less by one than that done by a superior planet.

84. Show that the angle between the direction of the motion of a planet and the radius vector joining it to the sun is given by $\tan^{-1} \dfrac{\sqrt{1-e^2}}{e \sin u}$, where u is its eccentric anomaly.

85. Prove the following graphical construction due to Adams, for finding a solution of the equation $m = u - e \sin u$. Draw the curve $y = \sin x$; from the origin take along the x axis $OM = m$; through M draw a straight line inclined to the axis at $\cot^{-1} e$ cutting the sine curve at P. The abscissa of P gives the value of u.

86. If m be the mean anomaly, v the true anomaly and e the eccentricity of the orbit of a planet, show that

$$\frac{m}{2} = (1-e)\left(\frac{1-e}{1+e}\right)^{1/2} \tan\frac{v}{2} - \frac{1-3e}{3}\left(\frac{1-e}{1+e}\right)^{3/2} \tan^3\frac{v}{2}$$

$$+ \frac{1-5e}{5}\left(\frac{1-e}{1+e}\right)^{5/2} \tan^5\frac{v}{2} - \ldots\ldots\ldots\ldots\ldots\ldots \text{(M. U.)}$$

87. Show that at any place the sidereal time is mean time added to the mean sun's R. A.

88. If r and s be the mean times of sunrise and sunset on any day and E the equation of time for that day, show that $2\,E = s - (12 - r)$ assuming the sun's declination to remain unaltered from sunrise to sunset. If the morning of any day is 10·4 mins. longer than the afternoon of the same day, find the equation of time for that day.

89. Find the mean time of sunset on any day when the true sun rises at 6 hrs. 50 min. mean time and crosses the meridian 16 min. 18 sec. earlier than the mean sun.

90. At two stations A and B on a certain day, the times of rising of the sun are 5 hrs. 38 min. and 4 hrs. 58 min. respectively. If the sun sets at 6 hrs. 46 min. at

A on that day, find the local mean time of sunset at B on the same day. (M. U.)

91. At mean noon on a given date the sidereal time is 14 hrs. 20 min. What will be the sidereal time 45 days later at mean noon at the same place?

92. Find the sidereal time corresponding to 4-30 P. M. on December 20th, the sidereal time of mean noon of the same date being 17 hrs. 55 min. 8 sec.

93. Find the sidereal time corresponding to mean noon (Indian Standard Time) on a certain date, when the sidereal time of Greenwich mean noon on the same date is 11 hrs. 2 min. 21 sec., the sun's hourly variation in R. A. being 9 sec.

94. Find the mean solar time corresponding to 5 hrs. 17 min. 25 sec. sidereal time at a place of east longitude $35^\circ - 30'$, the sidereal time of mean midnight at Greenwhich being 11 hrs. 50 min. 12 sec.

95. If the sidereal clock indicates 23 hrs. at 4 P. M. on a certain day, what is the time of the year and find approximately the sidereal time at 10 A. M. on the 1st of February.

96. Why should the mean solar time be devised? On a certain day the forenoon was 22 min. longer than the afternoon, and the night was 1 hr. 16 min. longer than the day. Find the time of sunset on that day.

97. Show how the time of sunset is earliest some days before and the time of sunrise latest some days after the shortest day.

98. If n is the number of mean solar days in the Tropical year, and s is the sidereal time at mean noon for

any place, show that the mean time of sidereal noon for the place is given by $(24^{h}-s)\frac{n}{n+1}$, and the corresponding quantity for a place L^{h} west of this place is obtained by subtracting $\frac{L}{n+1}$ from the above.

99. Mars rotates on its axis in 24 hrs. 37 min. and its sidereal period is 686 days. Find the length of the mean solar day for a Martian observer. Also find the difference between the mean solar day and sidereal day in Mars.

100. Find the R. A. of the true sun at true noon on November 25th from the following data :—

Equation of time on 25th November.	$=-12$ min, 45 sec.
Sidereal time of mean noon on 2nd September.	$=10$ hrs. 47 min. 26 sec.

101. If the eccentricity were zero, the equation of time in minutes would be $\frac{720}{\pi}\tan^{-1}\left(\frac{(1-\cos\omega)\tan L}{1+\cos\omega\tan^{2}L}\right)$ where ω is the obliquity and L the sun's longitude. (M. U.)

102. (i) Explain the necessity for a standard time over a certain country or area.

(ii) Find the standard time of the transit of Pollux at Madras on 20th January 1934, given :—

R. A. of Pollux	7 hrs. 41 min. 17 sec.
Mean time of the transit of the first point of Aries at Greenwich on June 19th 1934.	16 hrs. 6 min. 34 sec.

Do.	on 20th June 1934	16 hrs.	2 min.	38 m sec.	
Longitude of Madras		5 „	20 „	59·12 „	East

(M. U.)

103. Find the Indian Standard time of the transit of Jupiter at Bombay on 1st March 1936 from the following data :—

	hr.	min.	sec.	
Longitude of Bombay	4	51	15·6	East
R. A. of Jupiter at 0 hr. at Greenwich on 1st March 1936	17	25	8·5	
Variation of R. A. in 24 hrs.			30·5	
Mean time of sidereal noon at Greenwich on 1st March 1936	13	23	19·6	

(M. U.)

104. Find the equation of time at 8-32 P. M. on the 14th April at Greenwich from the following data. Find also the corresponding apparent time at a place in west longitude 182° – 30′.

At mean noon for Greenwich the equation of time to be subtracted from mean time

	min.	sec.
14th April	0	24·64
15th April	0	9·53

(M. U.)

105. At apparent noon at a certain place the Greenwich mean time is 5 hrs. 4 min. 30 sec. Find the longitude of the place, given the equation of time that day to be 16 min. 12 sec.

106. Show that the greatest value of the equation of time arising from eccentricity is $\frac{24e}{\pi}$ hours, and that the maximum value of the reduction to the equator is $\sin^{-1}\left(\tan^2\frac{\omega}{2}\right)$

107. Find the mean times at apparent noons on the equinoctial days.

108. When the equation of the centre has a turning value, show that the distance between the sun and the earth is the geometric mean between the semi-major and semi-minor axes of the earth's orbit round the sun.

109. Show that the reduction to the equator varies between $+(2°-28'-8'')$ and $-(2°-28'-8'')$ taking the mean value of the obliquity to be $23°-27'-4''$.

110. Obtain an expression for the hourly rate of change for the equation of time, the daily motion of the mean sun being $59'-8\cdot''32$, and find its value at the winter solstice.

111. At mean times T and T′, $\alpha°$ and $\alpha'°$ are the hour angles of the true sun; find the equation of time at the preceding and the following mean noons.

112. If the apse line of the earth's orbit be perpendicular to the equinoctial line, show that the sun's longitude when the equation of time due both to eccentricity and obliquity is numerically maximum, is given by

$$e \sin \mathrm{L} - \tan^2 \frac{\omega}{2} \cos 2\,\mathrm{L} = 0.$$

113. Taking into consideration, the forward motion of the apse line of the earth's orbit and the retrograde motion of the line of equinoxes, find the extreme limits to the lengths of the four seasons.

114. If t_1 and t_2 are the hourly variations in the equation of time when the sun is at perigee and apogee, show that the eccentricity of the earth's orbit is nearly $\frac{t_1 - t_2}{t_1 + t_2} \tan^2 \frac{\omega}{2}$ assuming the equinoctial line to be perpendicular to the apse line of the earth's orbit.

115. When the equation of time has a turning value, show that the projection of the sun's position on the equatorial plane lies on a circle of radius $\sqrt{ab\cos\omega}$ with the earth as the centre.

116. Show that if the sun be supposed to move uniformly along the ecliptic and a point to move at the same uniform rate along the equator, the difference between the R. A. of both will vanish four times a year provided the interval between their passages through the first point of Aries be less than $\dfrac{\sin^{-1}(\tan^2\omega/2)}{2\pi}$ of a year. (M. U.)

117. Find the years in this century having five Saturdays in February.

118. The sun's meridian altitude on December 4th is found to be 16° - $8'$ and the chronometer time at that moment is 6^h - 5^m - 12^s, its error being 3^m - 20^s. The sun's declination at the preceding mean noon was 22° - $19'$ - $25''$S and its hourly change in declination was $19''\cdot6$. Find the latitude of the place.

119. Given the time of the year when a known star rises with the sun, find the latitude of the place.

120. Show that the hour angle, altitude and declination of a star which is due east enable us to determine the latitude of the place.

· 121. The zenith distance of a star whose R. A. is 21 hours and declination 30° N when it crosses the prime vertical is observed to be 42°. Calculate the latitude. (M. U.)

122. If a star rises at a place at the northeast point and crosses the prime vertical at an altitude of 45°, find the latitude of the place.

123. When the sun's longitude is L, it is due east at an altitude a. Show that the latitude of the place satisfies the equation $\sin\varphi\sin a = \sin\omega\sin L$.

124. A star of known R. A. and declination rises at 45° north of east. Find the latitude of the place.

125. A star of hour angle h and declination δ has azimuth a, and its azimuth changes to $(180° + a)$ when the hour angle is h'. Find the latitude of the place of observation.

126. Two stars of R. A. and declinations (α, δ) and (α', δ') are observed to rise simultaneously. Show that the latitude of the place is given by the equation

$$\varphi = \cot^{-1} \frac{\sqrt{\tan^2 \delta + \tan^2 \delta' - 2 \tan \delta \tan \delta' \cos (\alpha - \alpha')}}{\sin (\alpha - \alpha')}$$

127. Two stars have the same azimuth, when their hour angles are h and h'. Show that the latitude of the place of observation is given by

$$\tan \varphi = \frac{\tan \delta \sin h' - \tan \delta' \sin h}{\sin (h' - h)}.$$

128. Two known stars are simultaneously due east at a place. Find the latitude of the place.

129. Two known stars have the same azimuth. Find the latitude of the place from the observed altitude of one of these.

130. Two places have the same latitude and the distance of the pole from the great circle through them is equal to the sun's declination on a particular day. Prove that at these places, the length of that particular night is equal to their difference of longitude.

131. Show how the latitude of a place can be determined by knowing the azimuths of the sun while rising on the solstitial days.

132. Explain how the 'position circle' is used to determine the position of a ship at sea. What further

data would be required to determine the latitude and longitude of the ship?

133. A known star crosses the southern meridian. Its altitude is observed to be h° and the time of transit to be T hours after noon (Greenwich mean time). From the Nautical Almanac, the mean sun's R. A. is found to be t hours at Greenwich noon on the day of observation. If α hours be the R. A. and δ° the declination of the sun, prove that the ship is in latitude $(90+\delta-h)$ and west longitude $15\left(t-\alpha+\mathrm{T}\,\dfrac{366\frac{1}{4}}{365\frac{1}{4}}\right)$ degrees.

134. Two known stars are seen at a given place on the same vertical when at another place they are seen to be rising together. Find the longitude of the latter place.

135. A star of known declination δ which never sets at a place is observed at that place to cross the same vertical circle at altitudes a_1 and a_2. Show that the latitude φ of the place is given by the equation

$$\sin\varphi\sin\frac{a_1+a_2}{2}=\sin\delta\cos\frac{a_1-a_2}{2}\quad\text{or}$$

$\sin\varphi\cos\dfrac{a_1-a_2}{2}=\sin\delta\sin\dfrac{a_1+a_2}{2}$ according as the small circle described daily by the star does or does not surround the zenith. (A. U.)

136. Two places on the globe have telegraphic communication. Show how their longitude difference can be determined. Mention the various errors to which the method is liable and show how they could be eliminated.

137. Describe a method of determining the longitude of a point in the sea by observing the altitude of the sun.

138. Find the difference of longitude between A and B from the following and point out which of them is the more eastern.

		H.	M.	S.
On 1st June the chronometer is faster than local time at	A by	5	45	7·5
On 8th June „	B	5	52	29·2
On 14th June „	B	5	52	49·0
On 21st June „	A	5	46	10·7

139. A ship leaving San Francisco (west long. 122°) on Friday morning at 6 A. M. on January 15th, reaches Yokohama (East long. 140°) after a passage of exactly 16 days. On what day of the month and of the week does she arrive? (M. U.)

140. At a certain place the sun has equal altitudes at 7 hrs. 20 min. and 9 hrs. 36 min. chronometer time, Find the longitude of the place, given the equation of time on that day to be 12 min. 11 sec. (M. U.)

141. Show that due to precession stars on the equinoctial colure undergo the same change both in R. A. and declination whereas stars on the solstitial colure do not change their declinations at all.

142. Show that precession increases the interval between two consecutive transits of a star by $0·00366 \sin \alpha \tan \delta$ over a sidereal day.

143. If the effect of precession is to change only the declination of a star, show that the celestial pole and the pole of the ecliptic subtend a right angle at the star.

144. Show that the stars undergoing no changes of R. A. due to precession and nutation lie on the conical surface $f\ (x^2+y^2)+\frac{1}{15}\,g\,z\ (x \sin G+y \cos\ G)=0$ where f, g and G are the *independant day numbers* for the day.

145. If the effects due to nutation be omitted in the preceding question, show that the stars will lie on an elliptic cone passing through the poles of the ecliptic and the equator.

146. If the constant of precession be given by the formula $50''\cdot2453 + 0''\cdot0002225\ t$ where t represents the number of years since 1850, find the period of retrograde motion of the first point of Aries round the ecliptic.

147. When the latitude of a star is greater than that of the celestial pole, show that there will be no difference between the sidereal day determined by the transit of the first point of Aries and that by the transit of the same star, if the longitude of the pole differs from that of the star by

$$\cos^{-1}\frac{\tan\ (\text{lat. of pole})}{\tan\ (\text{lat. of star})}$$

148. If K be the constant of precession and $\Delta\,L$ and $\Delta\,\omega$ be the nutations in longitude and obliquity, show that the effect of precession and nutation for t years will be the same as that produced by rotating the celestial sphere of stars by $\left\{(Kt + \Delta\,L)^2 + (\Delta\,\omega)^2\right\}^{\frac{1}{2}}$ in a retrograde direction about an axis passing through the point of zero longitude and $\tan^{-1}\left(\dfrac{Kt+\Delta\,L}{\Delta\,\omega}\right)$ latitude.

149. Find the maximum displacement possible for a star due to precession and nutation given in the previous question.

150. Prove that the stars on the celestial sphere whose declinations undergo the greatest change in a given period owing to the precession of the equinoxes, lie on two arcs of a great circle; and that the stars whose declinations are unchanged at the end of the period lie on another great circle.

(A. U.)

151. Find the aberration of a star X in any direction XX′.

152. Show that aberration shifts all stars on a great circle to a parallel small circle.

153. If the aberration of a star in R. A. is stationary, show that the R. A. of the star is the same as that of the sun.

154. When aberration of a star in declination attains its greatest numerical value for the year, the sun and the celestial pole subtend a right angle at the star.

155. Show that when the aberration in R. A. of a star on the equator is maximum, the sun's longitude is $\tan^{-1}(\tan \alpha \cos \omega)$; and find the corresponding value when the aberration in R. A. of the star is least.

156. Two stars having the same latitude λ and a mean value φ for their longitudes are separated by a distance θ. Find the apparent distance between them as altered by aberration.

157. Show that the aberrational ellipse of a star on the plane touching the celestial sphere at the mean place of the star is the orthogonal projection of a circle in the ecliptic plane.

158. Show that the phenomenon of annual aberration could be explained if the stars were assumed to move on small circles parallel to the ecliptic, the earth being taken to be at rest.

159. The apparent position of a point S, whose co-ordinates on the celestial sphere are (x, y) is displaced by a small distance S S′ along the great circle towards a fixed point Q, whose co-ordinates are (x_0, y_0) so that S S′ is equal to K sin S Q where K is small. Prove that the changes in the co-ordinates x and y are given by the equations

$$\Delta x = \text{K} \sin (x_0 - x) \cos y_0 \sec y$$

$$\text{and } \Delta y = \text{K} [\cos y \sin y_0 - \sin y \cos y_0 \cos (x_0 - x)]$$

Hence or otherwise find the changes in the R. A. and declination of a star due to aberration. (M. U.)

160. Show that owing to aberration, a star at the pole of the ecliptic appears to describe a circle and a star on the ecliptic appears to oscillate to and fro in a straight line in the course of a year.

161. Show that the locus of all stars whose zenith distances at a given instant in a given place, are unaffected by aberration, is an elliptic cone, one of whose circular sections is horizontal and the other is perpendicular to the ecliptic.

162. Prove that all stars whose aberration in R. A. is a maximum and whose aberration in declination is zero lie on the cone, $x^2+y^2+z^2-z(z-y\tan\omega)=0$, or on the solstitial colure.

163. Show that the effect of aberration on the distance θ between two stars is to reduce it by

$$K\tan\tfrac{1}{2}\theta\,[\cos\beta_1\sin(S-\lambda_1)+\cos\beta_2\sin(S-\lambda_2)]$$

where (β_1, λ_1), (β_2, λ_2) are the latitudes and longitudes of the stars and S the sun's longitude.

Hence or otherwise show that two very near stars on the ecliptic appear to approach and recede from one another during the course of the year.

164. The apparent positions of a star when the earth is in perihelion and aphelion are P and Q respectively. Show that the true position of the star is at R in PQ such that $\frac{PR}{RQ}=\frac{1+e}{1-e}$, e being the eccentricity of the earth's orbit. Show that PQ is conjugate to the diameter of the ellipse formed by drawing a great circle through its centre and the apses of the earth's orbit. (M. U.)

165. The meridian altitude of the sun's lower limb is observed to be 62°-24′-45″ south. If the dip of the horizon be 4′, the sun's declination 20°-55′-10″ N, its semi-diameter 15′-47″, the parallax at the observed altitude 5″ and the mean refraction at that altitude 30″, find the latitude of the place. (Parallax increases the zenith distance.)

166. The apparent zenith distances of a star at lower and upper culminations are $75°\text{-}3'\text{-}23''{\cdot}2$ and $1°\text{-}53'\text{-}18''{\cdot}6$ S, the amounts of refractions being $3'{\cdot}41''{\cdot}9$ and $1''{\cdot}9$ respectively. Find the declination of the star and also the latitude of the place. (M. U.)

167. At a place of latitude 43°, the meridian altitude of a star is found to be $67°\text{-}35'$. Find the declination of the star, if the coefficient of refraction is $58''$.

168. From the following meridian circle observations on β Ursa Minoris at its upper and lower culminations determine the latitude of the place and the declination of the star.

	Upper culmination.	Lower culmination
Altitude	$55°\text{-}48'\text{-}6''$.	$24°\text{-}58'\text{-}56''$
Temperature	30° F	25° F
Pressure	30·1 ins.	30·1 ins.

(The refraction correction for zenith distance z being given by the Comstock formula $r''=\dfrac{983\,b}{460+t}\tan z$, where b is the barometric pressure in inches and t the temperature in degrees Fahrenheit.)

169. If K be the coefficient of refraction, the sine of the zenith distance of an object will be reduced by refraction to $(1-K)$ of its true value.

170. Show that the azimuth of a star is maximum when its declination is unaffected by refraction, provided the latitude of the place is less than the star's declination.

171. If the law of refraction be given by the expression $\rho=58''{\cdot}294\tan z-0''{\cdot}0668\tan^3 z$, find from the formula of Simpson the value of the refractive index of the lowest stratum of the earth's atmosphere and obtain the approximate height of the effective atmosphere causing refraction.

172. Show that if the law of refraction be $\rho = K \tan z$ the apparent places of a star as seen by different observers on the same meridian would lie on a great circle.

173. Assuming that the refraction of any object S is equal to K tan ZS show that the resolved parts of the refraction in R. A. and north polar distance expressed respectively in seconds of time and seconds of arc are nearly $\frac{K}{15} \cdot \frac{\tan ZL}{\sin \Delta \cos (\Delta - PL)}$ and $K \tan (\Delta - PL)$ where Δ is the north polar distance of the object, P the pole, Z the zenith, and ZL an arc of a great circle drawn from Z perpendicular to P S. (M. U.)

174. If K be small in the previous question, show that the change in the hour angle of a circumpolar star is greatest if the north point of the horizon of the place and the pole subtend a right angle at the star and that the greatest value is $K \cos \varphi \sec \delta \sqrt{\sec z_1 . \sec z_2}$ where z_1 and z_2 are the greatest and least zenith distances of the star. (M. U.)

175. Show that the apparent place of a star describes each sidereal day a conic section, assuming the same law of refraction as in question (172).

176. Show how the parallax of the moon is determined by meridian observations. Why is not solar parallax determined in this way?

177. The sun's horizontal parallax is $8''.806$ and its angular diameter is $16' - 1''$. Find its diameter in miles.

178. Explain how the distance of the sun from the earth could be determined from observations on Eros.

179. Show that the geocentric parallax of the moon is $\frac{\sin P \sin z}{(1 - \sin P \cos z)}$, where P is its horizontal parallax and z its geocentric zenith distance.

180. Show that the distance of the moon is increased by parallax in the ratio $\sin z' : \sin (z' - \sin p)$ where p is the parallax at apparent zenith distance z' the earth being assumed to be spherical.

181. Show that if h' and δ' be the hour angle and declination of the moon as seen from the earth's surface at a place of geocentric latitude φ', and h, δ, the hour angle and declination as seen from the centre, then,
$\sin (h' - h) = a \sec \delta \sin h'$

$\tan \delta' \operatorname{cosec} h' = (1 - b \operatorname{cosec} \delta) \tan \delta \operatorname{cosec} h$

where $a = \sin \pi' \varphi \cos \varphi'$ and $b = \sin \pi' \varphi \sin \varphi'$,

$\pi' \varphi$ being the horizontal parallax of the moon at latitude φ. (M. U.)

182. Show that due to parallax, the time during which the sun is below the horizon at the pole is longer by 7 cosec ω minutes, the sun's horizontal parallax being $8''.8$.

183. Show that all bodies which have their parallaxes in R. A. the same at any instant, will lie on a right circular cylinder touching the meridian plane along the axis of the earth, provided their parallaxes are very small.

184. Explain what is meant by geocentric parallax and annual parallax; and obtain the fundamental equations to find out the former.

185. Show that the parallaxes in R. A. and declination of the moon are respectively

$$\pi' \alpha = - \tan^{-1} \left\{ a \sin h / (1 - a \cos h) \right\}$$

$$\pi' \delta = - \delta - \tan^{-1} \left\{ (\tan \delta - a \tan \varphi') \sin \pi' \alpha \,/ a \sin h \right\},$$

where $\pi' \varphi$ is the horizontal parallax, h the hour angle, φ' the geocentric latitude and $a = \sin \pi' \varphi \cos \varphi' \sec \delta$.

(M. U.)

186. If the annual parallax of α Centauri is $0''\cdot 75$, compare its distance with the distance of the sun from the earth.

187. Find roughly the distance of a star whose parallax is $0''\cdot 5$, given that the sun's parallax is $9''$ and the earth's radius is 4,000 miles.

188. Show that due to annual parallax, the sidereal time of transit of a star of R. A. α is different on different days and that the correction to the observed time due to annual parallax is maximum $\frac{1}{2\pi}\, 365\frac{1}{4}\, \tan^{-1}\left\{\sec \omega \tan \alpha\right\}$ days after a solstice, assuming the sun's motion on the ecliptic to be uniform.

189. Find the maximum effects of annual parallax on the R. A. and decl. of a star.

190. Show that due to annual parallax and aberration, the apparent place of a star is not shifted away from the aberrational ellipse.

191. Find the time of new moon from the following:—

	Sun's longitude.	Moon's longitude.
July 26th noon.	$125°—4'—30''$	$122°—0'—0''$
,, midnight	$125°—34'—30''$	$128°—41'—15''$.

192. Explain why moonlit nights are much brighter in India than in Europe. State in what part of the year, full moon nights are brightest.

193. The moon is seen at Trivandrum just rising at 2 A. M. on a particular day. Find the area of the illuminated portion of the moon's disc, assuming its radius to be a. When will the moon rise next day?

194. Describe how the earth would appear as seen by a lunar observer, stating its size, phases and motion relative to the lunar horizon.

195. Find the exact time of full moon from the following :—

	longitude of sun.	longitude of moon.
31st December noon	279°—36′—30″	95°—0′—30″
„ midnight	280°— 7′— 0″	102°—1′—30″

196. The moon is observed at a certain instant to be on the meridian three quarters full, and with the illuminated limb to the west. How old is it then, and what is the time of the day?

197. Show that the apparent area of the bright part of the moon is approximately proportional to $1-\cos$ (*moon's elongation*).

198. 3rd May 1924 and 24th March 1925 are both new moon days and there are 11 lunations between the two new moons. Find the sidereal period of the moon.

199. A satellite revolves in a circle of radius b about a primary, which revolves about a fixed centre in a circle of radius a, the angular velocity of the satellite being n times that of the primary. Show that the satellite as seen from the fixed centre, will have a certain part of its path convex, if a lies between b and $n\,b$ miles. Deduce that the path of the moon is everywhere concave to the sun. (M. U.)

200. The synodic period of Saturn is 378 days. Find the sidereal period of the planet and its mean distance from the sun, in terms of the earth's distance from the sun. Represent in a figure its geocentric motion during the sidereal period.

201. Show that the sidereal period of Jupiter is 11·86 years, if its distance from the sun is 5·2 astronomical units. Find the synodic period.

202. If the mean distance of Saturn from the sun be 9 astronomical units, find the time during which the

motion of Saturn is retrograde during a synodic period, the sidereal period being 29·5 years.

203. When two planets whose distances from the sun are c and d appear stationary, they subtend an angle of 60° at the sun. Show that $c^2+d^2=7\,cd$.

204. Mercury is at a distance of 0·39 of an astronomical unit. Show that when it is stationary, the line joining the earth and the planet subtends at the sun an angle $\cot^{-1} 3$, assuming the planets to be moving in circular and coplanar orbits.

205. Account for the fact that no inferior planet could be seen at Trivandrum or Madras, at midnight. Why is Mercury rarely visible to the naked eye?

206. Show that the apparent motion of Jupiter is retrograde when we are nearest to it and direct, when we are farthest.

207. An observer in Mars would find the nearer of the two satellites rising in the west and setting in the east. Explain this.

208. The greatest elongation of Venus is 45° and its sidereal period is $\frac{8}{13}$ of a year; show that it will be at its greatest elongation at intervals of $\frac{6}{5}$ and $\frac{2}{5}$ of a year.

209. The synodic period of an asteroid is 1·4 years. Find its mean distance from the sun.

210. On a certain day, when Saturn is in conjunction with the sun, its heliocentric longitude is 130°. If the synodic period of Saturn is 378 days, find its longitude 150 days later.

211. The synodic period of Mars is 780 days and the sidereal period is 1·52 years. Find the time taken by it to move from opposition to quadrature.

212. The sidereal period of Venus is 224·7 days and its maximum elongation is 45°. Find the interval between superior conjunction and the next maximum elongation of the planet.

213. Define the six elements of the orbit of a planet and show how from these, the position of a planet at any time may be known.

214. Write a note on the transits of Venus, their recurrence, and their application in determining the sun's parallax.

215. Show that two planets moving in coplanar and circular orbits of radii r_1 and r_2, will be stationary as seen from one another when their angular distances measured from a node are

$$\tan^{-1}\left(\frac{\sqrt{r_2}\,(r_1+r_2)^{\frac{1}{2}}}{r_1}\right) \text{ and } \tan^{-1}\left(\frac{\sqrt{r_1}\,(r_1+r_2)^{\frac{1}{2}}}{r_2}\right)$$

216. Assuming the earth and a planet to be moving in coplanar and circular orbits round the sun, show that the difference (u) between the geocentric longitudes of the sun and the planet changes at the rate given by $\frac{2\pi}{s}\left(1 \sim \frac{b}{d}\cos u\right)$, b being the radius of the earth's orbit and s the synodic period of the planet and d its distance from the earth then.

217. Assuming the earth and an inferior planet to be moving in circular orbits, show that the planet approaches the earth most rapidly when its elongation is greatest, and that its magnitude is the same as what would make it describe its own orbit in the synodic period.

218. If θ be the angle subtended at the earth by the sun and the stationary point of a planet's orbit and φ be

the greatest elongation of the planet, prove that
$2 \cot \theta = \sec \varphi/2 + \operatorname{cosec} \varphi/2$. (M. U.)

219. If E be the elongation of a planet from the sun at the moment when it is stationary, show that $\frac{b}{a} = \frac{1}{2} \tan^2 E + \frac{1}{2} \tan E \sqrt{4 + \tan^2 E}$, where a and b are the radii of the orbits of the earth and the planet which are coplanar and circular.

220. A planet rotates about its axis in x hours and revolves round the sun in y days. Find the length of a mean solar day for an observer on the planet.

221. Show that the apparent breadth of the illuminated surface of a planet is $r\,(1-\cos\theta)$ where r is the radius of the planet and θ the exterior angle subtended at the planet by the earth and the sun. How does this formula get modified in the case of the moon.

222. Show that, if a planet's motion is retrograde as seen from the earth, the earth's motion is retrograde as seen from the planet.

223. If x be the phase of the moon as seen from the earth and y that of the earth as seen from the moon, show that $a\,(y-2)=x\,\left\{(2b-a)-bx\right\}$ approximately, where a and b are respectively the orbital radii of the earth and the moon and 2 the phase of the full moon. (M. U.)

224. Show that the area of the moon's illuminated disc varies as the versine of the exterior angle of elongation and hence devise a method of determining the parallax of the sun in terms of that of the moon.

225. Show that the equation $\frac{3}{4}\cos^2\varphi + \frac{b}{a}\cos\varphi = 1$ where φ, b, a represent the elongation, the radii of the

orbits of Venus and the earth respectively, would give the elongation of Venus when it is brightest.

226. The orbits of the earth and Venus being assumed circular and coplanar, find the elongation of Venus when it is seen to appear brightest to sun ,the radius of its orbit being 0·7233 of the radius of the earth's orbit.

227. Assuming the orbits of both to be circular and coplanar, prove that the geocentric motion of a superior planet will be direct when the square of the distance of the earth from the planet is greater than

$\left(b^2-a^2\right)\dfrac{\left(b^{\frac{3}{2}}-a^{\frac{3}{2}}\right)}{b^{\frac{3}{2}}+a^{\frac{3}{2}}}$ where a and b are respectively the distances of the earth and the planet from the sun.

228. The mean distance of Venus from the sun is 0·72 of that of the earth. Determine the greatest altitude and the time of the year at which Venus, supposed to have a circular orbit in the plane of the ecliptic, will be visible after sunset in a given latitude.

229. (i) Find in terms of the solar and lunar parallaxes and the semi-diameter of the sun, the value of the diameter of the earth's shadow, where the moon crosses it at a lunar eclipse, and show that a total lunar eclipse can never last for more than 2 hours.

(ii) Why should not there be an annular eclipse of the moon? Explain why a solar eclipse may be partial, total or annular.

230. If a lunation be 29·5306 days and the period of the sidereal revolution of the moon's node 6798·3 days, prove that after a period of 14558 days eclipses may be expected to recur in an invariable order. (M. U.)

231. If a total lunar eclipse happens at the summer solstice, and at the middle of the eclipse, the moon is seen in the zenith, find the latitude of the place of observation and the local time of the middle of the eclipse.

232. Explain how some times three lunar eclipses occur within a month.

233. Find the synodic period of the moon's nodes, their daily retrograde motion being $3'-10''\cdot 64$ and the sun's direct motion being $59'-8''\cdot 25$.

234. What part do the nodes of the lunar orbit play in the occurrence and frequency of eclipses.

235. Show that eclipses occur in an invariable order after 223 lunations.

236. Explain the circumstances that determine the nature and duration of a lunar eclipse.

237. Show that an eclipse of the moon will occur, provided that at full moon the sun is within nine days of the moon's nodes.

238. If θ radians be the inclination of the moon's relative orbit to the ecliptic, $n°$ be the angle between the line of nodes and the axis of the earth's shadow, and $\mu°$, $\sigma°$ are respectively the semi-diameters of the moon and the section of the umbra, then the duration of the eclipse is approximately

$$4\,\frac{\sqrt{(\sigma+\mu)^2-n^2\theta^2}}{(\sigma+\mu)}\text{ hours}$$

and the fraction of the moon's diameter eclipsed is

$$\frac{(\sigma+\mu-n\,\theta)}{2\,\mu}. \qquad \text{(M. U.)}$$

239. On the 15th March in a particular year, the moon is full and is 8° behind a node. Show how to find the number of eclipses in that year from the following data:—

	Major	Minor
Lunar ecliptic limits	12°—5′	9°—30′
Solar Do.	18°—31	15°—21′

The sun separates from the line of nodes by 30°—36′ in a lunar month (A. U.)

240. Show that in order that a total eclipse of the sun visible somewhere on the earth's surface may occur, it is necessary that the moon's celestial latitude at the time of new moon shall not exceed
$\dfrac{(R-r+P)\ (n^2+n'^2-2\,n\,n'\cos i)^{\frac{1}{2}}}{n'\cos i-n}$ approximately, where
R, r are the angular radii of the apparent discs of the sun and the moon as seen from the centre of the earth, P the moon's horizontal parallax (the sun's parallax being neglected), n and n' the angular velocities with which the earth and the moon describe their orbits, and i the inclination of the plane of the moon's orbit to that of the ecliptic. (A. U.)

241. During a total eclipse of the moon, the moon's centre at a certain instant coincided with that of the earth's shadow. The angular diameter of the earth's shadow at the moon's distance was $1^\circ - 30'$. The angular diameter of the moon was $30'$. The apparent motion of the moon about the earth was $30'$ per hour. From the above facts find the duration of the eclipse and also the duration of totality. (M. U.)

242. (i) Show that eclipses occur within two periods of the year, each of a month's duration, and state how the position of these periods changes in the year.

(ii) State with reasons whether the eclipse begins on the eastern or western edge of the bright disc (i) in a lunar eclipse. (ii) in a solar eclipse. (M. U.)

243. Explain why more lunar eclipses are visible at a place than solar eclipses, although there are on the whole, more of the latter than of the former.

244. Calculate roughly the totality of a solar eclipse occurring at the time of the vernal equinox, as viewed from the equator, given the following:—

Moon's diameter = 2160 miles,

Sun's diameter $= 400 \times$ moon's diameter.
Distance of the moon from the earth $= 222000$ miles.
Distance of the sun from the earth $= 92,000,000$ „
Vel. of moon's shadow relative to an observer on the earth $= 1080$ miles per hour. (M. U.)

245. Name the chief circumpolar constellations, and represent their positions on the celestial sphere. (M. U.)

246. Describe the zenith telescope and mention some of its important uses.

247. Describe a sidereal clock. How does it differ from a mean time clock. What is meant by the error and rate of a clock and show how these can be determined.

248. Explain why the time given by a sun dial is in general different from that indicated by an ordinary clock.

249. Describe the equatorial and explain the purposes for which it may be used. Compare it with the meridian circle in respect of its usefulness.

250. Describe the method of fixing an equatorial at any place.

251. What are the errors of the transit circle and how are they determined for any instrument nearly fixed on the meridian?

252. Find the declination of a star from the following observations made with the transit instrument at a place of latitude $50°-28'-31''$ N.

Pointer reading $32°-10'$.
Microscope readings; $1'-12''$; $0'-50''$; $0'-46''$; $0'-58''$
Zenith reading $0'-16''$

253. Describe the sextant, explaining the principle on which it is based and compare its uses with those of the

transit circle. Explain how both are used to obtain the declination of the sun at noon.

254. Explain how the sextant can be used to determine the latitude of a place. Point out its advantages and disadvantages as a means of finding latitude.

255. Describe briefly the transit instrument and explain how it is used to find the R. A. and declination of a star.

256. An equatorial whose axis is adjusted to point to the apparent pole is pointed to a star very near the meridian. Show that if the telescope is to follow the star accurately, the rate of the clock must be diminished in the ratio $(1-k \cot \varphi \tan z): 1$ where k is the astronomical refraction, φ the latitude of the place, and z the zenith distance of the star.

257. (a) Establish a formula expressing the azimuthal deviation of the transit circle in terms of the observed interval of time between two successive passages of a circumpolar star over the central wire, the latitude of the place of observation and the star's declination.

(b) On 6th January the transit of Polaris (Decl. $88^\circ-30'-50''$) was observed at Greenwich (Lat. $51^\circ-28'-39''$) at 13 hrs. 4 min. 39·40 sec. and on 7th January at 1 hr. 4 min. 57·62 sec., the observations being corrected for collimation and level errors, rate of clock, and change of R. A. Find the azimuthal error.

258. If the deviation, level, and collimation errors of a transit instrument are a, b, c respectively, prove that the error in the time of transit due to these errors will be a minimum for a star of declination $\sin^{-1} \dfrac{a \cos \varphi - b \sin \varphi}{c}$, provided this angle is real, φ being the latitude of the Observatory.

259. An equatorial instrument being supposed to be in perfect adjustment except in that the polar axis

though in the meridian, has an inclination error θ, show that even if the equatorial clock is working perfectly, the apparent place of a circumpolar star instead of being permanently at the centre of the field of view, traces out an ellipse whose principal semi-axes are θ and $\theta \sin \delta$.

260. Describe the transit theodolite and explain how the meridian of a place can be fixed with the help of this instrument.

261. Show that, in general, two values of the declination of a star (δ_1, δ_2) can be found, for which the three errors of adjustment produce no error in its time of transit at a place and prove that the correction to be added to the observed time of transit of a star of declination δ at that place is given by the expression $2c \sin \frac{1}{2}(\delta - \delta_1) \sin \frac{1}{2}(\delta - \delta_2) \sec \delta . \sec \frac{1}{2}(\delta_1 - \delta_2)$ where c is the collimation error.

262. If at any instant the plane of vibration of a pendulum passes through a star near the horizon, show that it will remain so, as long as the star is near the horizon.

263. Show how to graduate a sun dial whose dial is to be vertical and facing south. On such a dial, the position of the end of the shadow cast by the style is marked every day at mean noon. Show that the curve on which these points lie, is nearly an inverted figure of eight.

264. Show that the locus traced out by the end of the shadow of the style of a horizontal sun dial is nearly a conic.

265. In a transit instrument of 12 ft. focal length which has a small collimation error, a star of declination 60° transits 2^s too early. To what length and in what direction should the cross wire be displaced to compensate for this error?

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APPENDIX A.

IMPORTANT ASTRONOMICAL CONSTANTS.

	h.	min.	sec.	
Length of a sidereal day =	23	56	4·091	in mean solar units.
Length of a mean solar day =	24	3	56·555	in sidereal units.

	days.	h.	min.	sec.	
Length of the sidereal month =	27	7	43	11·5	in mean solar units
.. synodic .. =	29	12	44	2·8	
.. tropical .. =	27	7	43	4·7	
.. anomalistic .. =	27	13	18	33·1	

Length of the sidereal year = 365·2564 mean solar days.
Length of the tropical year = 365·2422 ..
Length of the anomalistic year = 365·2596 ..
Equatorial radius of the earth = 3963·35 miles.
Polar .. .. = 3950·01 ..
Mean geocentric distance of the sun (or the astronomical unit) = 92,900,000 miles. or 149.500,000 k. m.
.. of the moon = 238,900 miles.
Solar parallax = 8″·80.
Moon's equatorial horizontal parallax = 57′-2″·70.
Velocity of light = 186,324 miles per second.
1 light year = $5·88 \times 10^{12}$ miles. = 63290 astronomical units = 0·3069 parsecs.
1 parsec = $19·16 \times 10^{12}$ miles = 3·259 light years. = 206265 astronomical units.
General precession = 50″·2564 + 0″·0222 T, where T is measured in Julian Centuries from 1900 A. D.
Obliquity of ecliptic = 23°-27′-8″·26 − 46″·84 T.
Constant of nutation = 9″·21.
Constant of aberration = 20″·47.

APPENDIX B.

NAKSHATRAS. *

	Name of Nakshatras in order.	Duration. °	′	Corresponding Greek Names.
1	Asvini.	0	0	Beta Arietis
2	Bharani.	13	20	41 Arietis
3	Krittika.	26	40	Eta Tauri (*Alcyone*).
4	Rohini.	40	0	Alpha Tauri (*Aldebaran*).
5	Mrigasira.	53	20	Lambda Orionis,
6	Ardra.	66	40	Alpha Orionis (*Betalguese*).
7	Punarvasu.	80	0	Beta Geminorum (*Pollux*).
8	Pushya.	93	20	Delta Cancri.
9	Aslesha.	106	40	Alpha Cancri.
10	Magha.	120	0	Alpha Leonis (*Regulus*).
11	Purva Phalguni.	133	20	Delta Leonis.
12	Uttara Phalguni.	146	40	Beta Leonis.
13	Hasta.	160	0	Delta Corvi.
14	Chitra.	173	20	Alpha Virginis (*Spica*).
15	Svati.	186	40	Alpha Bootis (*Arcturus*).
16	Visakha.	200	0	Iota Librae.
17	Anuradha.	213	20	Delta Scorpii.
18	Jyeshta.	226	40	Alpha Scorpii (*Antares*).
19	Mula.	240	0	Lambda Scorpii.
20	Purva Ashadha.	253	20	Delta Sagittarii.
21	Uttara Ashadha.	266	40	Sigma Sagittarii
22	Sravana.	280	0	Alpha Aquilae (*Altair*).
23	Dhanishta.	293	20	Alpha Delphini.
24	Satabhisha.	306	40	Lambda Aquarii.
25	Purva Bhadrapada.	320	0	Alpha Pegasi.
26	Uttara Bhadrapada.	333	20	Alpha Andromedae.
27	Revati.	346	40	Zeta Piscium.

* The mean duration of each Nakshatra is 1·01191 days, or 1 day and nearly 17 minutes. $2\frac{1}{4}$ Nakshatras make one Rasi or Sign.

APPENDIX C.

A FEW CONSTELLATIONS AND NOTABLE FEATURES CONNECTED WITH THEM.

Constellations.	Binaries or otherwise.	Nebulae.	Clusters.	Novae and dates of occurrence.	Variable Stars.	Radiant points of Meteoric showers.
Aries	Beta (S) Gamma (B) Pi (T)					
Taurus	Zeta (S)	N. G. C. 1952 (Crab nebula)	The Pleiades The Hyades		Lambda Tauri (Algol type) S. U. Tauri (Long Period type)	Taurid meteors of Nov. 20, radiate from a point north of Aldebaran.
Gemini	Castor (Q) Gamma (S) Eta (B)	N. G. C. 2392	N. G. C. 2168		Zeta (Cepheid type) U. (Irregular type)	The meteoric shower Geminids occurring about Dec. 10 has its radiant point in this constellation.
Cancer	Zeta (Q)		N. G. C. 2682 N. G. C. 2632		S (Algol type)	
Leo	Alpha (B) Gamma (B)					The radiant point of the meteors Leonids, seen during the middle of Nov. lies in this constellation. These meteors are connected with the comet 1866 I (Tempel's comet) Period 33.3 yrs.

APPENDIX C.—(*Continued*)

Constellations.	Binaries or otherwise.	Nebulae.	Clusters.	Novae and dates of occurrence.	Variable Stars.	Radiant points of Meteoric showers.
Virgo	Alpha (S) Gamma (B)	N. G. C. 4526 N. G. C. 4594 N. G. C. 4649				
Libra	Alpha (D)		N. G. C. 5904		Delta (Algol type)	
Scorpio	Alpha (B) Beta (S)		N. G. C. 6093 N. G. C. 6231			
Sagittarius	Beta (D) Delta (D) Zeta (B) Mu 1 (Triple)	N. G. C. 6822 N. G. C. 6514 N. G. C. 6523	N. G. C. 6626 N. G. C. 6656 N. G. C. 6864 N. G. C. 6531	Between July 16 and July 30, 1926		
Capricornus	Alpha (M) Alpha 1 (B) Alpha 2 (Triple) Beta 1 (B) Zeta (B)		N. G. C. 7099 N. G. C. 6981			

Aquarius	Alpha (B) Zeta (B)	N. G. C. 7009	N. G. C. 7089			Meteoric stream Delta Aquarids radiates from near Delta Aquari from 27th to 29th July. These are due to Winnecke's comet. Aquarids I of May 6 are due to Halley's comet.
Pisces	Alpha (B)					
Andromeda	Alpha (D) Gamma (T) Lambda (S) Nu (S) Pi (D)	N. G. C. 7662 N. G. C. 1023 N. G. C. 224	N. G. C. 752			Bielid meteors of Nov. 23 radiate from the vicinity of Gamma. These meteors are connected with the comet Biela.
Aquila	Eta (S) Theta (S)		N. G. C. 6705	(1) 1612 (2) April 1899 (3) 8th June 1918		
Auriga	Alpha (S) Beta (B) Zeta (S)		N. G. C. 1960 N. G. C. 2099 N. G. C. 1912	T. in 1892	R. T. (S) (cepheid type)	
Bootes	Alpha (S) Xi (B)					
Canes Venatici	Alpha II (S)	N. G. C. 5194	N. G. C. 5272			
Cassiopeia	Eta (B)		N. G. C. 7789	Nov. 11th 1572		

APPENDIX C.—(*Continued*)

Constellations.	Binaries or otherwise.	Nebulae.	Clusters.	Novae and dates of occurrence.	Variable Stars.	Radiant points of Meteoric showers.
Cepheus	Delta (S)	N. G. C. 4826	N. G. C. 7654 N. G. C. 5024 N. G. C. 4147		Beta (Cepheid type) Delta do. U. (Algol type) R. Z.	
Corona Borealis	Alpha (S) Beta (S) Gamma (B) Zeta (B) Eta (B)			May 12, 1866	U. (Algol type)	
Cygnus	Beta (D) Delta (B) Tau (B)			18th Aug. 1600 24th Nov. 1876 20th Aug. 1920.	Chi. S. S. Cygni. (irregular type)	
Delphinus	Beta (B) Gamma (B)				V. W. (Algol type.)	
Draco	Psi (B) Beta (B) Chi (S) Gamma (D) Epsilon (B) Eta (B) Omega (S) Theta (S) Sigma (B)	N. G. C. 6543			T.W. (Eclipsing type)	Zeta is the radiant point of the meteoric showers Draconids on Jan. 19 and March 28.

Hercules	Beta (S) Epsilon (S) Zeta (B) Mu (T) Upsilon (B) R. X. (B) Z (B)		N. G. C. 6205 N. G. C. 6229 N. G. C. 6341			
Lyra	Alpha (B) Beta (B) Delta (S) Epsilon (Q) Zeta (D)	Ring Nebula	N. G. C. 6779			The radiant point of the meteors Lyrids lies in this constellation. These are connected with the comet 1861—I and are seen in the latter half of April.
Ophiuchus	Rho (B) 36 (B) 70 (B)		N. G. C. 6273 N. G. C. 6333 N. G. C. 6402 N. G. C. 6494	123 A. D. 1230 A. D. 10th Oct. 1604 28th April 1848	V. Ophiuchi (eclipsing type.)	
Pegasus	Eta (S) Iota (S) Kappa (B)		N. G. C. 7078		Beta (irregular type.)	
Perseus	Alpha (S) Beta (T) Epsilon (B) Zeta (D) Eta (D) Mu (S)		N. G. C. 669 N. G. C. 884	Feb. 22nd 1901	Beta (Algol) eclipsing type	The meteoric shower Perseids has a radiant point in this constellation and they appear in the middle of August. They are connected with the comet 1862 III.

APPENDIX C.—(*Continued*)

Constellations.	Binaries or otherwise.	Nebulae.	Clusters.	Novae and dates of occurrence.	Variable Stars.	Radiant points of Meteoric showers.
Ursa Major	Beta (S) Epsilon (S) Zeta (B) Theta (B) Xi (B)	N. G. C. 3587 N. G. C. 3031			R. S, & T.	Eta marks the radiant point of a meteoric shower called the Winds of November 10
Ursa Minor	Polaris (T) Gamma (D)					
Vulpecula		N. G. C. 6853 (Dumb-bell Nebula)			T (cepheid type)	Meteoric stream Vulpeculoids appearing from June 13 to July 7 radiates from this constellation.
Argus	Alpha (B)	N. G. C. 3372	N. G. C. 2447 N. G. C. 2808 N. G. C. 3532		Eta	
Canis Major	Alpha (B)					
Centaurus	Alpha (B)		N. G. C. 5139 N. G. C. 3766 N. G. C. 5662	Between 14th June and 8th July 1895		
Cetus	Gamma (B)	N. G. C. 1068			Omiron. (irregular type)	

Corvus	Delta (D) Zeta (B)	N. G. C. 4361			R Corvi	
Crux	Alpha (Triple)				Gamma	
Eridanus	Omicron (T) Upsilon (S)					
Hydra	Epsilon (B)	N. G. C. 3242	N. G. C. 4590		SX Hydrae (eclipsing type) R. W. Hydrae (long period type)	
Lepus	Alpha (D) Beta (B) Gamma (D)		N. G. C. 1904		R. (long period type)	
Monoceros			N. G. C. 2323 N. G. C. 2244 N. G. C. 2301 N. G. C. 2548			
Orion	Beta (S) Delta (S) Zeta (Triple) Eta (S) Theta (M) Iota (S) Lambda (B) Sigma (M) Psi (S)	N. G. C. 1976 (Orion nebula)				The Orionids of Oct. 20 have their radiant point here.

N. B.—In the second column, (B) stands for binary, (S), for spectroscopic binary, (T) for ternary, (D) for double, (Q) for quaternary and (M) for multiple stars.

APPENDIX D.

LIST OF A FEW CONSPICUOUS STARS

STAR	MAGNITUDE	SPECTRAL TYPE
Sirius	—1·6	A_0
Canopus	—0·9	F_0
Vega	0·1	A_0
Capella	0·2	G_0
Rigel	0·3	B_8
Procyon	0·5	F_5
Achernar	0·6	B_5
Altair	0·9	A_5
Aldebaran	1·1	K_5
Spica	1·2	B_2
Fomalhaut	1·3	A_3
β Crucis	1·5	B_1
Castor	1·6	A_0
γ Orionis	1·7	B_2
β Tauri	1·8	B_8
α Persei	1·9	F_5
β Canis Majoris	2·0	B_1
Polaris	2·1	F_8
α Andromedæ	2·2	A_0
γ Cassiopeiæ	2·3	B_0
β Cassiopeiæ	2·4	F_5
δ Orionis	2·5	B_0
δ Leonis	2·6	A_2
β Arietis	2·7	A_5
δ Cassiopeiæ	2.8	A_5
ζ Persei	2·9	B_1
ε Persei	3·0	B_1

APPENDIX E

LIST OF SOME IMPORTANT COMETS THAT APPEARED SINCE 1800 A. D.

Year and No. of comet.	Discoverer.	Period in years or Eccentricity for hyperbolic orbits	Year and No. of comet.	Discoverer.	Period in years or Eccentricity for hyperbolic orbits	Year and No. of comet.	Discoverer.	Period in years or Eccentricity for hyperbolic orbits
1806	Biela	6·7	1895 I	Perrotin	3·3	1906 VI	Metcalf	8·2
1811 II	(seen for 17	754·5	1895 II	L. Swift	7·2	1907 II	Grigg, Mellish	164·32
1819 I	Encke [months)	3·3	1896 I	Perrine, Lamp	e=1·0036	1908 III	Morehouse	
1835 III	Halley	76·3	1896 II	Javelle		1909 I	Borrelly	2038
			(Faye)		7·4	(B. C. 137)		
1852 IV	Westphal	61·5	1896 VII	Perrine	6·5	1909 IV	Daniel	6·5
1858 VI	Donati		1897 I	Do	e=1·001	1910 I	Many people	364000
						(The great		
1859		e=1·00003	1897 II	Do.	6·7	day light		
						comet)		
1861 I	Many People	415	1898 I	Do	322·6	1910 II	Wolf	76·02
						(Halley)		
1869 III	Tempel-Swift	5·5	1898 II	Do	5·8	1914 V	Delavan	e=1·0016
			(Pons-			1916 III	Perrine	16·35
			Winnecke)					
1871 III	Tuttle	13·8	1899 I	L. Swift	e=1·00035	1917 I	Mellish	141·6
1881 I	(seen for 2½ yrs.)		1902 II	Grigg		1917 III	Wolf	
1884 III	Wolf	6·8	1903 V	Aitken	7·1	1921 I	Dubiago	79·5
1889	Many people	9740		(Brooks)		1922 I	Skjellerup	4·9
1889 V	Brooks	7·1	1905 II	Borrelly	6·9	(Grigg)		
1892 III	Holmes	6·9	1905 III	Giacobini	200	1923 I	Do	2174
1894 I	Denning	7·4	1905 IV	(seen for 3½ yrs)		1923 II	Reid	6·6
1894 IV	E. Swift	5·9	1905 VI	Brooks	e=1·0002	1925 I	Wolf	7·5

APPENDIX F.

HALLEY'S COMET.

The mean period of this comet is about 76 years. This is the most famous of all comets in its being the first comet whose return was predicted. Halley was able to predict its return as he found by calculation an identity between the orbits of the comets of 1682 A. D. and 1607 A. D. (seen by Kepler) and 1531 A. D. (seen by Apian). He also obtained records of the appearance of a very bright comet during the years 1456, 1301, 1145 and 1066. The deviation of the period between 1607 and 1682 from that between 1531 and 1607 was explained to be due to the perturbation caused by the planets Jupiter and Saturn. The perihelion passage of this bright comet has been traced up to 240 B. C. There are ancient records to verify the twenty seven returns between 87 B. C. and 1910 A. D. Due to the attraction caused by planets like Jupiter, the actual sidereal period at any revolution has varied up to 5 years from the mean period.

The latest return of the comet to perihelion was on April 20th, 1910. On May 13th, the length of the comet was about 50° in the sky. On and after May 16th, the head was too near the sun to be seen and the tail extended beyond 120° from the head and could be seen as a brilliant band of light illuminating the sky. On May 20th, the comet was at the minimum distance of 14,300,000 miles from the earth and on the following day, probably the tail of the comet either grazed or completely enveloped the earth.

APPENDIX G.

THE SIX ELEMENTS OF THE ORBIT OF A PLANET

All the planets of the solar system move round the sun in elliptic orbits in the west to east direction. To fix the orbit of a planet, the following six quantities have to be determined.

(i) The longitude of the *ascending node* of the planetary orbit. This is the point of intersection of the orbit of the planet with the ecliptic, where the planet crosses from the south to the north. The second point of intersection of the planet's orbit with the ecliptic is known as the *descending node.*

(ii) The inclination of the orbit of the planet to the ecliptic. This with the previous quantity determines the plane of motion of the planet.

(iii) The longitude of the perihelion of the planetary orbit, giving the orientation of the ellipse. It is usual to measure this quantity from the first point of Aries along the ecliptic up to the ascending node and then measure the same along the planetary orbit in the direction of its motion up to the perihelion.

(iv) The semi-major axis of the orbit, which is also the mean distance of the planet from the sun.

(v) The eccentricity of the elliptic orbit. This quantity and the previous one give the size and shape of the orbit.

(vi) The instant when the planet is at perihelion, which is known as the *epoch.* This gives the position of the planet at any time in its own orbit, so that at any other time its position can be predicted, with the knowledge of the other elements already known.

All the above six quantities are known by the name '*elements of a planetary orbit.*'

hrs.	m.	s.	Places.
−4	54		Maldive Islands.
−4	0		Mauritius Islands.
− 3	0		Madagascar, Socotra, Russia (from 40° E to 52°-30′ E)
2	0		East Europe (Roumania, Bulgaria, Turkey, Greece, Egypt, South Africa)
− 1	0		Mid Europe (Norway, Sweden, Denmark, Germany, Poland, Chechoslovakia, Austria, Switzerland, Italy, West Africa, Belgian Congo)
− 0	19	32·1	Holland.
− 0	0		(Greenwich) Great Britain. Ireland, France, Belgium, Spain, Portugal, Algeria, Morocco.
+ 1	0		Iceland; Portuguese and French Guinea.
2	0		Azores; Trinidad Islands (South Atlantic)
3	0		Eastern Brazil.
4	0		Canada, Bermuda, Trinidad, Central Brazil, Argentina.
5	0		Eastern Canada (from 68° to 89°), Eastern States of U. S. A., Cuba, Jamaica, Panama, Chile, Western Brazil.
6	0		Central Canada (from 89° to 103°), Central States of U. S. A, Mexico, Honduras, Costa Rica.
7	0		Canada from 103° to British Columbia, Arizona, Mexico.
8	0		(Pacific) British Columbia, California, Washington.
10	0		Alaska.
11	0		Aleutian Islands, West Coast of Alaska.
11	30		Western Samoa. (British)

TABLES

TABLE I.

For converting intervals of Mean Solar Time into equivalent intervals of Sidereal Time.

Hours.				Minutes.								Seconds.			
Hours of Mean Time.	Equivalents in Sidereal Time.			Minutes of Mean Time.	Equivalents in Sidereal Time.		Minutes of Mean Time.	Equivalents in Sidereal Time.		Seconds of Mean Time.	Equivalents in Sidereal Time.	Seconds of Mean Time.	Equivalents in Sidereal Time.		
	h.	m.	s.		m.	s.		m.	s.		s.		s.		
1	1	0	9.8565	1	1	0.1643	31	31	5.0925	1	1.0027	31	31.0849		
2	2	0	19.7130	2	2	0.3286	32	32	5.2568	2	2.0055	32	32.0876		
3	3	0	29.5694	3	3	0.4928	33	33	5.4211	3	3.0082	33	33.0904		
4	4	0	39.4259	4	4	0.6571	34	34	5.5853	4	4.0110	34	34.0931		
5	5	0	49.2824	5	5	0.8214	35	35	5.7496	5	5.0137	35	35.0958		
6	6	0	59.1388	6	6	0.9857	36	36	5.9139	6	6.0164	36	36.0986		
7	7	1	8.9953	7	7	1.1499	37	37	6.0782	7	7.0192	37	37.1013		
8	8	1	18.8518	8	8	1.3142	38	38	6.2424	8	8.0219	38	38.1040		
9	9	1	28.7083	9	9	1.4785	39	39	6.4067	9	9.0246	39	39.1068		

10	10	1	38.5647
11	11	1	48.4212
12	12	1	58.2777
13	13	2	8.1342
14	14	2	17.9906
15	15	2	27.8471
16	16	2	37.7036
17	17	2	47.5600
18	18	2	57.4165
19	19	3	7.2730
20	20	3	17.1295
21	21	3	26.9859
22	22	3	36.8424
23	23	3	46.6989
24	24	3	56.5554

10	10	1.6428	40	40	6.5710
11	11	1.8070	41	41	6.7353
12	12	1.9713	42	42	6.8995
13	13	2.1356	43	43	7.0638
14	14	2.2998	44	44	7.2281
15	15	2.4641	45	45	7.3924
16	16	2.6284	46	46	7.5566
17	17	2.7927	47	47	7.7209
18	18	2.9569	48	48	7.8852
19	19	3.1212	49	49	8.0495
20	20	3.2855	50	50	8.2137
21	21	3.4498	51	51	8.3780
22	22	3.6140	52	52	8.5423
23	23	3.7783	53	53	8.7066
24	24	3.9426	54	54	8.8708
25	25	4.1069	55	55	9.0351
26	26	4.2711	56	56	9.1994
27	27	4.4354	57	57	9.3637
28	28	4.5997	58	58	9.5279
29	29	4.7640	59	59	9.6922
30	30	4.9282	60	60	9.8565

10	10·0271	40	40.1095
11	11.0301	41	41.1123
12	12.0329	42	42.1150
13	13.0356	43	43.1177
14	14.0383	44	44.1205
15	15.0411	45	45.1232
16	16.0438	46	46.1259
17	17.0465	47	47.1287
18	18.0493	48	48.1314
19	19.0520	49	49.1342
20	20.0548	50	50.1369
21	21.0575	51	51.1396
22	22.0602	52	52.1424
23	23.0630	53	53.1451
24	24.0657	54	54.1479
25	25.0685	55	55.1506
26	26.0712	56	56.1533
27	27.0739	57	57.1561
28	28.0767	58	58.1588
29	29.0794	59	59.1615
30	30.0821	60	60.1643

TABLE II.

FOR CONVERTING INTERVALS OF SIDEREAL TIME INTO EQUIVALENT INTERVALS OF MEAN SOLAR TIME.

HOURS.				MINUTES.						SECONDS.			
Hours of Sidereal Time.	Equivalents in Mean Time.			Minutes of Sidereal Time.	Equivalents in Mean Time.		Minutes of Sidereal Time.	Equivalents in Mean Time.		Seconds of Sidereal Time.	Equivalents in Mean Time.	Seconds of Sidereal Time.	Equivalents in Mean Time.
	h	m	s		m	s		m	s		s		s
1	0	59	50·1704	1	0	59·8362	31	30	54·9214	1	0·9973	31	30·9154
2	1	59	40·3409	2	1	59·6723	32	31	54·7576	2	1·9945	32	31·9126
3	2	59	30·5113	3	2	59·5085	33	32	54·5937	3	2·9918	33	32·9099
4	3	59	20·6818	4	3	59·3447	34	33	54·4299	4	3·9891	34	33·9072
5	4	59	10·8522	5	4	59·1809	35	34	54·2661	5	4·9864	35	34·9045
6	5	59	1·0226	6	5	59·0170	36	35	54·1023	6	5·9836	36	35·9017
7	6	58	51·1931	7	6	58·8532	37	36	53·9384	7	6·9809	37	36·8990
8	7	58	41·3635	8	7	58·6894	38	37	53·7746	8	7·9782	38	37·8963
9	8	58	31·5340	9	8	58·5256	39	38	53·6108	9	8·9754	39	38·8935

10	9	58	21·7044
11	10	58	11·8748
12	11	58	2·0453
13	12	57	52·2157
14	13	57	42·3862
15	14	57	32·5566
16	15	57	22.7270
17	16	57	12·8975
18	17	57	3·0679
19	18	56	53·2384
20	19	56	43·4088
21	20	56	33·5792
22	21	56	23·7497
23	22	56	13·9201
24	23	56	4·0906

10	9	58·3617	40	39	53·4470
11	10	58·1979	41	40	53·2831
12	11	58·0341	42	41	53·1193
13	12	57·8703	43	42	52·9555
14	13	57·7064	44	43	52·7917
15	14	57·5426	45	44	52·6278
16	15	57·3788	46	45	52·4640
17	16	57·2150	47	46	52·3002
18	17	57·0511	48	47	52·1364
19	18	56·8873	49	48	51·9725
20	19	56·7235	50	49	51·8087
21	20	56·5597	51	50	51·6449
22	21	56·3958	52	51	51·4810
23	22	56·2320	53	52	51·3172
24	23	56·0682	54	53	51·1534
25	24	55·9044	55	54	50·9896
26	25	55·7405	56	55	50·8257
27	26	55·5767	57	56	50·6619
28	27	55·4129	58	57	50·4981
29	28	55·2489	59	58	50·3343
30	29	55·0852	60	59	50·1704

10	9·9727	40	39·8908
11	10·9700	41	40·8881
12	11·9672	42	41·8854
13	12·9645	43	42·8826
14	13·9618	44	43·8799
15	14·9591	45	44·8772
16	15·9563	46	45·8744
17	16·9536	47	46·8717
18	17·9509	48	47·8690
19	18·9481	49	48·8662
20	19·9454	50	49·8635
21	20·9427	51	50·8608
22	21·9399	52	51·8580
23	22·9372	53	52·8553
24	23·9345	54	53·8526
25	24·9318	55	54·8499
26	25·9290	56	55·8471
27	26·9263	57	56·8444
28	27·9236	58	57·8417
29	28·9208	59	58·8389
30	29·9181	60	59·8362

TABLE III.

For converting Space into Time.

Space.	Time.			Space.	Time.			Space.	Time.	
Deg= Min= Sec=	H. Min. Sec.	Min Sec.	Sec.	Deg. Min. Sec.	H Min. Sec.	Min. Sec.	Sec.	Deg.	H.	Min.
1	0	4	·07	31	2	4	·07	70	4	40
2	0	8	·13	32	2	8	·13	80	5	20
3	0	12	·20	33	2	12	·20	90	6	0
4	0	16	·27	34	2	16	·27	100	6	40
5	0	20	·33	35	2	20	·33	110	7	20
6	0	24	·40	36	2	24	·40	120	8	0
7	0	28	·47	37	2	28	·47	130	8	40
8	0	32	·53	38	2	32	·53	140	9	20
9	0	36	·60	39	2	36	·60	150	10	0
10	0	40	·67	40	2	40	·67	160	10	40
11	0	44	·73	41	2	44	·73	170	11	20
12	0	48	·80	42	2	48	·80	180	12	0
13	0	52	·87	43	2	52	·87	190	12	40
14	0	56	·93	44	2	56	·93	200	13	20
15	1	0	·00	45	3	0	·00	210	14	0
16	1	4	·07	46	3	4	·07	220	14	40
17	1	8	·13	47	3	8	·13	230	15	20
18	1	12	·20	48	3	12	·20	240	16	0
19	1	16	·27	49	3	16	·27	250	16	40
20	1	20	·33	50	3	20	·33	260	17	20
21	1	24	·40	51	3	24	·40	270	18	0
22	1	28	·47	52	3	28	·47	280	18	40
23	1	32	·53	53	3	32	·53	290	19	20
24	1	36	·60	54	3	36	·60	300	20	0
25	1	40	·67	55	3	40	·67	310	20	40
26	1	44	·73	56	3	44	·73	320	21	20
27	1	48	·80	57	3	48	·80	330	22	0
28	1	52	·87	58	3	52	·87	340	22	40
29	1	56	·93	59	3	56	·93	350	23	20
30	2	0	·00	60	4	0	·00	360	24	0

TABLE IV.

FOR CONVERTING TIME INTO SPACE.

TIME.	SPACE.	TIME.	SPACE.		TIME.	SPACE.		TIME.	SPACE.
H.	Deg.	Min. Sec.	Deg. Min.	Min. Sec.	Min. Sec.	Deg. Min.	Min. Sec.	For decimal parts of a second.	
1	15	1	0	15	31	7	45		
2	30	2	0	30	32	8	0		
3	45	3	0	45	33	8	15		
4	60	4	1	0	34	8	30		
5	75	5	1	15	35	8	45		
6	90	6	1	30	36	9	0		
7	105	7	1	45	37	9	15		
8	120	8	2	0	38	9	30		
9	135	9	2	15	39	9	45		
10	150	10	2	30	40	10	0		
11	165	11	2	45	41	10	15	·1	1 5
12	180	12	3	0	42	10	30	·2	3·0
13	195	13	3	15	43	10	45	·3	4·5
14	210	14	3	30	44	11	0	·4	6·0
15	225	15	3	45	45	11	15	·5	7·5
16	240	16	4	0	46	11	30	·6	9.0
17	255	17	4	15	47	11	45	·7	10·5
18	270	18	4	30	48	12	0	·8	12·0
19	285	19	4	45	49	12	15	·9	13·5
20	300	20	5	0	50	12	30		
21	315	21	5	15	51	12	45		
22	330	22	5	30	52	13	0		
23	345	23	5	45	53	13	15		
24	360	24	6	0	54	13	30		
25	375	25	6	15	55	13	45		
26	390	26	6	30	56	14	0		
27	405	27	6	45	57	14	15		
28	420	28	7	0	58	14	30		
29	435	29	7	15	59	14	45		
30	450	30	7	30	60	15	0		

TABLE V.

Dip of the Horizon

Height.	Dip.		Height.	Dip.	
Feet.	′	″	Feet.	′	″
1	0	58	19	4	11
	1	21	20	4	17
3	1	40	21	4	23
4	1	56	22	4	32
5	2	9	23	4	36
6	2	21	24	4	42
7	2	33	25	4	46
8	2	44	26	4	52
9	2	53	27	4	58
10	3	2	28	5	5
11	3	10	29	5	10
12	3	19	30	5	15
13	3	27	35	5	39
14	3	35	40	6	4
15	3	42	45	6	27
16	3	50	50	6	46
17	3	57	55	7	5
18	4	4	60	7	25

(Subtract from Altitude)

TABLE VI.

Sun' Parallax in Altitude.

Altitude.	Parallax. in seconds
°	
0	8·8
10	8·5
20	8
30	8
40	7
50	6
55	5
60	4
65	4
70	3
75	2
80	2
85	1
90	0

(Add to get true Altitude)

TABLE VII.

Diminution of the Vert. Semid. of Sun or Moon on account of Refraction.

Altitude.	Diminution of Semid.
°	″
5	25
6	19
7	14
8	11
9	9
10	8
11	7
12	6
13	5
14	4
15	4
16	3
18	3
20	2
30	1
45	1

(Add to get actual Semidiameter)

TABLE VIII.

TABLE OF MEAN REFRACTION OF THE ATMOSPHERE FOR A TEMPERATURE OF 50° F AND A BAROMETRIC PRESSURE OF 29·6 INCHES.

Zenith distance	Refraction	Zenith distance	Refraction
°	′ ″	°	′ ″
0	0 —0·0	72	2—55·5
5	0 —5·0	74	3—18·2
10	0—10·2	76	3—47·0
15	0—15·5	77	4— 4·4
20	0—20·9	78	4—24·5
25	0—26·8	79	4—47·7
30	0—33·2	80	5—16
35	0—40·3	81	5—49
40	0—48·3	82	6—29
45	0—57·6	83	7—19
50	1—08·6	84	8—23
55	1—22·1	85	9—45
60	1—39·5	86	11—37
62	1—48·0	87	14—13
64	1—57·6	88	18— 6
66	2— 8·6	89	24—22
68	2—21·6	90	34—50
70	2—37·0		

Note:—The refraction formula for variation of pressure and temperature is $\rho'' = \frac{983\ B}{460+T} \tan Z$, where B is the barometric pressure in inches, T the temperature in degrees Fahrenheit and Z the apparent zenith distance of the celestial object.

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ERRATA.

PAGE	LINE	ERROR	CORRECTION
30	last but one	the two	the angle between the two
36	5	on	delete
48	last line	$\cos_2 \varphi$	$\cos^2 \varphi$
56	3	between	in
67	27	$NS \simeq$	$nS \simeq$
68	1	$\because$	$\therefore$
,,	9	w	ω
,,	12	Fig. 38	Fig. 39.
,,	17	and ω	delete
80	7	NN'	NN_1
86	15	$\left(d\, r^2 \frac{d\,\theta}{d\,t}\right)$	$d\left(r^2 \frac{d\,\theta}{d\,t}\right)$
96	15	CV	CU
104	17	$3\frac{3}{4}$	$-3\frac{3}{4}$
,,	23	$45°$	$0°, 45°$
130	last line	ɩocal	local
142	30 & 32	independant	independent
153	17	then	delete
165	31	a media	media
170	18	$\Delta \mu$	$d \mu$
180	9	K	A
197	14	moon	body
200	30	above	following
208	last but one	aunnal	annual
209	20	parallatic	parallactic
215	13	is the	delete
221	16	sun	earth
233	2	between	at
235	23	astromer	astronomer
239	6	planets	planets, except Pluto

PAGE	LINE	ERROR	CORRECTION
239	9	Ceres	Ceres or Pluto
261	10	(1), (2), adding	(2), (1), subtracting
273	last but one	1766	1716
274	10	positios	positions
288	20	tan (S N)	sin (S N)
320	21	its	their
322	7	connetion	connection
,,	25	40″, Yerkes	18″, Chicago
,,	27	are	is
332	15	nebule	nebulae
333	11	densities	diameters
,,	12	Neptune	Neptune's orbit
336	24	might	night
341	18	and	and it is
344	5	centres	centre
356	28	magnificient	magnificent
359	28	three	delete
395	19	smaller	small
405	1	bill	hill